
Apertures, Part II

**ECE 6279: Spatial Array Processing
Fall 2013
Lecture 5**

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Where We Are in J&D

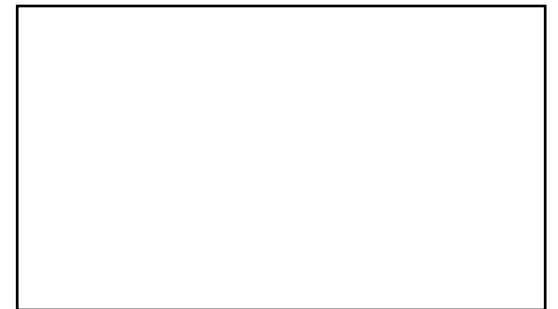
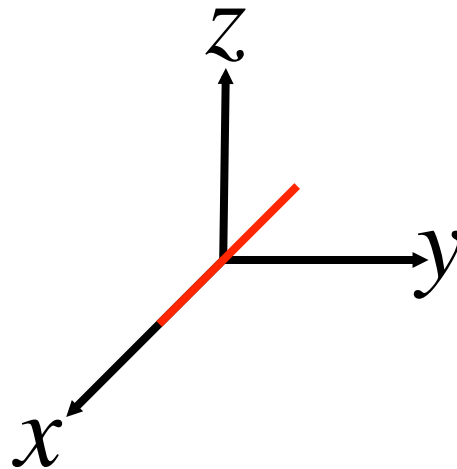
- **Lecture material drawn from:**
 - Sec. 3.3, 3.3.1-3.3.2
- **Might be a good idea to read through Sec. 3.2**
 - Reviews sampling theory



Last Time: Filled Linear Aperture

$$b(x) = \begin{cases} 1, & |x| \leq D/2 \\ 0, & \text{otherwise} \end{cases}$$

$$w(\vec{x}) = b(x)\delta(y)\delta(z)$$

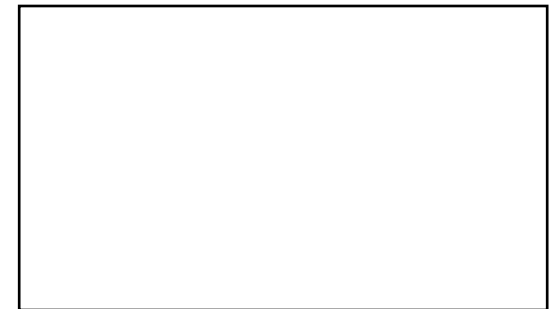
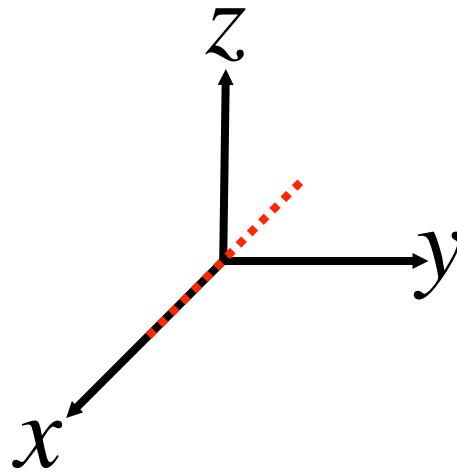


Regularly Sampled Linear Aperture

- Odd number of sensors M , w/spacing d
- Total length $D=Md$

$$b(x) = \sum_{m=-(M-1)/2}^{(M-1)/2} \delta(x - md)$$

$$w(\vec{x}) = b(x)\delta(y)\delta(z)$$



Aperture Smoothing Function (1)

$$\begin{aligned} W(k_x) &= \int_{-\infty}^{\infty} b(x) \exp(\oplus jk_x x) dx \\ &= \int_{-\infty}^{\infty} \sum_{m=-(M-1)/2}^{(M-1)/2} \delta(x - md) \exp(jk_x x) dx \\ &= \sum_{m=-(M-1)/2}^{(M-1)/2} \exp(jk_x md) \times \left[\exp\left(\frac{jk_x d}{2}\right) - \exp\left(-\frac{jk_x d}{2}\right) \right] \end{aligned}$$

$$\exp\left(\frac{jk_x d}{2}\right) - \exp\left(-\frac{jk_x d}{2}\right)$$



Aperture Smoothing Function (2)

$$\sum_{m=-\underbrace{(M-1)/2}_a}^{(M-1)/2} \exp(jk_x md) \times \left[\exp\left(\frac{jk_x d}{2}\right) - \exp\left(-\frac{jk_x d}{2}\right) \right]$$

- **Terms from first set:**

$$\left\{ a + \frac{1}{2} \right\} \left\{ \cancel{(a-1) + \frac{1}{2}} \right\} \cdots \left\{ \cancel{-(a-1) + \frac{1}{2}} \right\} \left\{ \cancel{-a + \frac{1}{2}} \right\}$$

- **Subtract terms from second set:**

$$\left\{ \cancel{a - \frac{1}{2}} \right\} \left\{ \cancel{(a-1) - \frac{1}{2}} \right\} \cdots \left\{ \cancel{-(a-1) - \frac{1}{2}} \right\} \left\{ -a - \frac{1}{2} \right\}$$

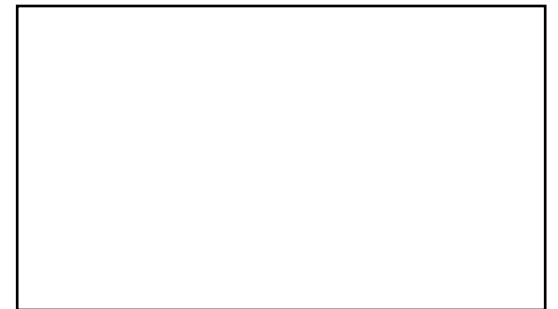


Our Old Friend, the *Sind*

$$W(k_x) = \frac{\exp\left(\frac{jk_x Md}{2}\right) - \exp\left(-\frac{jk_x Md}{2}\right)}{\exp\left(\frac{jk_x d}{2}\right) - \exp\left(-\frac{jk_x d}{2}\right)}$$

$$= \frac{\sin\left(\frac{k_x Md}{2}\right)}{\sin\left(\frac{k_x d}{2}\right)}$$

$$W(0) \equiv M$$



In Polar Wavenumber Coordinates

$$\frac{\sin\left(\frac{k_x M d}{2}\right)}{\sin\left(\frac{k_x d}{2}\right)} = \frac{\sin\left(\frac{k_r M d \sin\phi \cos\theta}{2}\right)}{\underbrace{\sin\left(\frac{k_r d \sin\phi \cos\theta}{2}\right)}_{W(k_r, \phi, \theta)}}$$

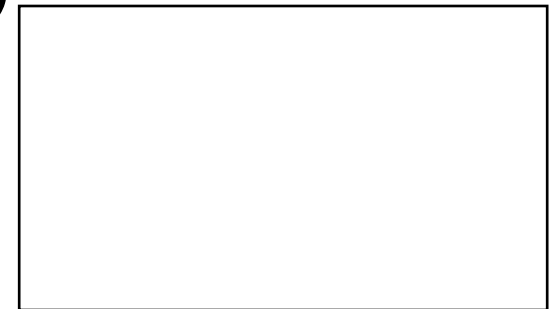
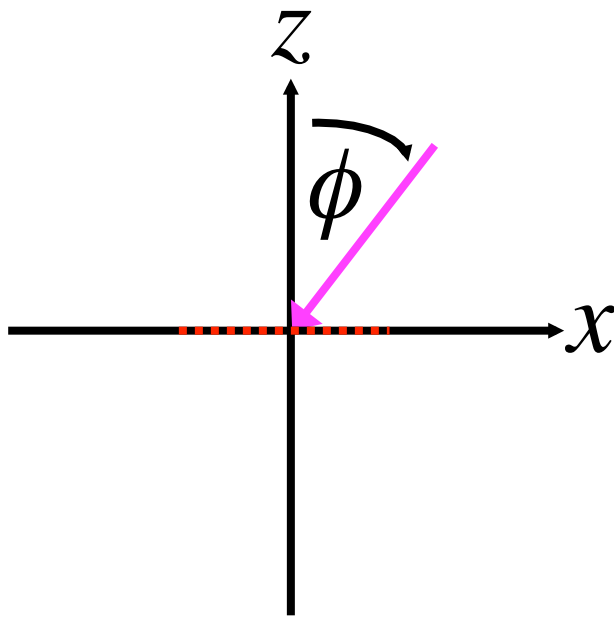
$$k_x = -k_r \sin\phi \cos\theta$$



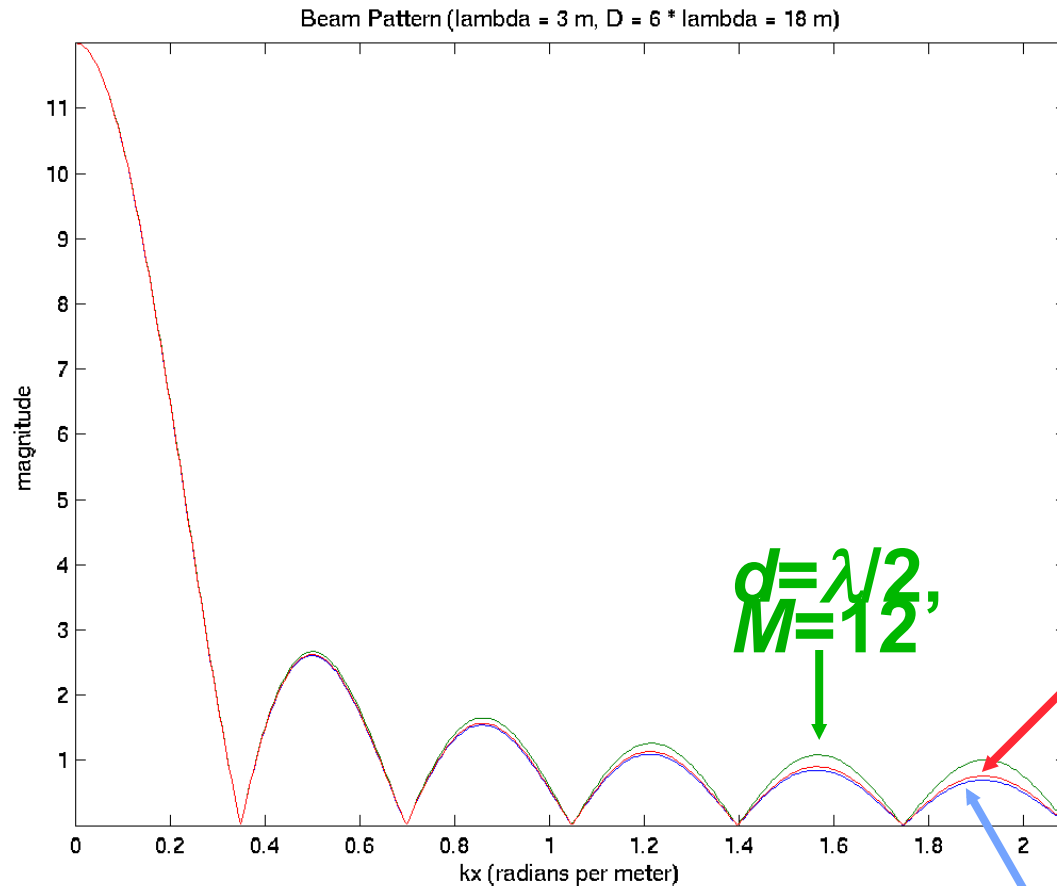
Limiting to the X-Z Plane

- **Suppose $\theta = 0$**

$$W\left(\frac{2\pi}{\lambda}, \phi, 0\right) = \frac{\sin\left(\frac{Md\pi}{\lambda} \sin \phi\right)}{\underbrace{\sin\left(\frac{d\pi}{\lambda} \sin \phi\right)}_{W(\phi)}}$$



Comparison (Wavenumber)

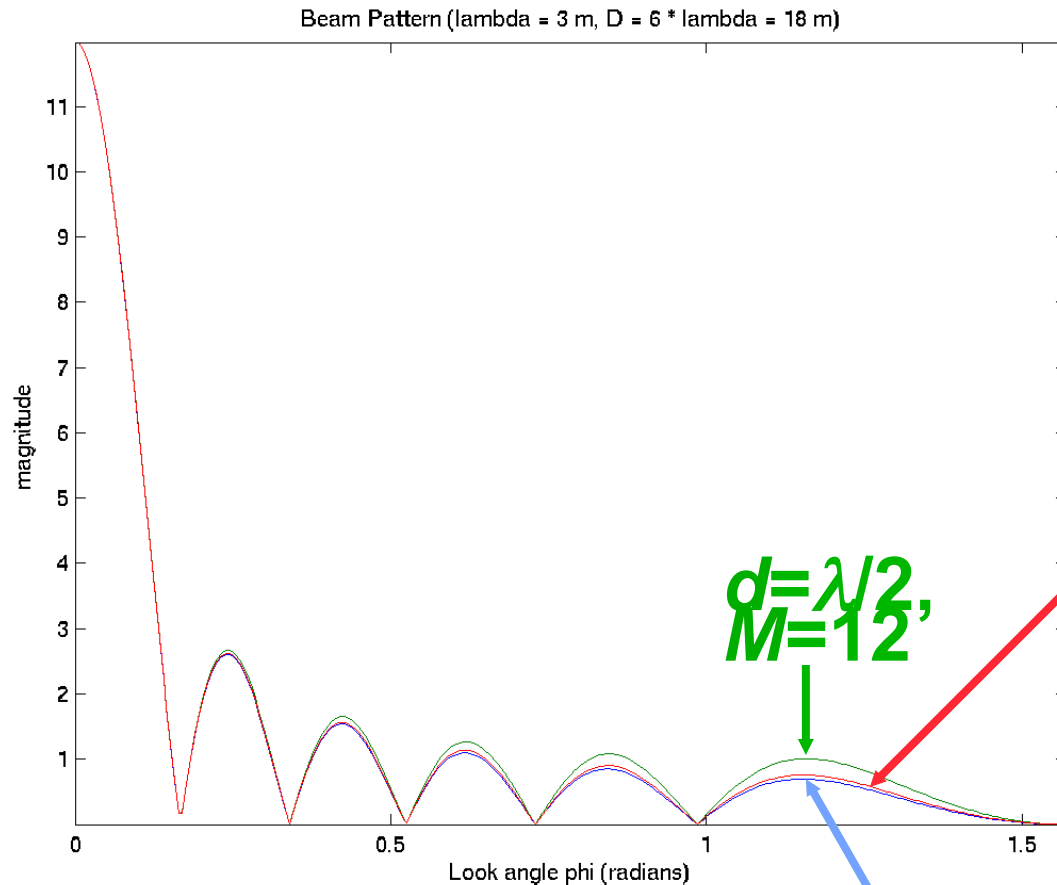


To study changes in overall shape, all plotted curves are artificially normalized to have gain 12 at angle 0

- 100 MHz wave
- 18 meter long aperture



Comparison (Phi)

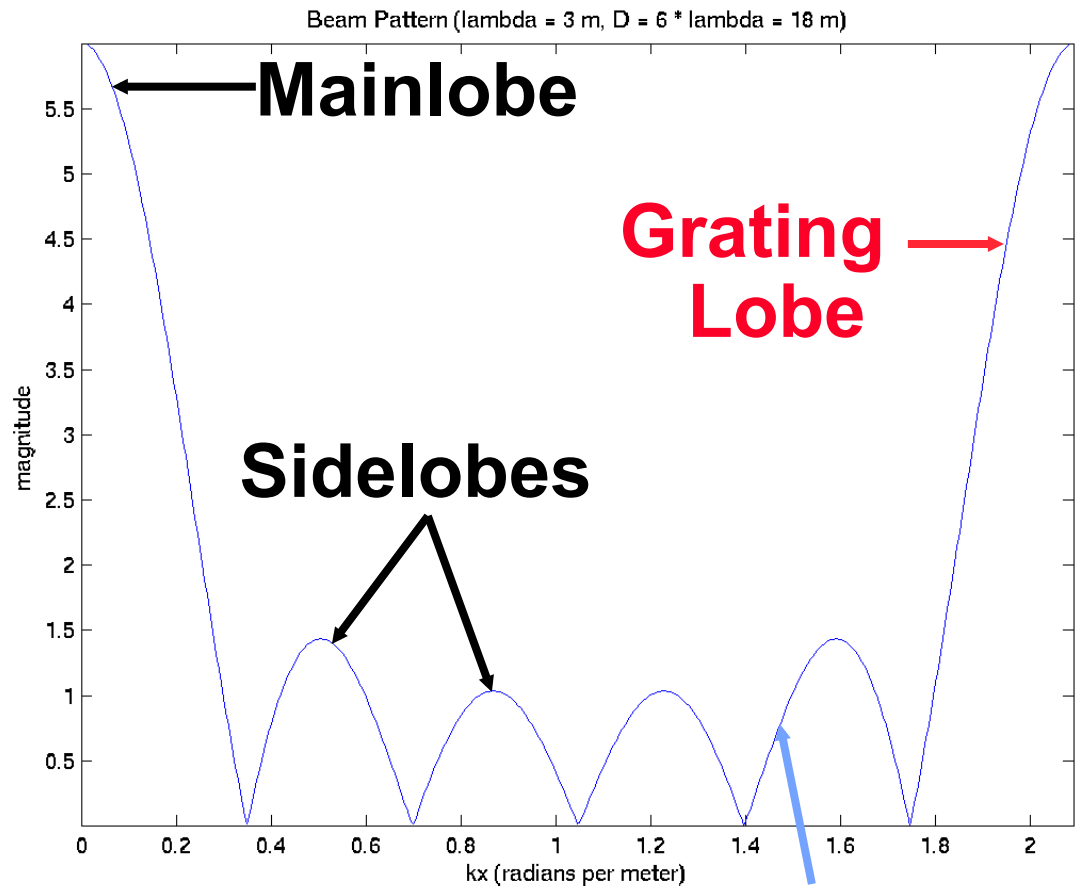


To study changes in overall shape, all plotted curves are artificially normalized to have gain 12 at angle 0

- 100 MHz wave
- 18 meter long aperture



Grating Lobes

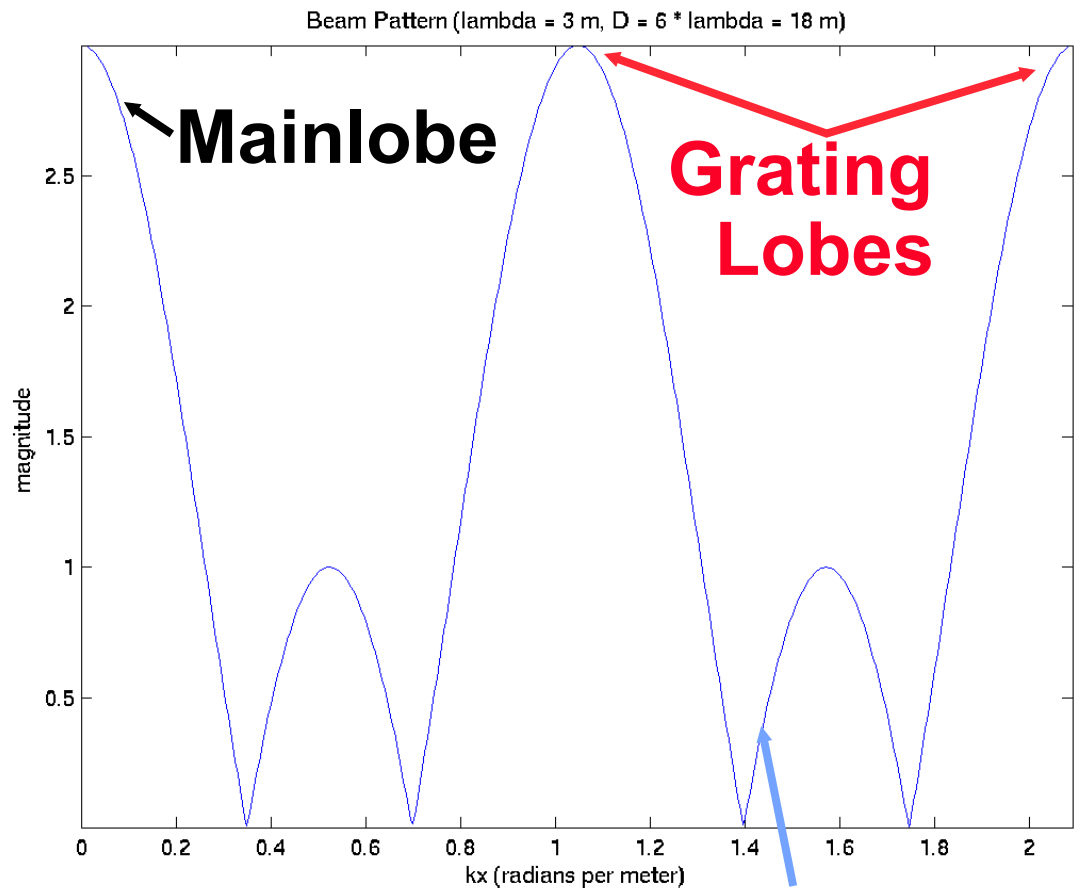


- 100 MHz wave
- 18 meter long aperture

$$d = \lambda, M = 6$$



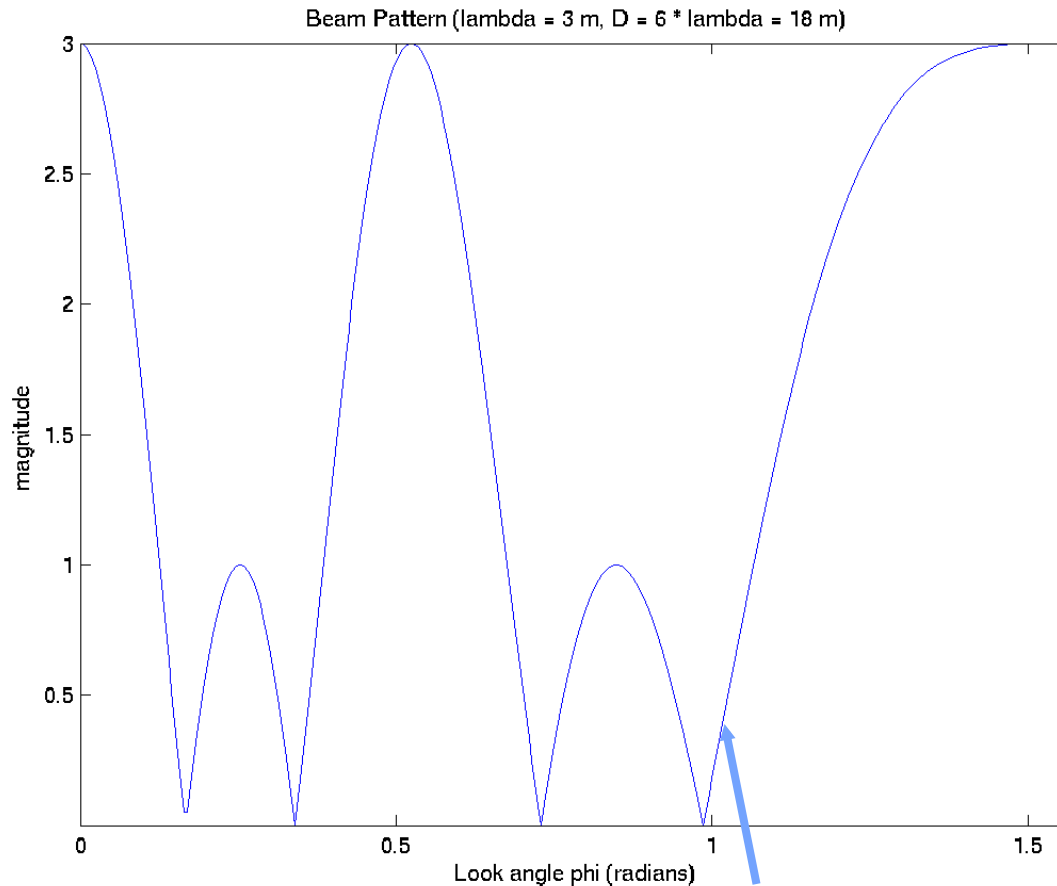
More Grating Lobes



- 100 MHz wave
- 18 meter long aperture



Stretching of Grating Lobes



- 100 MHz wave
- 18 meter long aperture

$$d=2\lambda, M=6$$



Sampling Issues

- **When designing a uniform linear array, generally try to pick $d \leq \lambda / 2$**
- **Can sometimes get away $d > \lambda / 2$ with if you have side information**
 - Detect a target with a low frequency
 - **Long wavelength**
 - Then track it with a high frequency
 - **Short wavelength**
 - **Use knowledge from detector to disambiguate aliases**
- **Most papers will assume $d \leq \lambda / 2$**
 - Usually assume $d = \lambda / 2$



2-D & 3-D Apertures

- **Covered in detail by VT-IV, Chapter 4**
- **Can solve “cone of ambiguity” problem**
 - Careful: some 2-D apertures have their own kinds of cones of ambiguity
 - **i.e. circular disk**
- **Sampling issues get trickier to analyze**
 - Theory from ECE6258: Digital Image Processing applies here
 - **Hexagonal sampling, etc.**
 - Discussed in Sec. 3.2 of J&D



Windowing

- **By weighting an aperture by a window, we can**
 - Reduce sidelobe levels...
 - ...at the expense of a wider mainlobe

- **All your favorites from ECE4270 apply**
 - Bartlett, Dolph-Chevshev, Hamming, Hann, Kaiser, etc.
 - Just use them in space (ECE6279) instead of time (ECE4270)



Integrating Across Apertures

- Here's one way aperture smoothing functions show up
- Typically integrate across the aperture

$$z(t) = \int_{-\infty}^{\infty} w(\vec{x}) f(\vec{x}, t) d\vec{x}$$

- “Input” a monochromatic plane wave to the “system”

$$f(x, t) = \exp\{j(\omega_0 t - \vec{k}^0 \cdot \vec{x})\}$$

$$z(t) = \exp(j\omega_0 t) \underbrace{\int_{-\infty}^{\infty} w(\vec{x}) \exp(-j\vec{k}^0 \cdot \vec{x}) d\vec{x}}_{W(-\vec{k}^0)}$$

