Fall 2013, Aaron Lanterman

ECE 6279: Spatial Array Processing Homework 8

Due date: Friday, November 15, at 4:30 PM under my Van Leer 431 office door.

This homework is pretty small, and hence is only worth half the number of points as a usual homework.

You are welcome to discuss approaches to the problems and solutions to difficulties you encounter with one another and with others outside the class. You can and should learn from each other as much as, and even more than, you learn from me. However, **your solutions should be your own work and should be written up by yourself**; feel free to discuss things, but **don't be looking at someone else's paper when you are writing your solution**. It's too easy to freeload that way and not learn anything. See the class website for more guidelines.

Looking at solutions to homeworks and quizzes from previous offerings of ECE6279 is expressly forbidden. Look, here I am expressing how forbidden it is. Forbidden! Forbidden!!!

1 Required Problems

1. In Lecture 19, we derived an expression for the optimal weights derived from constrained optimization algorithms that employ uncertainty constraints on the signal model:

$$\mathbf{w}_\diamond = -\lambda_1 \left(\mathbf{R} - \frac{\lambda_1^2}{\lambda_2} \mathbf{I} \right)^{-1} \mathbf{e} \tag{1}$$

However, we did not fully explore the difficulties of explicitly finding the Lagrange multipliers associated with this problem.

   (a) Show that $\mathbf{w}_\diamond^H \mathbf{R} \mathbf{w}_\diamond = -\lambda_1$, thereby indicating that this multiplier is negative. (Hint: it's easier to use the gradients of the Lagrangian and the constraints than it is to use equation (1).)

   (b) Assuming that the spatial correlation matrix has the simple form $\mathbf{R} = \sigma^2 \mathbf{I}$, use part (a) and equation (1) to show that

$$M\sigma^2 = \left(\sigma^2 - \frac{\lambda_1^2}{\lambda_2} \right)^2 / (-\lambda_1).$$

   (c) Use the constraints and the gradients of the Lagrangian to show that under the same conditions as in part (b), the other simultaneous equation for the Lagrange multipliers is

$$\lambda_1 = -\sigma^2 \epsilon^2 \left(\frac{\lambda_2^2}{\lambda_1^2} \right).$$