Fall 2013, Aaron Lanterman

ECE 6279: Spatial Array Processing
Homework 10

Due date: Friday, December 6, at 4:30 PM, in my Van Leer 431 office. If I am not there you may slip it under my office door.

You are welcome to discuss approaches to the problems and solutions to difficulties you encounter with one another and with others outside the class. You can and should learn from each other as much as, and even more than, you learn from me. However, **your solutions should be your own work and should be written up by yourself**; feel free to discuss things, but **don't be looking at someone else's paper when you are writing your solution**. It's too easy to freeload that way and not learn anything. See the class website for more guidelines.

Looking at solutions to homeworks and quizzes from previous offerings of ECE6279 is expressly forbidden. Look, here I am expressing how forbidden it is. Forbidden! Forbidden!!!

1 Required Problems

1. In Lecture 24, we derived an expression for the Cramér-Rao bound on the parameter γ under the "deterministic" Gaussian model, where γ is the "electrical angle" specified by $\gamma = (2\pi d/\lambda)\sin(\phi)$ $(\theta = 0)$ for the case of an equally-spaced linear array of M elements with spacing d lying along the x-axis. Here is the exact result:

$$CRB(\gamma) = \frac{6}{L(SNR)} \frac{1}{M(M^2-1)}$$

   SNR denotes the signal power to noise power ratio. The expression acts as you might expect; achieving a better SNR or adding more elements gives you a lower CRB. Interestingly, that the CRB on the electrical angle is not actually a function of the true electrical angle.

   In this problem, we'll consider an alternate formulation where we take $\phi = \pi/2$, and consider the azimuth angle θ. Hence, in this problem, we will let $\gamma = (2\pi d/\lambda)\cos(\theta)$.

   (a) We'd like a CR bound in terms of θ instead of γ. Slide 5 of Lecture 25 shows you two formulas for computing the CR bound on a parameter θ (I used ϕ in the example in the lecture slides, but the same idea applies) that's defined by a functional mapping $\theta = f(\gamma)$, given the CR bound on γ. Using these formulas, compute the CR bound on θ using the CR bound on γ given above for an M-element linear array. **Try both formulas, and make sure you get the same answer with each!** (Basically, you will redo Slides 5 through 10 of Lecture 25 using $\cos(\theta)$ at the beginning instead of $\sin(\phi)$).

   (b) What happens to the CRB on θ as $\theta \to 0°$?

2. The extremely simple formula for the CRB used above required two simplifying assumptions: a linear array assumption and a single-target assumption. In this problem, we will keep using a linear array, but we will explore the CRB for a two-target problem. To do this more complicated computation, you will need to pull out MATLAB.

   Consider a linear array of 32 equally spaced elements (with half-wavelength spacing) along the x-axis, centered around the origin (although, as usual, it doesn't really matter where it is centered for this problem.) We will characterize things in terms of the electrical angle γ. Recall the CRB on γ is constant in the single-target case; we will see if this is still true in the two-target case. Use MATLAB to do the hard work; just code up the Fisher information matrix formula on Slide 9 of Lecture 24, without trying to do any serious symbolic simplification on your own.

   Assume we only have one snapshot, and that $\sigma^2 = 1$. Let the true γ_2 for source 2 correspond to a real angle of $\phi_2 = \pi/4$. Plot the square root of the CRB on γ_1 for source 1 vs. the true γ_1 for γ_1 corresponding to a sweep of ϕ_1 between $-\pi/2 \leq \phi \leq \pi/2$. This will be the upper-left entry of the inverse of the Fisher information matrix. Make plots for two cases: $s_1 = 1$ and $s_2 = 0.2$ (s_2 much weaker), and $s_1 = 1$ and $s_2 = 1$ (both the same strength).

   Comment on interesting effects you observe. How does the CRB change with the strength of s_2? How does it change as γ_1 approaches γ_2? (Note: You may need to tweak the plot if something numerically strange goes on at certain points. Or, you might not.)