ECE 7251: Signal Detection and Estimation

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Lecture 17, 2/15/02:
Variations of the Kalman Filter

The Setup

$$\Theta_{k+1} = f(\Theta_k) + U_k \quad \text{State equation}$$

$$Y_k = h(\Theta_k) + W_k \quad \text{Measurement eqn.}$$

$U_k \sim \mathcal{N}(0, \underbrace{K_U}_{\text{Process "noise" covariance}})$

$W_k \sim \mathcal{N}(0, \underbrace{K_W}_{\text{Measurement noise covariance}})$

$\Theta_1 \sim \mathcal{N}(\hat{q}_{1|0}, P_{1|0})$

$\hat{q}_{1|0}$: Initial guess, before taking any data

$P_{1|0}$: Covariance indicating confidence of initial guess

$U_k,\ W_k,\ \Theta_1$ are uncorrelated with each other and for different k

The Extended Kalman Filter

- In the state update, go ahead and use the nonlinear models for predicting the state and data

$$\hat{q}_{k+1} = f(\hat{q}_k) + L_{k+1}(\hat{q}_k)(y_{k+1} - h(f(\hat{q}_k)))$$

- Use a local linearization of f and h in computing the covariance update and the Kalman gain:

$$L_{k+1} = P_{k+1|k} C^T(\hat{q}_{k+1})(C(\hat{q}_{k+1}) P_{k+1|k} C^T(\hat{q}_{k+1}) + K_W)^{-1}$$

$$P_{k+1|k} = A(\hat{q}_k) P_{k|k} A^T(\hat{q}_k) + K_U$$

$$P_{k+1|k+1} = P_{k+1|k} - P_{k+1|k} C^T(\hat{q}_{k+1}) \times (C(\hat{q}_{k+1}) P_{k+1|k} C^T(\hat{q}_{k+1}) + K_W)^{-1} C(\hat{q}_k) P_{k+1|k}$$

Linearizing the Dynamics Model

- Slope of tangent plane for the dynamics:

$$A(\hat{q}) = \begin{bmatrix} \frac{\partial f_1(q)}{\partial q_1} & \frac{\partial f_1(q)}{\partial q_2} & \cdots & \frac{\partial f_1(q)}{\partial q_m} \\ \frac{\partial f_2(q)}{\partial q_1} & \frac{\partial f_2(q)}{\partial q_2} & & \frac{\partial f_2(q)}{\partial q_m} \\ \vdots & & \ddots & \vdots \\ \frac{\partial f_m(q)}{\partial q_1} & \frac{\partial f_m(q)}{\partial q_2} & \cdots & \frac{\partial f_m(q)}{\partial q_m} \end{bmatrix}_{q=\hat{q}}$$

Linearizing the Measurements

- Slope of tangent plane for the measurement:

$$C(\hat{q}) = \begin{bmatrix} \frac{\partial h_1(q)}{\partial q_1} & \frac{\partial h_1(q)}{\partial q_2} & \cdots & \frac{\partial h_1(q)}{\partial q_m} \\ \frac{\partial h_2(q)}{\partial q_1} & \frac{\partial h_2(q)}{\partial q_2} & & \frac{\partial h_2(q)}{\partial q_m} \\ \vdots & & \ddots & \vdots \\ \frac{\partial h_n(q)}{\partial q_1} & \frac{\partial h_n(q)}{\partial q_2} & \cdots & \frac{\partial h_n(q)}{\partial q_m} \end{bmatrix}_{q=\hat{q}}$$

Ex: Target Tracking

- "Constant-velocity" dynamics model:

$$\begin{bmatrix} p_x \\ v_x \\ p_y \\ v_y \end{bmatrix}_{k+1} = \underbrace{\begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{A} \begin{bmatrix} p_x \\ v_x \\ p_y \\ v_y \end{bmatrix}_k + U_k$$

- Could also use a six-parameter "constant acceleration" model

Two Options

- Option 1:
  1. Take all data points, and convert them from polar to cartesian coordinates
  2. Use formulas from slides on transforming CR bounds to transform error covariance in polar coordinates to error covariance in cartesian coordinates
  3. Use linear Kalman filter from last lecture on the transformed data
- Option 2:
  - Try an EKF

Using the Radar Model in the EKF

- Linearization of measurements:

$$C(q) = \begin{bmatrix} \frac{X}{\sqrt{X^2+Y^2}} & 0 & \frac{Y}{\sqrt{X^2+Y^2}} & 0 \\ \frac{-Y}{X^2+Y^2} & 0 & \frac{}{X^2+Y^2} & 0 \end{bmatrix}$$

(Aside: I think there is an error on p. 292 of Kamen & Su, "Introduction to Optimal Estimation)

Problems with the EKF

- Even if process noise and measurement noise are Gaussian, since there are nonlinear transformations, the posterior density isn't Gaussian
- No longer optimal in the linear MMSE sense either!
- Only as good as the linearization; if the estimate gets too far off, the linearization sucks, and you get EKF divergence which is bad, bad, bad.
- Covariance estimates are often overoptimistic

Multitarget Tracking

- Problem: Don't know which measurements go which which targets
- Can have false alarms and missing detections
- Optimal solution has O(n!) type complexity
- Suboptimal solutions used in practice
  - Multiple Hypothesis Testing – hard assignments via pruning a search tree
  - Joint Probabilistic Data Association – soft assignments, many detections contribute to each target update (Yaakov Bar-Shalom)
  - Symmetric Measurement Equations (Ed Kamen)
  - Multiple Frame Assignment – "Lagrangian Relaxation" (Aubrey Poor)