STUDYPALOOZA!
PHYS 2211
(CLASSICAL)
Welcome to Studypalooza my Physics Phriends, brought to you by the Sinister Six (can you tell I'm excited for the new Spider-Man movie?!!) This problem set is organized into six groups of problems:

1. Static and Dynamic Equilibrium
2. Work and Energy
3. Circular Motion
4. Momentum, Impulse, and Collisions
5. Rotational Energy and Torque
6. Torque and Angular Momentum

Various consultants (PLUS Leaders, 1-to-1 Tutors, TAs, etc) will be around the room to help answer questions and guide your problem-solving process! You may work alone or in groups, whatever helps you study best!

Happy studying, and best of luck on your finals!
Static and Dynamic Equilibrium
Problem 1

Equation for (X):

\[ T_2 = T_1 \frac{\sin \theta_1}{\sin \theta_2} \]

where \( \sin \theta_1 = \sin 30^\circ = \frac{1}{2} \)
and \( \sin \theta_2 = \sin 60^\circ = \sqrt{3}/2 \)

\[ T_2 = \frac{1}{\sqrt{3}} T_1 \]

Equation for (Y):

\[ T_1 \cos \theta_1 + T_2 \cos \theta_2 = mg + mg/3 = \frac{4}{3} mg \]

where \( \cos \theta_1 = \frac{\sqrt{3}}{2} \) and \( \cos \theta_2 = \frac{1}{2} \)

\[ T_1 \left( \frac{\sqrt{3}}{2} \right) + \left( \frac{1}{\sqrt{3}} T_1 \right) \left( \frac{1}{2} \right) = \frac{4}{3} mg \]

\[ T_1 \left[ \frac{\sqrt{3}}{2} \left( 1 + \frac{1}{3} \right) \right] = \frac{4}{3} mg \rightarrow T_1 = \frac{2}{3} mg = 1.15 mg \]

Then \( T_2 = \frac{1}{\sqrt{3}} T_1 = \frac{1}{\sqrt{3}} \left( \frac{2}{3} mg \right) \rightarrow T_2 = \frac{2}{3} mg = 0.667 mg \)
Problem 2

[II] (20 points) In the figure at right, a crate rests on a steep frictionless ramp inclined at an angle $\theta = 56.5^\circ$ above the horizontal. The block is held in equilibrium by a cord that is attached to the ramp in such a way that the cord is exactly horizontal.

Determine the tension in the cord and the normal force exerted by the ramp on the block. Express each force as some multiple of $mg$.

The quality and clarity of your free body diagrams will graded, as part of your work on this problem.

Equilibrium Problem — any choice of coordinate systems is allowed

→ since two forces are vertical or horizontal, choose conventional $xy$ axes

\[ \sum F_y = m\vec{g} \text{ zero in equilibrium} \]
\[ \langle +N\cos\theta \rangle + \langle -mg \rangle = 0 \]
\[ N = \frac{mg}{\cos\theta} = 1.81\, mg \]

\[ \sum F_x = m\vec{a} \text{ zero} \]
\[ \langle +T \rangle + \langle -N\sin\theta \rangle = 0 \]
\[ T = N\sin\theta = \frac{(mg)\sin\theta}{\cos\theta} \]
\[ T = mg\tan\theta = 1.51\, mg \]
A block of mass $m$ is suspended motionless by a string and three identical springs as shown in the figure. Each spring has stiffness $k$ and relaxed length $L_0$. The spring from above forms an angle $\theta$ with the horizontal; the string has an unknown tension and the Earth’s gravity points down. Recall that the force for an ideal spring is given by $\vec{F} = -k(|\vec{L}| - L_0)\vec{L}$ where $\vec{L}$ points from the attachment point to the block.

(a 10pts) Determine the tension $T$ in the string if the stretched length of the upper spring is $3L_0$. It will help if you start by identifying the forces acting on the system and sketch a corresponding force diagram.

\[ \vec{F}_{\text{net}} \cdot \hat{x} = \vec{T}_x - \vec{F}_1 \cdot \hat{x} = 0 \]

(b 5pts) If you treat the two lower springs as a single spring, what is the effective spring constant $k_{\text{eff}}$?

\[ T = \vec{F}_1 \cos \theta \]

\[ \Rightarrow T = F_1 \cos \theta = k(L - L_0) \cos \theta = k(3L_0 - L_0) \cos \theta = k(2L_0) \cos \theta \]

\[ \Rightarrow T = 2kL_0 \cos \theta \]
Problem 4

I. (16 points) A block with mass \( m = 10.0 \text{ kg} \) is on a plane inclined \( \theta = 30.0^\circ \) to the horizontal, as shown. A balloon is attached to the block to exert a constant upward force \( F_B = 9.8 \text{ N} \). If the block moves down the plane with a constant velocity, what is the coefficient of kinetic friction \( \mu_k \) between the block and plane? (On Earth.)

Use Newton’s Second Law. Sketch a Free Body Diagram of the block. There is a gravitational force \( W \) downward, a balloon force \( F_B \) upward, a normal force \( n \) up away from the plane, and a kinetic friction force \( f_k \) up along the plane. Choose a coordinate system. I’ll choose \( x \) down the plane, and \( y \) up away from the plane. Write out Newton’s Second Law for each axis.

I’ll start with the \( x \) axis, showing signs explicitly, so symbols represent magnitudes. The velocity is constant, so the acceleration is zero.

\[
\sum F_x = W_x - F_{B_x} - f_k = m a_x = 0 \quad \Rightarrow \quad mg \sin \theta - F_B \sin \theta = \mu_k n
\]

So

\[
\mu_k = \frac{(mg - F_B) \sin \theta}{n}
\]

Now for the \( y \) axis.

\[
\sum F_y = n + F_{B_y} - W_y = m a_y = 0 \quad \Rightarrow \quad n = mg \cos \theta - F_B \cos \theta = (mg - F_B) \cos \theta
\]

Substitute this expression for \( n \) into the expression for \( \mu_k \).

\[
\mu_k = \frac{(mg - F_B) \sin \theta}{(mg - F_B) \cos \theta} = \frac{\sin \theta}{\cos \theta} = \tan \theta = \tan (30.0^\circ) = 0.577
\]
1. (16 points) Corentine is driving her car of mass \( m \) around a curve when suddenly, all systems fail! The engine quits, she can't brake, she can't steer, and the car coasts straight off the side of the road, as pictured. Fortunately, the car goes into a plowed field. As the tires sink deeper into the soft dirt, the frictional force magnitude \( f \) on the car increases according to

\[
f = f_0 \frac{x}{x_0}
\]

where \( x \) is the distance from where the car left the road, and \( f_0 \) and \( x_0 \) are positive constants. If the car travels a distance \( d \) before stopping in the field, how fast was the car going when it left the road? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants. (On Earth.)

Use the Energy Principle.

\[
W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{th}}
\]

Since friction will produce a thermal energy change, choose a system that includes both surfaces, the car and the Earth. With that choice, there is no work done by external forces, and there are no conservative internal forces producing potential energy changes. The Earth's kinetic energy change is negligible. The internal dissipative force varies with position, so an integral must be evaluated to find the thermal energy change.

\[
0 = \Delta K + 0 + \Delta E_{\text{th}} = \left( \frac{1}{2}mv_1^2 - \frac{1}{2}mv_i^2 \right) + \int f \, dx
\]

The car travels from \( x = 0 \) to \( x = d \), then comes to a stop with a final speed of zero. Substitute the expression for the friction force.

\[
\frac{1}{2}mv_i^2 = \int_0^d f_0 \frac{x}{x_0} \, dx = f_0 \int_0^d \frac{x}{x_0} \, dx = f_0 \frac{x_0}{x_0} \left[ \frac{x^2}{2} \right]_0^d = f_0 \frac{d^2}{2x_0}
\]

Solve for the initial speed.

\[
v_i^2 = \frac{f_0 d^2}{mx_0} \quad \Rightarrow \quad v_i = d \sqrt{\frac{f_0}{mx_0}}
\]
Problem 2

Normal force does zero work: \( \hat{N} \perp \Delta \vec{r} \) (vertical) \( \perp \) (horizontal)

Ditto for gravitational force

\[ W_N = 0 \quad W_{\text{grav}} = 0 \]

For pushing force \( P \):

\[ W_P = P \cdot \Delta \vec{r} = P_x \Delta x = (P \cos \theta)(+D) \]

\[ W_P = +PD \cos \theta = +2mgD \cos \theta = \sqrt{3}mgD \]

Finally, work done by friction

\[ f_k = \mu_k N \]

\[ \Sigma \vec{F}_y = 0 = (+N) + (-mg) + (-Ps \sin \theta) \]

\[ N = mg + (2mg) \sin \theta = mg(1 + 2 \sin \theta) = 2mg \]

So:

\[ f_k = \mu_k mg(1 + 2 \sin \theta) \]

Since \( \vec{F}_k \) is opposite to \( \Delta \vec{r} \), dot product gives factor (-1)

\[ W_f = -\mu_k mg D(1 + 2 \sin \theta) = -0.5mgD \]

\[ W_{\text{tot}} = \Delta K \]

\[ W_f + W_g + W_P + W_f = K_f - K_i = 0 \]

\[ 0 = (\sqrt{3}mgD) + (-\frac{1}{2}mgD) = \frac{1}{2}mV_f^2 \]

\[ V_f = \sqrt{2(\sqrt{3} - 1)gD} = 6.0 \text{ m/s} \]
Problem 3

Use Newton’s Second Law to find the minimum speed that the cart must have at the top of the loop. A Free Body Diagram will have the gravitational force $mg$ downward, and no other force at the minimum speed. Choosing an axis that points downward toward the center of the loop,

\[
\sum F_c = mg = ma_c = m\frac{v^2}{r} \quad \Rightarrow \quad g = \frac{v^2}{h} \quad \Rightarrow \quad v = \sqrt{gh}
\]

Now use the Energy Principle.

\[W_{ext} = \Delta K + \Delta U + \Delta E_{th}\]

I’ll choose a system that includes the cart, the Earth, and the spring. With that choice, no external forces do work on the system, and no internal dissipative forces change its thermal energy. The Earth’s kinetic energy change is negligible. Both an internal gravitational force and in internal spring force change the system’s potential energy.

\[0 = \Delta K + \Delta U_g + \Delta U_s + 0 = \left(\frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2\right) + (mgh_f - mgh_i) + \left(\frac{1}{2}k\Delta s_f^2 - \frac{1}{2}k\Delta s_i^2\right)\]

The car isn’t moving before it is launched ($v_i = 0$). The spring is relaxed after the car is launched ($\Delta s_f$). I’ll choose the initial height to be zero ($h_i = 0$ and $h_f = h$).

\[0 = \left(\frac{1}{2}mv_i^2 - 0\right) + (mgh - 0) + (0 - \frac{1}{2}k\Delta s_i^2)\]

Substitute the expression for the speed at the top of the loop found above, and solve for the initial compression.

\[
\frac{1}{2}k\Delta s_i^2 = \frac{1}{2}m\left(\sqrt{gh}\right)^2 + mgh \quad \Rightarrow \quad k\Delta s_i^2 = mgh + 2mgh = 3mgh \quad \Rightarrow \quad \Delta s_i = \sqrt{\frac{3mgh}{k}}
\]
Problem 4

[Hint: start by finding an expression for the spring constant $k$ in terms of the parameters $g$, $m$, and $l$.]

1. Frictionless ramp: $E = K + U = \text{constant}$
2. Begins and ends at rest: $K_i = K_f = 0$

\[ U_{gi} + U_{f} = U_{af} + U_{sf} \]

\[ + mgL \sin \theta = -mgd \sin \theta + \frac{1}{2}kd^2 \]

\[ \Rightarrow \frac{1}{2}kd^2 = mg(l+d) \sin \theta \]

Now, use $d = \frac{L}{2}$:

\[ \frac{1}{2}k \left( \frac{L}{2} \right)^2 = mg \left( \frac{3}{2}l \right) \sin \theta \]

\[ k = \frac{12mg}{L} \sin \theta = \frac{12mg}{L} \cdot \frac{1}{2} = \frac{6mg}{L} = k \]

Now—same problem, but let $y_i = 2L \sin \theta$, $y_f = -5 \sin \theta$, $s = \text{unknown}$

\[ U_{gi} + U_{f} = U_{af} + U_{sf} \]

\[ + mg(2L) \sin \theta = -mg \cdot 5 \sin \theta + \frac{1}{2}ks^2 \]

\[ mgL = -\frac{mg}{2} \cdot 5 + \frac{1}{2} \left( \frac{6mg}{L} \right)s^2 \]

\[ \Rightarrow \text{multiply through by } 2L: \]

\[ 2L^2 = -5Ls + 6s^2 \Rightarrow 6s^2 - 5Ls - 2L^2 = 0 \]

\[ s = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 6 \cdot (-2)}}{2(6)} \]

\[ s = \frac{L \pm \sqrt{49L^2}}{12} = \frac{L \pm 7L}{12} \]

by definition, $s$ must be positively valued, so choose pos root

\[ s = \frac{8L}{12} = \frac{2L}{3} \]
Circular Motion
(16 points) In an amusement park ride called The Roundup, passengers stand inside a ring of radius \( R \). After the ring has acquired sufficient angular speed, it tilts into a vertical plane, as shown. If the wall of the ride exerts a normal force \( \frac{mg}{3} \) on a passenger of mass \( m \) at the topmost position, what normal force is exerted on that same passenger at the bottom position? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants. (On Earth.)

Use Newton's Second Law. Sketch a Free Body Diagram of the passenger at the top. There will be a gravitational force \( mg \) downward, and a normal force \( n \) downward. Choose an axis that points in the direction of the acceleration, which is down in this case. Write Newton's Second Law for this passenger at the top. I'll show signs explicitly, so symbols represent magnitudes.

\[
\sum F_c = n + mg = ma_c \quad \Rightarrow \quad ma_c = \frac{mg}{3} + mg = \frac{4}{3}mg
\]

Now sketch a Free Body Diagram of the passenger at the bottom. There will be a gravitational force \( mg \) downward, and a normal force \( n \) upward. Choose an axis that points in the direction of the acceleration, which is up in this case. Write Newton's Second Law for this passenger at the bottom. Note that the magnitude of the acceleration is now known.

\[
\sum F_c = n - mg = ma_c \quad \Rightarrow \quad n = mg + ma_c = mg + \frac{4}{3}mg = \frac{7}{3}mg
\]
Use Newton’s Second Law. Sketch a Free Body Diagram of the car—a “rear view” is most helpful. There will be a gravitational force \( mg \) downward, a static frictional force \( f_s \) down the slope (as we’re concerned with the maximum speed of the car—friction prevents it from sliding to the outside of the curve), and a normal force \( n \) perpendicular to the road surface. Choose a coordinate system. Since the acceleration of the car must be toward the center of the curve (horizontal!) I’ll choose that as the positive \( c \) direction. The other axis must be perpendicular, so I’ll choose the positive \( y \) direction to be straight upward.

Write Newton’s Second Law for each axis. I’ll show signs explicitly, so symbols represent magnitudes.

\[
\sum F_y = n \cos \theta - f_s \sin \theta - mg = ma_y = 0 \quad \Rightarrow \quad n \cos \theta - \mu_s n \sin \theta = mg
\]

\[
\Rightarrow \quad n = \frac{mg}{\cos \theta - \mu_s \sin \theta}
\]

Then

\[
\sum F_c = n \sin \theta + f_s \cos \theta = ma_c = m \frac{v^2}{r} \quad \Rightarrow \quad n \sin \theta + \mu_s n \cos \theta = m \frac{v^2}{R} \quad \Rightarrow \quad n (\sin \theta + \mu_s \cos \theta) = m \frac{v^2}{R}
\]

Eliminate the unknown normal force by substitution, and solve for \( v \).

\[
\left( \frac{mg}{\cos \theta - \mu_s \sin \theta} \right) (\sin \theta + \mu_s \cos \theta) = m \frac{v^2}{R} \quad \Rightarrow \quad v^2 = gR \left( \frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta} \right)
\]

\[
v = \sqrt{gR \left( \frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta} \right)}
\]
Problem 3

(20 points) A water-filled bucket has true total weight $W$. A physics instructor then spins the bucket in a vertical circle at constant speed $v$, using a cord of length $L$. When the bucket is inverted at the top of its loop, the instructor feels that the apparent weight of the bucket is $2W$. What will be the apparent weight of the bucket when it is at the bottom of the loop? Express your answer as a numerical multiple of $W$.

1. Note that perceived/apparent weight in this case, is found as the magnitude of the tension force in the rope. Given that $T_{top} = 2W$, what is $T_{bottom}$?

2. Bucket rotates at constant speed $v$ so the magnitude of the radial acceleration $a_r = \frac{v^2}{L}$ = constant

3. Bucket at top of loop:
   $$\sum F_r = m \ddot{a}_r$$ with down=positive
   $$+ T_e + W = m(\ddot{a}_r)$$
   $$\therefore a_r = \text{the same in both equations, so}$$
   $$T_e + W = m \ddot{a}_r = T_b - W$$
   $$\therefore T_b = T_e + 2W$$

4. Bucket at bottom of loop:
   $$\sum F_r = m \ddot{a}_r$$ with up=positive
   $$+ T_b - W = m(\ddot{a}_r)$$
   $$\therefore T_b = 4W$$
Problem 4

Determine the speed $v_T$ of the roller-coaster car as it passes through the bottom of the loop, expressed as a multiple of $v_B$.

1. **Inverted at top of loop:**
   \[ \Sigma F_r = m \ddot{a}_r \]
   \[ \Rightarrow \Sigma F_r = m (-g) \]
   \[ m \ddot{a}_r = -mg \]
   \[ \ddot{a}_r = -g \]
   \[ \Rightarrow a_r = \frac{V_T^2}{R} \]
   \[ N_T = mg \]
   \[ 2mg = \frac{mV_T^2}{R} \]
   \[ V_T^2 = 2gR \]

2. **Rightside-up at bottom of loop:**
   \[ \Sigma F_r = m \ddot{a}_r \]
   \[ \Rightarrow \Sigma F_r = m (\frac{V_B^2}{3R}) \]
   \[ m \ddot{a}_r = \frac{m V_B^2}{3R} \]
   \[ \ddot{a}_r = \frac{V_B^2}{3R} \]
   \[ \Rightarrow N_B = 3mg \]
   \[ \Rightarrow \frac{V_B^2}{3R} \]
   \[ \Rightarrow \frac{V_B^2}{6gR} = \frac{1}{3} \]
   \[ V_B^2 = 6gR \]

3. **Comparing:**
   \[ \frac{V_T^2}{V_B^2} = \frac{2gR}{6gR} = \frac{1}{3} \]
   \[ V_B^2 = 3V_T^2 \]
   \[ V_B = \sqrt{3} V_T \]
Momentum, Impulse, and Collisions
Problem 1

(A) (12 points) Determine the velocity of both the sumo and the flatcar, as measured by an observer on the ground. (You may presume that there is negligible rolling friction between the flatcar's wheels and the ground.)

\[ \text{let } v_1 = \text{speed of sumo relative to ground: } \quad \mathbf{v}_{1g} = \mathbf{v}_{1g} \]

\[ \text{let } v_2 = \text{speed of flatcar relative to ground: } \quad \mathbf{v}_{2g} = \mathbf{v}_{2g} \]

- Relative velocity: \( \mathbf{v}_{1g} = \mathbf{v}_{sc} + \mathbf{v}_{1g} = \mathbf{v}_{1g} \)
- \( \mathbf{v}_{2g} = \mathbf{v}_{sc} + \mathbf{v}_{2g} = \mathbf{v}_{2g} \)

\[ \mathbf{v}_{1} = \mathbf{v}_{1g} = \mathbf{v}_{1g} = \mathbf{v}_{1g} \quad \text{or} \quad \mathbf{v}_{2} = \mathbf{v}_{2g} = \mathbf{v}_{2g} = \mathbf{v}_{2g} \]

We now have two equations in three unknowns:

\[ v_1 = 4v_2 = 4(v_0 - v_1) \]
\[ 5v_2 = 4v_0 \]

\[ v_1 = \frac{4}{5}v_0 \]

So:

\[ \mathbf{v}_{1g} = \left( \frac{4}{5}v_0 \right) \]

and:

\[ \mathbf{v}_{2g} = \left( \frac{4}{5}v_0 \right) \]

(B) (9 points) How far has the flatcar rolled at the moment the sumo reaches the other end of the car? How far is the sumo from his starting position, at that moment? (No, it is not simply \( L \))

Note: sumo moved distance \( L \) relative to car, at speed \( v_0 \) relative to car.

\[ \Delta x = v_0 \Delta t \]

\[ L = v_0 \Delta t \]

\[ \Delta t = \frac{L}{v_0} \]

hence, displacement of sumo along ground is:

\[ \Delta x_{sumo} = \mathbf{v}_{1g} \Delta t = \left( \frac{4}{5}v_0 \right) \cdot \frac{L}{v_0} \]

Displacement of car is similarly:

\[ \Delta x_{car} = \left( \frac{4}{5}v_0 \right) \frac{L}{v_0} \]

"How far" is a distance question, so scalar answers are okay: \( 0_1 = \frac{L}{5}, 0_2 = \frac{3L}{5} \)
Problem 2

A simple device called a "ballistic pendulum" consisting of a wooden block suspended from thin strings can be used to measure the speed of a bullet. The wooden block has a mass of 3 kg. The strings are 0.8 m long and have negligible mass.

(a) A bullet is fired into the block and becomes embedded in it. Is the collision elastic or inelastic?

(b) The bullet weighs 4 grams and travels at 650 m/s before striking the block. What is the speed of the block immediately after the bullet becomes embedded in the block?

\[
\begin{align*}
\Delta p_{\text{bullet}} + \Delta p_{\text{block}} &= 0 \\
\Rightarrow & \quad \Delta p_{\text{bullet}} = -\Delta p_{\text{block}} \\
\Rightarrow & \quad (3 \text{ kg} + 0.004 \text{ kg}) \Delta v_e = (0.004 \text{ kg})(650 \text{ m/s}) \\
\Rightarrow & \quad \Delta v_e = \frac{(0.004 \text{ kg})(650 \text{ m/s})}{(3.004 \text{ kg})} = 0.87 \text{ m/s}
\end{align*}
\]

(c) How much kinetic energy is converted to thermal energy during the collision?

\[
\begin{align*}
\Delta K &= \Delta E_{\text{thermal}} \\
\Rightarrow & \quad \Delta E_{\text{thermal}} = \frac{1}{2} m_{\text{bullet}} v_{e,i}^2 - \left( \frac{1}{2} m_{\text{bullet}} v_{e,i}^2 + \frac{1}{2} m_{\text{block}} v_{e,i}^2 \right) \\
\Rightarrow & \quad \Delta E_{\text{thermal}} = \frac{1}{2} (0.004 \text{ kg})(0.87 \text{ m/s})^2 - \frac{1}{2} (0.004 \text{ kg})(650 \text{ m/s})^2 = 8.44 \text{ J}
\end{align*}
\]

(d) After the collision, the block swings upward. What is the maximum height (relative to its original position) reached by the block?

\[
\begin{align*}
\Delta K + \Delta U &= 0 \\
\Rightarrow & \quad k_f - k_i + U_f - U_i = 0 \\
\Rightarrow & \quad U_f = k_i \\
\Rightarrow & \quad m g h = \frac{1}{2} m v^2 \\
\Rightarrow & \quad h = \frac{v^2}{2 g} \\
\Rightarrow & \quad h = \frac{(0.87 \text{ m/s})^2}{2 (9.8 \text{ m/s}^2)} = 0.038 \text{ m}
\end{align*}
\]
Consider first the energy of a system consisting of the bob and the Earth after the bullet as passed through the bob. In the Energy Principle,

$$W_{ext} = \Delta K + \Delta U + \Delta E_{th}$$

no external forces do work (the tension in the string is perpendicular to the bob's displacement at every instant) so $W_{ext} = 0$. There are no internal dissipative forces, so $\Delta E_{th} = 0$. The only internal conservative force is that due to gravity, as $\Delta U = Mgh_f - Mgh_i$, where "$f$" and "$i$" denote final and initial states, respectively. The Earth's velocity change is negligible, so

$$\Delta K = \frac{1}{2} MV_f^2 - \frac{1}{2} MV_i^2.$$  Then

$$0 = \left(\frac{1}{2} MV_f^2 - \frac{1}{2} MV_i^2\right) + (Mgh_f - Mgh_i) + 0$$

If we choose $h_i = 0$ then $h_f = L(1 - \cos \theta)$. When the bob reaches its highest point, its speed is zero, so $V_f = 0$. Therefore,

$$0 = \left(0 - \frac{1}{2} MV_i^2\right) + (MgL(1 - \cos \theta) - 0)$$

Solve for $V_i$, the speed of the bob just after the bullet passes through it.

$$\frac{1}{2} MV_i^2 = MgL (1 - \cos \theta) \quad \Rightarrow \quad V_i = \sqrt{2gL (1 - \cos \theta)}$$

Consider next the momentum of a system consisting of the bob and the bullet while the bullet is passing through the bob. During this brief time, external forces are negligible in comparison to the internal forces between the bullet and the bob. Momentum, therefore, is conserved in this system during the interaction.

$$\Delta \vec{p} = 0 \quad \Rightarrow \quad \vec{p}_i = \vec{p}_f$$

$mV_i = mV_i' + MV_f'$

where the motion is in one dimension so signs will indicate direction. The bob is stationary before the bullet hits it, so $V_i = 0$. The velocity of the bob just after the bullet leaves it, is the velocity of the bob just before it swings up, so $V_f = V_i$, from above.

$$mv_0 + 0 = mv_f + M \sqrt{2gL (1 - \cos \theta)} \quad \Rightarrow \quad v_f = v_0 - \frac{M}{m} \sqrt{2gL (1 - \cos \theta)}$$
Problem 4

No significant external forces act on the object during the explosion, so momentum is conserved. Since momentum is a vector, when it is conserved, it must be conserved independently on each axis.

\[ \Delta \vec{P} = 0 \quad \vec{P}_1 = \vec{P}_1 \quad P_{1x} = P_{1x} \quad P_{1y} = P_{1y} \]

Let \( M \) represent the total mass. Considering the \( x \) direction,

\[ M v_{0x} = m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x} \]

Solve for \( v_{0x} \), noting that \( v_{3x} = 0 \).

\[ v_{0x} = \frac{m_1 v_{1x} + m_3 v_{3x}}{M} = \frac{(2.1 \text{ kg})(-1.25 \text{ m/s}) + (3.2 \text{ kg})(5.75 \text{ m/s} \cos(-22^\circ))}{2.1 \text{ kg} + 1.4 \text{ kg} + 3.2 \text{ kg}} = +2.2 \text{ m/s} \]

Looking next at the \( y \) direction,

\[ M v_{0y} = m_1 v_{1y} + m_2 v_{2y} + m_3 v_{3y} \]

Solve for \( v_{0y} \), noting that \( v_{1y} = 0 \).

\[ v_{0y} = \frac{m_2 v_{2y} + m_3 v_{3y}}{M} = \frac{(1.4 \text{ kg})(2.75 \text{ m/s}) + (3.2 \text{ kg})(5.75 \text{ m/s} \sin(-22^\circ))}{2.1 \text{ kg} + 1.4 \text{ kg} + 3.2 \text{ kg}} = -0.45 \text{ m/s} \]

So

\[ \vec{v}_0 = 2.2 \hat{x} - 0.45 \hat{y} \text{ m/s} \]

1. (6 points) After the explosion, the first piece (2.1 kg traveling at \( v_1 = -1.25 \hat{x} \text{ m/s} \)) has a collision with a stationary 625 kg truck (on a frictionless surface). Assuming the collision is completely elastic, what is the velocity of the 2.1 kg object immediately following the collision? (Hint: You don’t need to do any calculations.)

As the mass of the truck is much greater than the mass of the first piece, the piece essentially bounces back in an elastic collision, at

\[ 1.21 \text{ m/s} \]
Rotational Energy and Torque
Problem 1

\[ W = \Delta K_{rot} = K_f - K_{rot} = \frac{1}{2} I_{left} \omega^2 \]

for pivot at left side:

\[ I_{left} = \sum m_i r_i^2 = (2m)(0)^2 + (3m)(L)^2 = 3mL^2 \]

\[ \rightarrow W_{left} = \frac{3}{2} mL^2 \omega^2 \]

Now, locate actual CM

\[ x_{cm} = \frac{1}{M_{rot}} \sum m_i x_i = \frac{1}{2m + 3m} [2m \cdot 0 + 3m \cdot L] \]

\[ x_{cm} = \frac{3mL}{5m} \]

\[ x_{cm} = \frac{3}{5} L \]

So, with pivot at CM, \( I_{cm} = 2m(\frac{2}{5}L)^2 + 3m(\frac{3}{5}L)^2 \)

The distance from 3m to CM

\[ I_{cm} = \frac{18}{25} mL^2 + \frac{12}{25} mL^2 \]

\[ I_{cm} = \frac{30}{25} mL^2 = \frac{6}{5} mL^2 \]

Then \( W_{cm} = \Delta K_{rot, cm} = K_f - K_{cm} = \frac{1}{2} I_{cm} \omega^2 - 0 \)

\[ \Rightarrow \frac{W_{cm}}{W_{left}} = \frac{\frac{1}{2} I_{cm} \omega^2}{\frac{1}{2} I_{left} \omega^2} = \frac{I_{cm}}{I_{left}} = \frac{\frac{6}{5} mL^2}{3 mL^2} = \frac{2}{5} \]

One could also use parallel axis theorem

\[ I_{left} = I_{cm} + (5m)(\frac{2}{5}L)^2 \]

\[ \rightarrow \text{distance from CM to axis at left} \]

Then

\[ I_{cm} = I_{left} - \frac{9}{5} mL^2 = \frac{15}{5} mL^2 - \frac{9}{5} mL^2 = \frac{6}{5} mL^2 \]

\[ W_{cm} = \frac{3}{5} W_{left} = 0.4 W_{left} \]
Problem 3

(20 points) A string is wrapped around a pulley wheel of mass \(m\) and radius \(R\), as shown in the figure. The pulley can rotate freely about its axle, without any friction. The loose end of the string is attached to a block of mass \(2m\). You may assume that the spokes of the pulley wheel have negligible mass, so that it may be treated an ideal hoop, with \(I_m = mR^2\).

Find an expression for the tension in the cord when the block is released. Express your answer as a multiple of \(mg\).

Free Body Diagrams:

\[
\begin{align*}
\sum F_y &= m\ddot{a}, \\
\langle +T \rangle + \langle -2mg \rangle &= 2m\langle -\dot{a} \rangle
\end{align*}
\]

\[
\begin{align*}
\sum \vec{Z_p} &= I_p \ddot{\alpha}, \\
\langle -RT \rangle &= (me^2)(-\dot{\alpha}) \\
RT &= mR^2 \left( \frac{a}{R} \right) = mRA \\
a &= \frac{T}{m}
\end{align*}
\]

\[
T - 2mg > 2m\left(-\frac{T}{m}\right) = -2T
\]

\[
3T = 2mg
\]

\[
T = \frac{2}{3} mg
\]
Problem 4

Choose pivot at hinge - only T, W generate torque about H
- Tension acts ccw, with \( r_1 = L \cos 30° \)
- Weight acts cw, with \( r_2 = \frac{L}{2} \cos 30° \)

So \( \Sigma F_H = 0 = \langle +L \cos 30° T \rangle + \langle -\frac{L}{2} \cos 30° W \rangle \) → \( T = \frac{W}{2} \)

Now, require \( \Sigma F_x = 0 \) and \( \Sigma F_y = 0 \)

- First, through express \( T \) in cartesian form

\[
\overrightarrow{T} = \langle -T \cos 30° \rangle \hat{i} + \langle +T \sin 30° \rangle \hat{j}
\]

\[
= \langle -\frac{W}{4} \rangle \hat{i} + \langle +\frac{W}{4} \rangle \hat{j}
\]

\[
\overrightarrow{T} = \langle -\frac{W}{4} \rangle \hat{i} + \langle +\frac{W}{4} \rangle \hat{j}
\]

Then \( \Sigma F_x = \langle +H_x \rangle + \langle -T_x \rangle \) → \( H_x = T_x = \frac{\sqrt{3}}{4} W \)

\( \Sigma F_y = \langle +H_y \rangle + \langle +T_y \rangle + \langle -W \rangle \) → \( H_y = W - T_y = W - \frac{W}{4} \) → \( H_y = \frac{3W}{4} \)

\( |\overrightarrow{H}| = \sqrt{\left(\frac{3W}{4}\right)^2 + \left(\frac{W}{4}\right)^2} = W \sqrt{\frac{9}{16} + \frac{1}{16}} = W \sqrt{\frac{10}{16}} = \frac{W \sqrt{10}}{4} \)

\( |\overrightarrow{T}| = \frac{\sqrt{3} W}{4} \)

Directed at angle
\( \phi = \tan^{-1}\left(\frac{H_y}{H_x}\right) = \tan^{-1}\left(\frac{\frac{3W}{4}}{\frac{W}{4}}\right) = \tan^{-1}(\sqrt{3}) \)

\( \phi = 60° \) above horizontal
Torque and Angular Momentum
Problem 1

\[ \frac{1}{M_s^f} = -4 m R^2 \frac{\omega}{M_c^2} \]

\[ \omega = \frac{-m R^2 \omega_y}{2 M_c^2} \]

\[ I \omega_i^f = I \omega_{ef} + I \omega_s^f \]

\[ I_s = I_s^f + I_s^e \]

\[ \frac{1}{L_s} = \frac{1}{L_s^f} + \frac{1}{L_s^e} \]

\[ \omega_i^f = \omega_{ef} + \omega_s^f \]

\[ \omega = (m R^2) \omega_y + (\frac{1}{2 M_c^2}) \omega_s^f \]

\[ \omega_y = \frac{-m R^2 \omega_y}{2 M_c^2} \]

direction: clockwise (-\( \dot{\gamma} \))
Problem 1, Con’t

\[ \frac{1}{2} I_i \omega_i + \int F \cdot d\mathbf{r} = \frac{1}{2} I_f \omega_f \]

Initial: \[ I_i = I_{wi} + I_{si} = \text{Initial from part (a)} \]
\[ = -mR^2 \omega \hat{\mathbf{j}} \]

Final: \[ I_f = I_{wr,f} + I_{we,f} + I_{sw,f} = (mR^2 \omega)(\hat{\mathbf{j}}) + (F \cdot \mathbf{r}) \omega + \frac{1}{2} Mc^2 \omega_{sf} \]
\[ \Rightarrow \omega_f = \frac{F \cdot \mathbf{r}}{dm \omega} = \frac{dm \omega}{md} \omega_{sf} = \frac{md^2 \omega_{sf}}{md^2} \]

\[ \Rightarrow I_f = mR^2 \omega(\hat{\mathbf{j}}) + (md^2 + \frac{1}{2} Mc^2) \omega_{sf} \]

Conservation of angular momentum \[ \Rightarrow I_i = I_f \]

\[ -mR^2 \omega \hat{\mathbf{j}} = mR^2 \omega(\hat{\mathbf{j}}) + (md^2 + \frac{1}{2} Mc^2) \omega_{sf} \]
\[ -2mR^2 \omega \hat{\mathbf{j}} = (md^2 + \frac{1}{2} Mc^2) \omega_{sf} \]

\[ \omega_{sf} = \frac{-2mR^2 \omega}{md^2 + \frac{1}{2} Mc^2} \hat{\mathbf{j}} \rightarrow \text{magnitude: } \frac{2mR^2 \omega}{md^2 + \frac{1}{2} Mc^2} \]
\[ \text{direction: clockwise } (-\hat{\mathbf{j}}) \]

(c. 5pts) Consider the situation immediately after part (b). A tangential friction force \( F \) is now applied between the wheel and its axle at a radius \( a \) from the center of the wheel. This frictional force is constant. How long does it take the wheel to come to rest with respect to its center of mass (to stop rotating)?

\[ \Delta t = \frac{2mR^2 \omega}{aF} \rightarrow 3.0 \text{ s} \]
\[ |I_{fr} - I_{fr}| = aF \Delta t \]
\[ (mR^2 \omega) = aF \Delta t \]
\[ \Rightarrow \Delta t = \frac{mR^2 \omega}{aF} \rightarrow 2.0 \text{ s} \]
Problem 2

Two small objects each of mass \( m \) are connected by a lightweight rod of length \( L \). At a particular instant they have velocities as shown in the diagram and the whole system is moving in outer space far from any other objects.

In the following questions, express any vector quantities in your answer in three-component vector notation.

(a 10pts) What is the total angular momentum \( \vec{L}_A \) of the system relative to point \( A \) in the diagram?

\[
\vec{L}_A = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2
\]

\[
\vec{L}_A = \langle -d, h+L, 0 \rangle \times \langle mv_1, 0, 0 \rangle + \langle -d, h, 0 \rangle \times \langle mv_2, 0, 0 \rangle
\]

\[
\vec{L}_A = \langle 0, 0, -(h+L)(mv_1) \rangle + \langle 0, 0, -h(mv_2) \rangle
\]

\[
\vec{L}_A = \langle 0, 0, -hm(v_1+v_2) - Lmv_1 \rangle
\]
Problem 3

A barbell consists of two small balls, each with mass $m$, at the ends of a very low mass rod of length $d$. The center of mass for the barbell is mounted on the end of a low mass rigid rod of length $b$. As shown in the diagram, the rod rotates clockwise with angular speed $\omega$. In addition, the barbell rotates counterclockwise about its own center, with an unknown angular speed.

(a 10pts) Determine the translational angular momentum $L_{\text{trans,B}}$ (magnitude and direction) for the barbell.

$$ L_{\text{trans,B}} = I \mathbf{\Omega} = 2mb^2 \omega \hat{z} $$

(b 5pts) If the total angular momentum for this system (about the point B) is zero, calculate the rotational angular momentum for the barbell about its center of mass. $L_{\text{rot, cm}}$ (magnitude and direction)

$$ L_{\text{rot, cm}} = -L_{\text{trans,B}} = -2mb^2 \omega \hat{z} $$

(c 5pts) Calculate the moment of inertia $I$ for the barbell about its center of mass.

$$ I_{\text{barbell}} = (2m)(d/2)^2 = (2m)(d^2/4) $$

(d 5pts) Determine the unknown angular speed of the barbell about its center of mass.

$$ L_{\text{rot, cm}} = I_{\text{barbell}} \mathbf{\Omega} = \frac{1}{2} md^2 \omega \hat{z} $$

$$ \Rightarrow \frac{1}{2} md^2 \omega \hat{z} = 2m \omega (r_{cm}^2 \hat{z}) $$

$$ \Rightarrow \frac{1}{2} md^2 \omega = 2m (r_{cm}^2)$$

$$ \therefore \omega = \frac{4b^2 \omega}{d^2} \rightarrow 2\text{pts} $$