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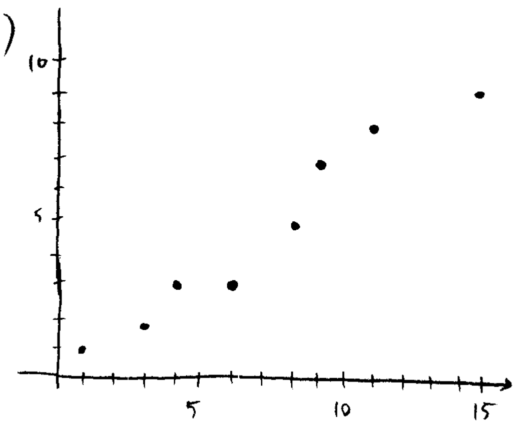
ECE 3075

HW #9

Solution

Problem 9.1 (4-6.1) Note that the corrected y-values are: 1, 2, 3, 3, 5, 7, 8, 9

(a)



(b) $b = \dfrac{8 \times 354 - 56 \times 38}{8 \cdot 524 - 56^2} = \boxed{0.6667}$

$a = \dfrac{38 - 0.6667 \times 56}{8} = \boxed{0.0833}$

Problem 9.2 (4-6.3)

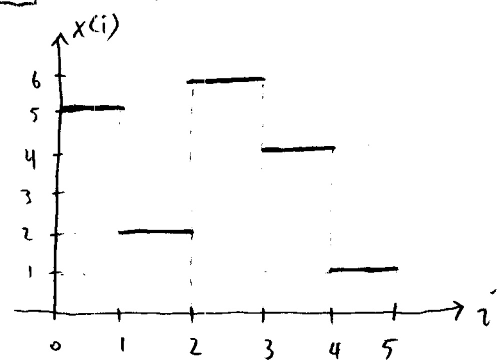
$g = \frac{1}{y} = a + bx$ ⇒ Construct another set of data $w = \frac{1}{y}$ and fit a straight line to the set w.

⇒ Using formulae 40-23, $\boxed{a = 1.1585}$

$\boxed{b = 1.5660}$

Problem 9.3 (5-1.1)

(a)



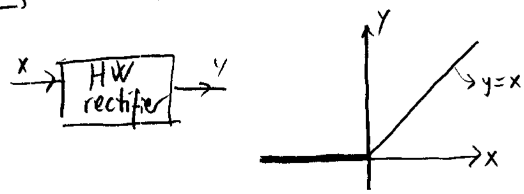
(b) $6^5 = \boxed{7776}$

(c) $\boxed{\dfrac{1}{7776}}$

(d) $\boxed{\dfrac{1}{7776}}$

Problem 9.4 (5-2.2)

(a)



$X(t)$: random variable for a <u>specific</u> t.

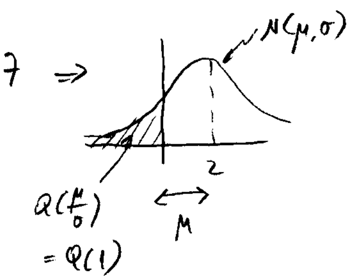
$X_p(t)$: output of the HW rectifier, which is $X(t)$ transformed according to the above y-x relation.

$$X_p(t) = \begin{cases} X(t), & X(t) > 0 \\ 0, & X(t) < 0. \end{cases}$$

Note that the time variable has nothing to do with our formulation. We could have denoted $X(t)$ by W and $X_p(t)$ by Y. The HW rectifier transforms W to Y.

For $Y<0$, there is no corresponding $W \Rightarrow f_Y(y) = 0$.

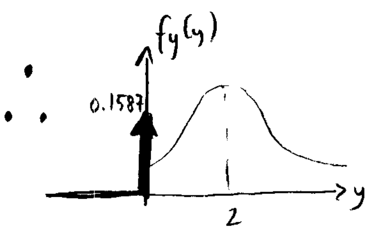
For $Y=0$, $\Pr\{Y=0\} = \Pr\{W<0\} = Q\left(\frac{\mu}{\sigma}\right) = Q(1) = 0.1587 \Rightarrow$



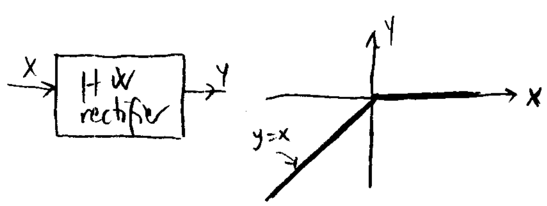
For $Y>0$, $\Pr\{Y \le y\} = \Pr\{X \le y\} \Rightarrow F_Y(y) = F_X(y)$

$$\Rightarrow f_Y(y) = f_X(y) = \frac{1}{\sqrt{8\pi}} e^{-\frac{(y-2)^2}{8}}$$

$\therefore$



(b)

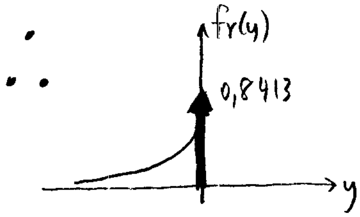


For $Y>0$, $f_Y(y) = 0$

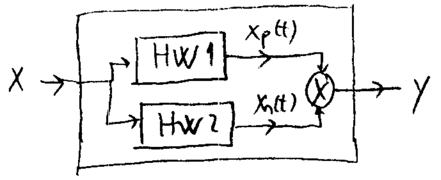
For $Y=0$, $\Pr\{Y=0\} = \Pr\{X>0\} = 1-Q(1)$

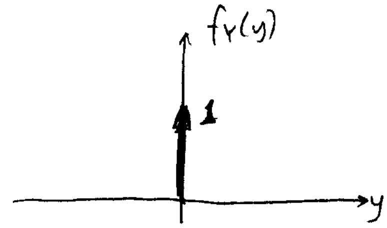
For $Y<0$, $f_Y(y) = f_X(y) = \frac{1}{\sqrt{8\pi}} e^{-\frac{(y-2)^2}{8}}$

$\therefore$



(c) Either $X_p(t)$ or $X_n(t)$ is always zero $\Rightarrow X_p(t)X_n(t)=0$.





Problem 9.5 (5-3.2)

(a) $E[X(t)] = E[At]+E[B] = t\cdot E[A]+E[B] = \boxed{3}$

(b) Since A and B are independent, At and B are also independent.

$\Rightarrow Var[X(t)] = Var[At] + Var[B]$ (from 3-28 , with $\rho=0$)

$$= t^2\cdot 9 + \frac{1}{12}\cdot 6^2 = \boxed{9t^2+3}$$

(c) $\left.\begin{matrix} 2A+B=10 \\ 4A+B=20 \end{matrix}\right\}$ $\left.\begin{matrix} B=0 \\ A=5 \end{matrix}\right\}$ at $t=8$: $\boxed{X(t)=40}$

Problem 9.6 (5-4.2)

(a)
$$E[X(t+\tau)X(t)] = A^2 E[\cos(\omega(t+\tau)+\theta)\cos(\omega t+\theta)]$$
$$= \frac{A^2}{2}E[\cos(\omega(t+\tau)+\theta+\omega t+\theta)+\cos(\omega(t+\tau)+\theta-\omega t-\theta)]$$
$$= \frac{A^2}{2}E[\cos(2\theta+2\omega t+\omega\tau)+\cos\omega\tau]$$
$$= \frac{A^2}{2}\cos\omega\tau + \frac{A^2}{2}E[\cos(2\theta+2\omega t+\omega\tau)]$$

Since θ is uniform between 0 and 2π,

$$E[\cos(2\theta+2\omega t+\omega\tau)] = \int_0^{2\pi}\frac{1}{2\pi}\cos(2\theta+2\omega t+\omega\tau)\,d\theta$$
$$= \underline{\underline{0}}$$

$$\Rightarrow \left.\begin{matrix} E[X(t+\tau)X(t)] = \frac{A^2}{2}\cos\omega\tau \\ E[X(t)] = A\int_0^{2\pi}\frac{1}{2\pi}\cos(\omega t+\theta)\,d\theta = 0 \end{matrix}\right\}$$ Do not depend on t $\Rightarrow$ $\underline{\underline{WSS}}$.

(b) $E[\cos(2\theta+2\omega t+\omega\tau)] = \int_0^{\alpha}\frac{1}{\alpha}\cos(2\theta+2\omega t+\omega\tau)\,d\theta = \frac{1}{2\alpha}\left(\sin(2\alpha+2\omega t+\omega\tau)-\sin(2\omega t+\omega\tau)\right) \neq 0$

for $\alpha\neq 2n\pi$