Ilker R Capoglu

ECE 3075

HW #8

Solution

Problem 8.1 (4-2.1)

(a) $\hat{\bar{X}} = \frac{1}{9}\sum_{i=1}^{9} X_i = \boxed{0,3727}$

(b) $\mathrm{var}(\hat{\bar{X}}) = \frac{\sigma_X^2}{9} = \frac{\frac{1}{12}(0.999-0.000)^2}{9} = \boxed{9.24.10^{-3}}$

(c) $(0.01)^2 < \frac{\frac{1}{12}(0.999-0.000)^2}{n} \Rightarrow \boxed{n \geqslant 832}$

Problem 8.2 (4-2.5)

(a) $N=50,\ n=10$: $\mathrm{var}(\hat{\bar{X}}) = \frac{12}{10}\left(\frac{50-10}{50-1}\right) = 0.9796$

$\sigma^2 = 12$

$\sigma_{\hat{\bar{X}}} = \sqrt{\mathrm{var}(\hat{\bar{X}})} = \boxed{0,9897}$

(b) $\mathrm{var}(\hat{\bar{X}}) = 1 = \frac{12}{n}\left(\frac{50-n}{50-1}\right) \Rightarrow 61n = 600 \Rightarrow n = 9,84$

$\Rightarrow \boxed{n=10}$

(c) $\mathrm{var}(\hat{\bar{X}}) = (0.01 . 70)^2 = \frac{12}{n}\left(\frac{50-n}{50-1}\right) \Rightarrow n = 16,66$

$\Rightarrow \boxed{n=17}$

Problem 8.3 (4-3.2)

$\mathrm{var}(\tilde{S}^2) = (0.02 . \underbrace{\sigma^2}_{\text{true variance}})^2 = \frac{n(\mu_4 - \sigma^4)}{(n-1)^2} = \frac{n . 2\sigma^4}{(n-1)^2} \Rightarrow \frac{2n}{(n-1)^2} = (0.02)^2$

$\Rightarrow n^2 - 5002n + 1 = 0$

$\Rightarrow \boxed{n = 5002}$

$\boxed{\text{Problem 8.4}}$ (4-4.2)

(a) $\hat{X}$: sample mean.

$$\mu_{\hat{X}} = \mu_X = 120$$

$$\sigma_{\hat{X}}^2 = \frac{\sigma_X^2}{150} = \frac{100}{150} = 0.67 \Rightarrow \sigma_{\hat{X}} = 0{,}816$$

$q\% = 90\% \Rightarrow k = 1.64$ (from Table 4.1)

$$\Rightarrow \mu_{\hat{X}} - 1.64\,\sigma_{\hat{X}} \le \hat{X} \le \mu_{\hat{X}} + 1.64\,\sigma_{\hat{X}}$$

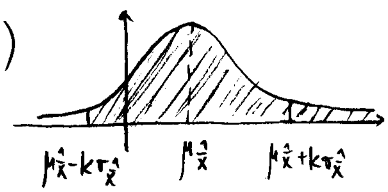
$$\boxed{118.66 \le \hat{X} \le 121.34}$$

(b) $\sigma_{\hat{X}}^2 = \frac{\sigma_X^2}{21} = \frac{100}{21} = 4.76 \Rightarrow \sigma_{\hat{X}} = 2.182$

$$\Rightarrow \mu_{\hat{X}} - 1.64\,\sigma_{\hat{X}} \le \hat{X} \le \mu_{\hat{X}} + 1.64\,\sigma_{\hat{X}} \Rightarrow \boxed{116.42 \le \hat{X} \le 123.58}$$

$\boxed{\text{Problem 8.5}}$ (4-4.3)

(a)



$$\Pr\{\hat{X} \ge \mu_{\hat{X}} - k\sigma_{\hat{X}}\} = 0.9$$

$$\int_{\mu_{\hat{X}} - k\sigma_{\hat{X}}}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_{\hat{X}}} e^{-\frac{(x-\mu_{\hat{X}})^2}{2\sigma_{\hat{X}}^2}}\,dx = 0.9$$

let $y = \frac{x - \mu_{\hat{X}}}{\sigma_{\hat{X}}} \Rightarrow dx = \sigma_{\hat{X}}\,dy \Rightarrow \int_{-k}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}\,dy = 0.9$

$$\Rightarrow Q(-k) = 1 - Q(k) = 0.9 \Rightarrow Q(k) = 0.1$$

$$\Rightarrow k = 1.28$$

$$\Rightarrow \mu_{\hat{X}} - 1.28\,\sigma_{\hat{X}} \le \hat{X} < \infty$$

$$\Rightarrow \boxed{118.95 \le \hat{X} < \infty}$$

(b) $\mu_{\hat{X}} - 1.28\,\sigma_{\hat{X}} \le \hat{X} < \infty$

$$\Rightarrow \boxed{117.21 \le \hat{X} < \infty}$$