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ECE 3075
HW #3
Solution

Problem 3.1 (2-2.2)

(a) The probability that a continuous r.v. (meaning that its distribution function is continuous) assumes a single value is always $\boxed{\text{zero}}$ We can only consider probabilities of "being in an interval".

(b) $P\{X > \frac{3}{4}\} = 1 - P\{X \le \frac{3}{4}\} = 1 - F_X(\frac{3}{4}) = 1 - (\frac{1}{2} + \frac{1}{2}\cdot\frac{3}{4}) = \frac{1}{8} = \boxed{0{,}125}$

(c) $P\{-\frac{1}{2} < X \le \frac{1}{2}\} = P\{X \le \frac{1}{2}\} - P\{X \le -\frac{1}{2}\} = F_X(\frac{1}{2}) - F_X(-\frac{1}{2}) = (\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}) - (\frac{1}{2} + \frac{1}{2}(-\frac{1}{2}))$
$= \boxed{0{,}5}$

Problem 3.2 (2-2.3)

(a) $F_X(x)$ must be equal to 1 at $x \to \infty$.

$\lim_{x\to\infty} F_X(x) = \boxed{A = 1}$

(b) $$\left.\begin{aligned} F_X(x) &= 1 - \exp[-(x-1)] &&, 1 < x < \infty \\ &= 0 &&, -\infty < x < 1 \end{aligned}\right\} F_X(2) = 1 - \exp[-1] = \boxed{0{,}6321}$$

(c) $F_X(\infty) - F_X(2) = 1 - F_X(2) = 1 - 0{,}6321 = \boxed{0{,}3679}$

(d) $F_X(3) - F_X(1) = F_X(3) - 0 = 1 - \exp[-2] = \boxed{0{,}8647}$

Problem 3.3 (2-2.4)

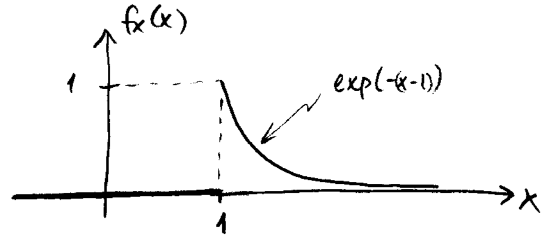
(a) $$\left.\begin{aligned} F_X(-2) &= A(1 - \sin 2b) = 0 \\ F_X(2) &= A(1 + \sin 2b) = 1 \end{aligned}\right\} \boxed{A = \frac{1}{2}}, \quad \sin 2b = 1 \Rightarrow 2b = \frac{\pi}{2} \Rightarrow \boxed{b = \frac{\pi}{4}}$$

(b) $P\{X > 1\} = 1 - P\{X \le 1\} = 1 - F_X(1) = 1 - \frac{1}{2}(1 - \sin\frac{\pi}{4}) = \boxed{\frac{1}{2} - \frac{\sqrt{2}}{4}}$

(c) $P\{X < 0\} = F_X(0) = \frac{1}{2}(1 - \sin 0) = \boxed{\frac{1}{2}}$

Problem 3.4 (2-3.2)

(a) $f_X(x) = \frac{d}{dx} F_X(x) = \begin{cases} \exp(-(x-1)) & , 1 < x < \infty \\ 0 & , -\infty < x < 1 \end{cases}$



(b) $P\{2 < X \leq 3\} = \int_2^3 f_X(x)\,dx$

$= \int_2^3 \exp(-(x-1))dx = -\exp(-(x-1))\Big|_2^3 = e^{-1} - e^{-2} = \boxed{0{,}2325}$

(c) $P\{X < 2\} = \int_{-\infty}^{2} f_X(x)dx = \int_{-\infty}^{1} 0 \cdot dx + \int_1^2 \exp(-(x-1))dx$

$= -\exp(-(x-1))\Big|_1^2 = 1 - e^{-1} = \boxed{0{,}6321}$