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ECE 3075

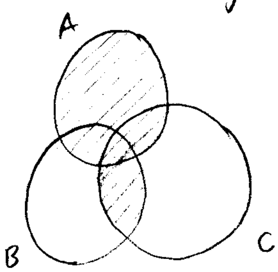
Homework #2

Solution

(1.5.4)

Problem 2.1 (a) Since $A \cap B \subset A$, it is obvious that $A \cup (A \cap B) = A$.

(b) In the most general case, A, B and C all intersect each other:



The shaded region is equal to both $A \cup (B \cap C)$ and $(A \cup B) \cap (A \cup C)$.

(c) $A \cup (\bar{A} \cap B) = (A \cup \bar{A}) \cap (A \cup B) = S \cap (A \cup B) = \underline{\underline{A \cup B}}$

(d) $(A \cap B) \cup (A \cap \bar{B}) \cup (\bar{A} \cap B) = [A \cap (B \cup \bar{B})] \cup (\bar{A} \cap B) = [A \cap S] \cup (\bar{A} \cap B)$

$= A \cup (\bar{A} \cap B) = \underline{\underline{A \cup B}}$ (from (c)).

The question in the book is wrong. The correct result is as above.

(1.6.3)

Problem 2.2 (a) $Pr[A \cap \bar{B}] = Pr[A]\, Pr[\bar{B} | A] = \frac{1}{13} \cdot \frac{48}{51} = \frac{48}{663} = \boxed{0{,}0724}$

since $P[\bar{B}|A] = Pr\{$not king on second draw, given that we have king on first draw$\}$

(b) $Pr[A \cup B] = Pr[A] + Pr[B] - Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A]\, Pr[B|A]$

$= \frac{1}{13} + \frac{1}{13} - \frac{1}{13} \cdot \frac{3}{51} = \frac{33}{221} = \boxed{0{,}14932}$

(c) $Pr[\bar{A} \cup \bar{B}] = Pr[\overline{(A \cap B)}] = 1 - Pr[A \cap B] = 1 - \frac{1}{13} \cdot \frac{3}{51} = \frac{220}{221} = \boxed{0{,}99548}$

(d) $Pr[\bar{A} \cap \bar{B} \cap \bar{C}] = Pr[\bar{A}]\, Pr[\bar{B} | \bar{A}]\, Pr[\bar{C} | (\bar{A}, \bar{B})]$

$= \frac{48}{52} \cdot \frac{47}{51} \cdot \frac{46}{50} = \boxed{0{,}78262}$

(e) $Pr[(A \cap B) \cup (\bar{B} \cap C)] = Pr[A \cap B] + Pr[\bar{B} \cap C] - Pr[\overbrace{A \cap \underbrace{B \cap \bar{B}}_{\phi} \cap C}^{\phi}]$

$= Pr[A \cap B] + \underbrace{Pr[\bar{B} \cap C]}_{Pr[\bar{B}]\, Pr[C|\bar{B}]} = \frac{4}{52} \cdot \frac{3}{51} + \frac{48}{52} \cdot \frac{4}{51} = \frac{1}{13} = \boxed{0{,}0769}$

(f) $Pr[\bar{A} \cup B \cup C] = 1 - Pr[A \cap \bar{B} \cap \bar{C}] = 1 - Pr[A]\,Pr[\bar{B}|A]\,Pr[\bar{C}|\bar{B},A]$

$$= 1 - \frac{4}{52} \cdot \frac{48}{51} \cdot \frac{47}{50} = \boxed{0{,}93194}$$

(1.7.5)

Problem 2.3 Pr[successful transmission from A to B]

$= 1 - Pr$[all paths fail $\Rightarrow$ no path to transmit message]

$$= 1 - (1-(0{,}9)^2)(1-0{,}9)(1-(0{,}9)^2) = \boxed{0{,}99639}$$

(1.8.4)

Problem 2.4 (a) First, note that the sets $A \cap B$ and $A \cap \bar{B}$ are disjoint:

$$A \cap B \cap A \cap \bar{B} = A \cap A \cap \underbrace{B \cap \bar{B}}_{\phi} = \phi$$

The union of these sets is the set A:

$$(A \cap B) \cup (A \cap \bar{B}) = (A \cup A) \cap (B \cup \bar{B}) = A \cap S = A$$

Therefore, $Pr[A \cap B] + Pr[A \cap \bar{B}] = Pr[A]$.

$\Rightarrow Pr[A \cap \bar{B}] = Pr[A] - Pr[A \cap B] = Pr[A] - Pr[A]\,Pr[B]$ (since A & B are indep.)

$= Pr[A](1 - P[B])$

$= Pr[A]\,Pr[\bar{B}]$ Q.E.D.

(b) $Pr[\bar{A} \cap \bar{B}] = Pr[(\overline{A \cup B})] = 1 - Pr[A \cup B] = 1 - Pr[A] - Pr[B] + Pr[A \cap B]$

$= 1 - Pr[A] - Pr[B] + Pr[A]\,Pr[B]$

$= (1 - Pr[A])(1 - Pr[B])$

$= Pr[\bar{A}]\,Pr[\bar{B}]$ Q.E.D.

(1.10.8)

Problem 2.5 (a) Pr[no errors] $= Pr$[all characters correct] $= [1 - 0.001]^{10{,}000} = \boxed{4{,}517 \cdot 10^{-5}}$

(b) $$Pr[10 \text{ errors}] = \binom{10{,}000}{10}(0{,}001)^{10}(1-0{,}001)^{10000-10} \simeq \frac{1}{\sqrt{2\pi \cdot 10^4 (0{,}001)(1-0.001)}}\, e^{-\frac{(10 - 10^4(0{,}001))^2}{2 \cdot 10^4 (0{,}001)(1-0{,}001)}} = \boxed{0{,}1262}$$

(c) $Pr[\text{no error}] = (1-p)^{10{,}000} > 0{,}99 \Rightarrow \boxed{p < 1{,}005 \cdot 10^{-6}}$