Ilker R Capoglu

ECE 3075

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HW #13

Solution

Problem 1 (7-6.2)

(a) $R_X(\tau) = \underbrace{16\, e^{-5|\tau|} \cos 20\pi\tau}_{1} + \underbrace{8\cos 10\pi\tau}_{2}$

$x(t)$ consists of 2 parts: 1 $\rightarrow$ can be shown to be ergodic $\rightarrow \eta_1 = R_{X_1}(\infty) = 0$

2 $\rightarrow$ corresponds to sinusoid with random phase $\rightarrow \eta_2 = 0$

$\Rightarrow E[X] = \eta = \eta_1 + \eta_2 = \underline{0}$

$E[X^2] = R_X(0) = 16 + 8 = \underline{24}$

$\boxed{Var[X] = 24}$

(b) $S_X(\omega) = \frac{16}{2\pi}\left(\frac{10}{25+\omega^2}\right) * \left(\pi\delta(\omega - 20\pi) + \pi\delta(\omega+20\pi)\right) + 8\pi\left(\delta(\omega-10\pi) + \delta(\omega+10\pi)\right)$

$= \boxed{16\left(\frac{5}{25+(\omega-20\pi)^2} + \frac{5}{25+(\omega+20\pi)^2}\right) + 8\pi\left(\delta(\omega-10\pi)+\delta(\omega+10\pi)\right)}$

(c) $\boxed{S_X(0) = 0.0403}$

Problem 2 (7-6.3)

(a) $\overline{X^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(\omega)\, d\omega = \frac{1}{2\pi}\int_{10}^{20} 5\, d\omega = \boxed{\frac{50}{\pi} = 15.91}$

(b) $R_X(\tau) = \frac{1}{2\pi}\int_{-20}^{-10} 5 e^{j\omega\tau} d\omega + \frac{1}{2\pi}\int_{10}^{20} 5 e^{-j\omega\tau} d\omega = \frac{5}{2\pi}\left[\frac{e^{j20\tau} - e^{-j20\tau}}{j\tau} - \frac{e^{j10\tau} - e^{-j10\tau}}{j\tau}\right]$

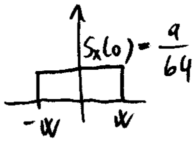
$= \frac{5}{2\pi}\left[\frac{\sin 20\tau}{\tau} - \frac{\sin 10\tau}{\tau}\right] = \boxed{\frac{50}{\pi}\left[\frac{\sin 5\tau}{5\tau}\right]\cos 15\tau}$

(c) $R_X(0) = \boxed{\frac{50}{\pi} = 15.91}$

Problem 3 (7-7.1)

$S_X(0) = \underline{\underline{\frac{9}{64}}}$, $\overline{X^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{9}{\omega^2+64}\,d\omega = \frac{1}{2\pi}\,\frac{9}{8}\tan^{-1}\omega\Big|_{-\infty}^{\infty} = \underline{\underline{\frac{9}{16}}}$

(a)



$\overline{X^2} = \frac{1}{2\pi}\int_{-W}^{W}\frac{9}{64}\,d\omega = \frac{9}{64\pi}W = \frac{9}{16} \Rightarrow W = 4\pi$

$\Rightarrow \boxed{S_N(\omega) = \begin{cases} \frac{9}{64}, & |\omega| \le 4\pi \\ 0, & \text{else} \end{cases}}$

(b) $R_X(\tau) = \mathcal{F}^{-1}\left\{\frac{9}{\omega^2+64}\right\} = \boxed{\frac{9}{16}e^{-8|\tau|}}$

(c) $R_N(\tau) = \mathcal{F}^{-1}\{S_N(\omega)\} = \boxed{\frac{9}{16}\cdot\frac{\sin 4\pi\tau}{4\pi\tau}}$

(d) $R_X(0) = \frac{9}{16} = R_N(0)$

$\text{Area}(R_X) = S_X(0) = S_N(0) = \text{Area}(R_N)$

Problem 4 (7-8.1)

(a) $U(t) = X(t) + Y(t) \Rightarrow R_U(\tau) = R_X(\tau) + R_Y(\tau)$, since X & Y are indep. and zero mean.

$\Rightarrow \boxed{S_U(\omega) = S_X(\omega) + S_Y(\omega) = 1}$

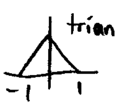
(b) $R_{XY}(\tau) = 0 \Rightarrow \boxed{S_{XY}(\omega) = 0}$

(c) $R_{XU}(\tau) = E[X(t)(X(t+\tau) + Y(t+\tau))] = E[X(t)X(t+\tau)] = R_X(\tau)$

$\Rightarrow \boxed{S_{XU}(\omega) = S_X(\omega) = \frac{16}{\omega^2+16}}$

Problem 5 (7-9.1)

$1-|t| \longleftrightarrow \text{sinc}^2 f$



$\Rightarrow 1-\frac{|t|}{\tau_m} \longleftrightarrow \tau_m \text{sinc}^2(\tau_m f)$: