Solutions for Extra Problems in Chap 7

7-2.3

$E\{X^2(t)\} = R_X(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(\omega)\,d\omega = 4.$

(a)

Let $S_Y(\omega) = 4 S_X(\omega)$

$\therefore \boxed{E\{Y^2(t)\}} = 4\,E\{X^2(t)\} = \boxed{16}$

(b) Let $S_Y(\omega) = S_X(4\omega)$

$\boxed{E\{Y^2(t)\}} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_Y(\omega)\,d\omega$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(4\omega)\,d\omega \quad (s = 4\omega)$

$= \frac{1}{4}\cdot\frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(s)\,ds$

$= \frac{1}{4}\cdot 4 = \boxed{1}$

(c) Let $S_Y(\omega) = S_X(\omega/4)$

$\boxed{E\{Y^2(t)\}} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(\omega/4)\,d\omega \quad (s = \omega/4)$

$= 4\cdot\frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(s)\,ds$

$= 4\cdot 4 = \boxed{16}$

(d) Let $S_Y(\omega) = 4 S_X(4\omega)$

From (a) and (b),

$\boxed{E\{Y^2(t)\}} = \boxed{4}$

7-3.1

(a) $f(\omega) = \frac{1}{\omega^2 + 3\omega + 1}$

Not even $\therefore$ $\boxed{\text{Not Valid}}$

(b) $f(\omega) = \frac{\omega^2 + 16}{\omega^4 + 9\omega^2 + 18}$

$\boxed{\text{Valid}}$

(c) $f(\omega) = 10 e^{-\omega^2}$

$\boxed{\text{Valid}}$

(d) $f(\omega) = \frac{\omega^2 + 4}{\omega^4 - 4\omega^2 + 1}$

$f(1) = \frac{1^2 + 4}{1^4 - 4\cdot 1^2 + 1} = \frac{5}{-2} < 0$

$\therefore$ $\boxed{\text{Not Valid}}$

(e) $f(\omega) = \left(\frac{1 - \cos\omega}{\omega}\right)^2$

$\boxed{\text{Valid}}$

(f) $f(\omega) = \delta(\omega) + \frac{\omega^3}{\omega^4 + 1}.$

Not even $\Rightarrow$ $\boxed{\text{Not Valid}}$.

7-4.1

$$S_x(\omega) = \frac{16(\omega^2+36)}{\omega^4+13\omega^2+36}$$

(a) $s = j\omega \quad \therefore \quad \omega^2 = -s^2$.

$$S_x(s) = \frac{16(36-s^2)}{s^4-13s^2+36}$$

(b)

$$S_x(s) = \frac{16(6+s)(6-s)}{(s^2-4)(s^2-9)}$$

$$= \frac{-16(s+6)(s-6)}{(s+2)(s-2)(s+3)(s-3)}$$

Poles @ ± 2, ± 3

zeros @ ± 6.

(c) when $f = 1\,Hz$, i.e.

($\omega = 2\pi f = 2\pi$ rad/s,)

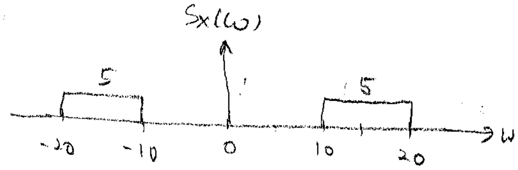
$$\boxed{S_x(2\pi)} = \frac{16[(2\pi)^2+36]}{(2\pi)^4+13(2\pi)^2+36} = \boxed{0.573}$$

(d) let $s' = s/100$

$$\boxed{S_x(s')} = \frac{-16(\frac{s}{100}+6)(\frac{s}{100}-6)}{(\frac{s}{100}+2)(\frac{s}{100}-2)(\frac{s}{100}+3)(\frac{s}{100}-3)}$$

$$= \boxed{\frac{-16\times 10^4(s+600)(s-600)}{(s+200)(s-200)(s+300)(s-300)}}$$

7-6.3



(a) $\boxed{E\{X^2(t)\}} = R_x(0)$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_x(\omega)\,d\omega$$

$$= \frac{1}{2\pi}\cdot(50+50) = \boxed{\frac{50}{\pi}}$$

(b) $\boxed{R_x(\tau)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_x(\omega)e^{j\omega\tau}\,d\omega$

$$= \frac{1}{2\pi}\int_{-20}^{-10} 5e^{j\omega\tau}\,d\omega + \frac{1}{2\pi}\int_{10}^{20} 5e^{j\omega\tau}\,d\omega$$

$$= \frac{5}{2\pi}\left[\frac{e^{-j10\tau}-e^{-j20\tau}}{j\tau}\right] + \frac{5}{2\pi}\left[\frac{e^{20\tau}-e^{10\tau}}{j\tau}\right]$$

$$= \boxed{\frac{5}{\pi}\cdot\left[\frac{\sin 20\tau}{\tau} - \frac{\sin 10\tau}{\tau}\right]}$$

(c) $\boxed{R_x(0)} = \frac{5}{\pi}[20-10] = \boxed{\frac{50}{\pi}}$

$\boxed{\text{n-n.1}}$

$$S_X(\omega) = \frac{9}{\omega^2+64}.$$

$$S_X(0) = 9/64.$$

$$E\{X^2(t)\} = R_X(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{9}{\omega^2+64}\, d\omega$$

let $\omega = 8\tan\theta$

$d\omega = 8\sec^2\theta \cdot d\theta$.

$$\int_{-\infty}^{\infty} \frac{1}{\omega^2+64}\, d\omega = \int_{-\pi/2}^{\pi/2} \frac{1}{64\cdot\sec^2\theta}\cdot 8\cdot\sec^2\theta\cdot d\theta$$

$$= \frac{1}{8}\cdot\pi$$

$$\therefore E\{X^2(t)\} = \frac{1}{2\pi}\cdot 9\cdot\frac{\pi}{8} = \frac{9}{16}.$$

(a) Let $\boxed{S_N(\omega)} = \boxed{\begin{cases} N_0 & |\omega| \le W \\ 0 & \text{o.w} \end{cases}}$

$$E\{N^2(t)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_N(\omega)\, d\omega$$

$$= \frac{1}{2\pi}\cdot N_0 \cdot 2W = \frac{N_0\cdot W}{\pi} = \frac{9}{16}$$

$$S_N(0) = \boxed{N_0} = \boxed{\frac{9}{64}}$$

$$\therefore \boxed{W} = \frac{9}{16}\cdot\frac{\pi}{N_0} = \frac{9}{16}\cdot\frac{64}{9}\cdot\pi = \boxed{4}$$

(b)

$$e^{-\alpha|t|} \leftrightarrow \frac{2\alpha}{\alpha^2+\omega^2}$$

$$\boxed{R_X(t)} = \mathcal{F}^{-1}\left\{\frac{9}{8^2+\omega^2}\right\} = \mathcal{F}^{-1}\left\{\frac{9}{16}\cdot\frac{16}{8^2+\omega^2}\right\}$$

$$= \boxed{\frac{9}{16}\cdot e^{-8|\tau|}}$$

(c) $R_N(\tau) = \mathcal{F}^{-1}\{S_N(\omega)\}$

$$= \mathcal{F}^{-1}\{N_0\cdot \mathrm{rect}(t/2W)\}$$

use

$\mathrm{rect}(t/\alpha) \Leftrightarrow \alpha\, Sa\left(\frac{\omega\alpha}{2}\right)$

$\alpha\, Sa\left(\frac{\alpha t}{2}\right) \Leftrightarrow 2\pi\, \underline{\mathrm{rect}(\omega/\alpha)}$.

$$\therefore \mathcal{F}^{-1}\{\mathrm{rect}(t/2W)\} \qquad \alpha = 2W.$$

$$= \frac{1}{2\pi}\cdot 2W\; Sa\left(\frac{2W\cdot t}{2}\right)$$

$$= \frac{W}{\pi}\, Sa(W\tau)$$

$$\therefore \boxed{R_N(\tau)} = N_0\frac{W}{\pi}\, Sa(W\tau)$$

$$= \frac{9}{64}\cdot\frac{4}{\pi}\, Sa(4\tau)$$

$$= \boxed{\frac{9}{16\pi}\cdot\frac{\sin(4\tau)}{(4\tau)}}$$

(d) $R_X(0) = \frac{9}{16} = R_N(0)$.

Since $E\{X^2(t)\} = E\{N^2(t)\}$, two autocorrelation functions have same areae.

$\boxed{7\text{-}8.2}$

$$V(t) = X(t) - Y(t),$$

$$U(t) = X(t) + Y(t).$$

$$\boxed{R_{UV}(\tau)} = E\{U(t)\,V(t+\tau)\}$$

$$= E\{[X(t)+Y(t)][X(t+\tau) - Y(t+\tau)]\}$$

$$= E\{X(t)X(t+\tau) - X(t)Y(t+\tau) + Y(t)X(t+\tau) - Y(t)Y(t+\tau)\}$$

$$= R_X(\tau) - \cancel{R_{XY}(\tau)} + \cancel{R_{YX}(\tau)} - R_Y(\tau).$$

$$= \boxed{R_X(\tau) - R_Y(\tau)}$$

$$\therefore \boxed{S_{UV}(\omega)} = \mathcal{F}\{R_{UV}(\tau)\} = S_X(\omega) - S_Y(\omega) = \boxed{\frac{16-\omega^2}{16+\omega^2}}$$