✓

5.5.3) $X(t) = \sum_{n=-\infty}^{\infty} A f_1(t - nT - t_0)$

$f(t_0) = \frac{1}{T}$ $\quad 0 < t < T$

$= 0$ $\quad$ elsewhere

$f_1(t) = 1$ $\quad 0 < t < T/2$

$= 0$ $\quad$ elsewhere

a)

For any t there is a value of n that will cause argument of $f_1(\cdot)$ to be in interval $0, T$. The r.v. t_0 will then cause argument to vary through an interval T seconds long. Therefore

$$\bar{X}(t) = E\{\sum A f_1(t - nT - t_0)\} = E\{A f_1(t_0)\}$$

$$= \int_0^T A f_1(t_0) \cdot \frac{1}{T} dt = A/2$$

$$\overline{X^2} = \int_0^T A^2 f_1(t_0) \cdot \frac{1}{T} dt_0 = A^2/2$$

b) $\langle x(t) \rangle = \frac{1}{T} \int_0^T A(t - t_0) dt = A/2$

$\langle x^2(t) \rangle = \frac{1}{T} \int_0^T A^2(t - t_0) dt = A^2/2$

c) yes $\quad$ $d \} \gg a$

✓

5.6.1)

a) $\hat{X} = \frac{1}{21} \sum_{i=0}^{21} x_i = 0.0362$

b) $\mathrm{Var}[\hat{X}] = \frac{1}{N} \sigma_X^2 + (\bar{X})^2 - (\bar{X})^2 = \frac{1}{N} \sigma_X^2 = 1/21$

6.1.1 $\sigma_X^2 = R_X(0) = 5$

a) $b = \frac{R_X(\tau)}{\sigma_X^2} = \frac{R_X(-0.1)}{5} = 0.606$

b) $\sigma_Y^2 = \sigma_X^2 + 2b\,R_X(-0.1) + b^2\sigma_X^2 = 5 + \frac{2R_X^2(-0.1)}{5} + 0.606^2 \times 5 = 10.51$

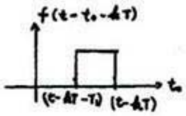
c) When $b = -1$

$\sigma_Y^2 = 5 + 2R_X(-0.1) + 5 = 16.07$

6.2.1 $X(t) = \sum_{k=-\infty}^{\infty} A\, f(t - t_0 - kT)$ where $f(t) = u(t) - u(t - T_1)$, $A = 0$ or 1.

a) $\bar{X} = E\left[\sum_{k=-\infty}^{\infty} A\, f(t - t_0 - kT)\right] = \sum_{k=-\infty}^{\infty} E[A] \cdot E[f(t - t_0 - kT)]$

$E[A] = \frac{1}{2}$ $E[f(t - t_0 - kT)] = \int_{t-kT-T_1}^{t-kT} \frac{1}{T}\, dt_0 = \frac{T_1}{T}$



For a given t, only one k will yield a non zero value

$\Rightarrow \bar{X} = \frac{1}{2} \cdot \frac{T_1}{T} = T_1/2T$

$\overline{X^2} = E[A^2] \cdot E[f^2(t - t_0 - kT)]$ for any t

$E[A^2] = \frac{1}{2}$

Since $f^2(t - t_0 - kT) = f(t - t_0 - kT)$

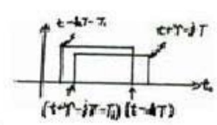
$E[f^2(t - t_0 - kT)] = E[f(t - t_0 - kT)] = \frac{T_1}{T}$

$\therefore \overline{X^2} = T_1/2T$

b) $R_X(\tau) = E[X(t)X(t+\tau)] = E\left[\sum_{k=-\infty}^{\infty}\sum_{j=-\infty}^{\infty} A_k A_j f(t - t_0 - kT) f(t + \tau - t_0 - jT)\right]$

$E[A_k A_j] = \frac{1}{2}$ $j = k$

$= \frac{1}{4}$ $j \neq k$



For $\tau > 0$ $E[f(t - t_0 - kT) f(t + \tau - t_0 - jT)]$

$= \int_{t+\tau-jT-T_1}^{t-kT} \frac{1}{T}\, dt_0 = \frac{T_1 - [\tau - (j-k)T]}{T}$

where $\tau - (j-k)T \le T$

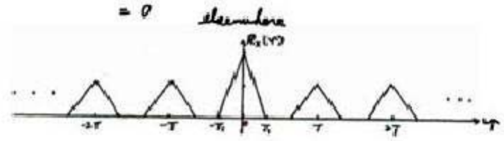
Similar for $\tau < 0$

For any τ, only one value of $(j-k)$ contribute

$\therefore R_X(\tau) = \frac{1}{2}\frac{T_1}{T}\left[1 - \frac{|\tau|}{T_1}\right]$, $j = k$ $|\tau| \le T_1$

$= \frac{1}{4}\frac{T_1}{T}\left[1 - \frac{|\tau - (j-k)T|}{T_1}\right]$, $j \neq k$, $|\tau - (j-k)T| \le T_1$

$= 0$ elsewhere



6.3.2

a) This process is stationary but not ergodic.

b) $\overline{X(t)} = E[X(t)] = E[Y(\cos w_0 t \cos\theta - \sin w_0 t \sin\theta)]$

$= E[Y] \cdot \{E[\cos w_0 t] \cdot E[\cos\theta] - E[\sin w_0 t] \cdot E[\sin\theta]\}$

$= 0$

$\overline{X^2(t)} = E[X^2(t)] = E[Y^2[\tfrac{1}{2}(1 + \cos 2w_0 t \cos 2\theta - \sin 2w_0 t \sin 2\theta)]]$

$= \tfrac{1}{2} E[Y^2] = 9$

c) $E[X(t)\,X(t+\tau)] = E[Y^2 \cos(w_0 t + \theta)\cos(w_0 t + w_0\tau + \theta)]$

$= E[Y^2] \cdot E[\cos w_0 t \cos(w_0 t + w_0\tau)\cos^2\theta + \sin w_0 t \sin(w_0 t + w_0\tau)\sin^2\theta$

$\quad - \sin w_0 t \cos(w_0 t + w_0\tau)\sin\theta\cos\theta - \cos w_0 t \sin(w_0 t + w_0\tau)\cos\theta\sin\theta]$

$= E[Y^2] \cdot E[\tfrac{1}{2}\cos w_0 t \cos(w_0 t + w_0\tau) + \tfrac{1}{2}\sin w_0 t \sin(w_0 t + w_0\tau)$

$\quad - \sin\theta\cos\theta[\sin(2w_0 t + w_0\tau)]]$

$= E[Y^2] \cdot E[\tfrac{1}{2}\cos w_0\tau]$

$= 18 \cdot \tfrac{1}{2} \cdot \tfrac{1}{6\tau}\sin 6\tau = \tfrac{3}{2\tau}\sin 6\tau$

6.3.3 $R_X(\tau) = 100 e^{-\tau^2}\cos 2\pi\tau + 10\cos 6\pi\tau + 36$

a) $\bar{X} = \pm 6$, $\overline{X^2} = R_X(0) = 146$, $\sigma_X^2 = 146 - 6^2 = 110$

b) Frequencies $=$ 0; 1, 3, Hz

c) First zero crossing of $R_X(\tau)$

$R_X(\tau) = 0 \Rightarrow \tau = 0.318$ by trial and error.