## ECE 3075A
***Random Signals***

**Lecture 6**

# Conditional Probability, Applications & Examples

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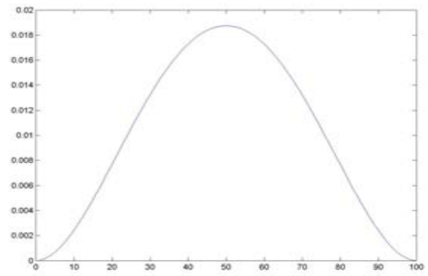
Fall, 2003



## Example 1

- Based on mortality records, a functional model that fits the age of death is given as

$$\beta(t) = 3\times 10^{-9} t^2 (100-t)^2 \qquad \text{for} \quad 0 \le t \le 100 \text{ (year)}$$



How to read the chart:

1.875% of the population die at age 50; or
A person will die at age 50 with probability 0.01875

Note: This is only a rough model for illustration.



## The Mortality Model - Example

- What is the probability that a person will die between the age of 20 and 30?

$$\Pr\{20 \le t \le 30\} = \int_{20}^{30} \beta(t)dt = 3\times 10^{-9} \int_{20}^{30} t^2 (100-t)^2 dt = 0.105$$

- What is the probability that a person will die within 10 years after he has celebrated his 30th birthday?

$$\Pr\{t \le 30\} = \int_{0}^{30} \beta(t)dt = 3\times 10^{-9} \int_{0}^{30} t^2 (100-t)^2 dt = 0.1631$$

$$\Pr\{t > 30\} = 1 - \Pr\{t \le 30\} = 1 - 0.1631 = 0.8369$$

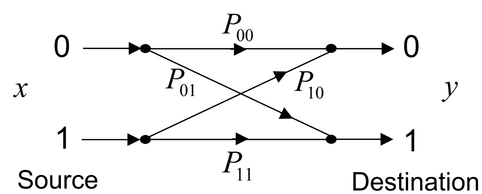
$$\Pr\{30 < t \le 40\} = \int_{30}^{40} \beta(t)dt = 3\times 10^{-9} \int_{30}^{40} t^2 (100-t)^2 dt = 0.164$$

$$\Pr\{30 < t \le 40 \mid 30 < t\} = \frac{\Pr\{30 < t \le 40\}}{\Pr\{30 < t\}} = \frac{0.164}{0.837} = 0.196$$



## Probability & Information Transmission

A binary channel




At the source,
$\Pr(x=0) = p$ and
$\Pr(x=1) = 1-p$,
$\Pr(x=0)$ and $\Pr(x=1)$
are called **a priori** probabilities.

The channel, being non-ideal (causing confusions), is characterized by the four conditional probabilities: $P_{00}, P_{01}, P_{10}$, and $P_{11}$

$P_{ij} = \Pr(y=j \mid x=i) =$ (probability of $j$ received at the destination, i.e., $y=j$, when $i$ was actually sent by the source, i.e., $x=i$)

**A posteriori probability** –
The probability that $i$ was sent at the source, given that $j$ is received at the destination.

$$\Pr(x=i \mid y=j) = \frac{\Pr(y=j \mid x=i)\Pr(x=i)}{\Pr(y=j)}$$

# Bayes Formula

Is used to find a posteriori probability

$$\Pr(x=i \mid y=j) = \frac{\Pr(y=j \mid x=i)\Pr(x=i)}{\Pr(y=j)}$$

More precisely,

$$\Pr(x=i \mid y=j) = \Pr(\text{symbol } i \text{ was sent, given that } j \text{ is received})$$

$$= \frac{\Pr(y=j \mid x=i)\Pr(x=i)}{\sum_{all\, i} \Pr(y=j \mid x=i)\Pr(x=i)}$$

$$= \frac{P_{ij}\Pr(x=i)}{\sum_{all\, i} P_{ij}\Pr(x=i)} = \frac{P_{ij}\Pr(x=i)}{\Pr(y=j)} =$$ the source portion of contributions that led to the reception of *j*



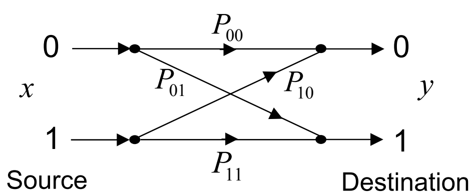
# Applications in Information Transmission

- When symbol 0 is sent, it has a probability of $P_{01}$ being received as symbol 1 – i.e., error occurs.
- When symbol 1 is sent, it has a probability of $P_{10}$ being received as symbol 0 – i.e., error occurs.
- What is the probability of error, regardless of the transmitted symbol? $pP_{01} + (1-p)P_{10}$
- If we want to make sure of the delivery of information by repeating the same symbol $n$ times, and our decision policy is to go with the majority of the $n$ received symbols, what would be the probability of error in information reception?



# Applications in Information Transmission

Repetitive transmission – For every source symbol, we send it three times, i.e., $n$ = 3




$x$=0 $P_{00} = P_{11} = 0.9$, $P_{01} = P_{10} = 0.1$

| | | | |
|---|---|---|---|
| 000 | $P_{00}P_{00}P_{00}$ | $y$ =0 | 0.729 |
| 001 | $P_{00}P_{00}P_{01}$ | 0 | 0.081 |
| 010 | $P_{00}P_{01}P_{00}$ | 0 | 0.081 |
| 100 | $P_{01}P_{00}P_{00}$ | 0 | 0.081 |
| 011 | $P_{00}P_{01}P_{01}$ | 1 | 0.009 |
| 101 | $P_{01}P_{00}P_{01}$ | 1 | 0.009 |
| 110 | $P_{01}P_{01}P_{00}$ | 1 | 0.009 |
| 111 | $P_{01}P_{01}P_{01}$ | 1 | 0.001 |

+ = 0.972 Probability of correct reception

+ = 0.028 Probability of incorrect reception

Same situation with $x$=1

The error probability is 0.028, compared to 0.1 with single transmission.

Efficiency of repetitive code is low, though. Other error correction code can have much higher performance.