A Network-based Column Generation Approach for Volumetric Modulated Arc Therapy

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Abstract

Background
- Radiation therapy (RT) is used in more than 50% of cancer treatment cases
- **Goal:** irradiating the tumor while protecting adjacent organs

Volumetric Modulated Arc Therapy
- **VMAT:** a fast RT modality with continuously rotating beam around patient
- Apertures change by a MLC to avoid irradiating critical organs

Purpose
- We model the clinical and deliverabilty constraints by a large-scale MIP and solve it

Method
- We develop a network-based column-generation algorithm which optimizes the intensity and shape of apertures.

Problem statement

Standard assumption
- We approximate the continuous path by discrete control points (CPs) and solve a DAO problem at each CP

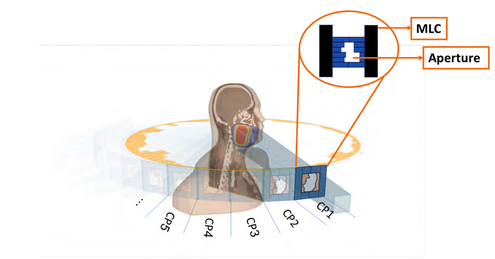
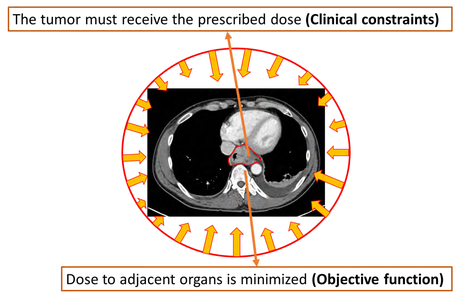


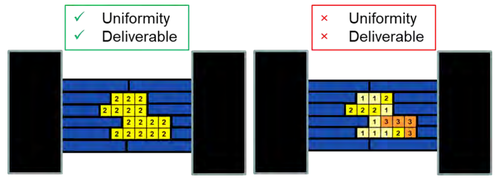
Image from www.rayresearchlab.com

Model



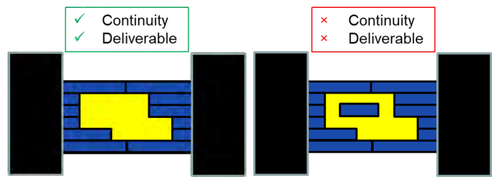
Deliverability constraints

1.Uniformity constraints: Apertures deliver uniform dose



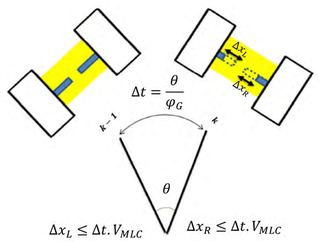
Uniform vs non-uniform non-deliverable aperture

2.Continuity constraints: Apertures are continuous



Continuous vs non-deliverable aperture with island

3.Leaf motion constraints: Within an arc θ, MLC leaves can move no more than Δx



Maximum leaf motion (MLM) constraints

Challenges

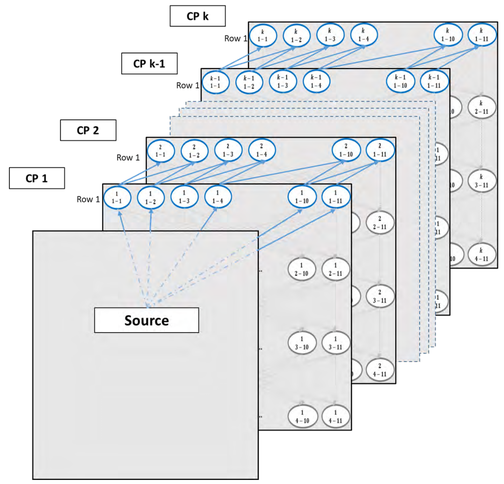
Advantages	Challenges
✓ Fast delivery	**Practical**
✓ Less patient discomfort	❖ Feasible solution (Not optimal)
✓ Less dispositioning error	❖ Limited usage
✓ Less waiting time	
✓ More machine availability	**Mathematical**
✓ Potential for higher quality	❖ Excessively large problem
	❖ MIP constraints

VMAT advantages and challenges

Column generation

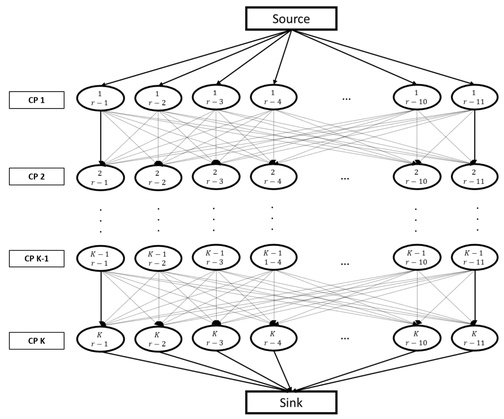
Master problem (MP)
- Optimizes the intensity over the available set of apertures
- A linear FMO problem

Pricing problem (PP)
- Optimizes the shape of apertures
- MIP PP is modeled by a network



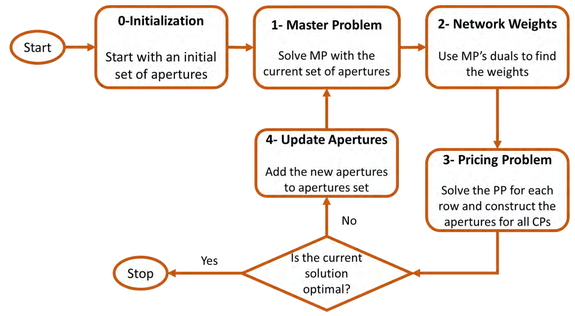
Modeling the MLC at each CP by a network

- Leaves in MLC can move independently.
- Position of each leaf pairs is dependent to that of the adjacent CP.
- A shortest path problem is solved for each MLC row.



The network for the first row of MLC

Algorithm



Results

A numerical example
- An example with 5 CPs, constant speed and dose rate
- For dose of 200 (Gy) and $\Delta x = 1$
- Optimal solution after 7 iterations

Iteration	Arc	Optimal Intensity (Gy)	Objective
0		118	1633
1		173	1106
...	...	...	...
7		215	745

CG iteration for the example

Conclusion and future works

Conclusion
- Our CG converges to optimal solution very fast
- At each iteration the hard Deliverability constraints in PP is modeled and solved by a network efficiently

Future works
- We incorporate uncertainty in our VMAT model using robust optimization
- We will incorporate more complicated row-dependent deliverability constraints and modify our network to solve PP