**GEORGIA INSTITUTE OF TECHNOLOGY**

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

# QUIZ #4

DATE: 22-April-11 COURSE: ECE-2025

NAME: ______________________ GT username: ______________
LAST, FIRST (ex: gpburdell3)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

| | | | |
|---|---|---|---|
| | L05:Tues-Noon (Stüber) | | L06:Thur-Noon (Bhatti) |
| | L07:Tues-1:30pm (Stüber) | | L08:Thur-1:30pm (Bhatti) |
| L01:M-3pm (McClellan) | L09:Tues-3pm (Lee) | L02:W-3pm (Chang) | L10:Thur-3pm (Madisetti) |
| L03:M-4:30pm (Lee) | L11:Tues-4:30pm (Lee) | L04:W-4:30pm (Chang) | |

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space .is. needed for scratch work, use the backs of previous pages.

| *Problem* | *Value* | *Score* |
|---|---|---|
| 1 | 35 | |
| 2 | 30 | |
| 3 | 15 | |
| 4 | 20 | |
| No/Wrong Rec | −3 | |

**PROBLEM s-11-Q.4.1:**

For each of the following expressions, reduce the expression to the simplest possible form.

*Provide justification or intermediate steps.* (The operator $*$ denotes convolution.)

(a) $\left\{e^{-5(t-1)}u(t-1)\right\} * \delta(t-5)$

(b) $\displaystyle\int_{-\infty}^{0} 5\delta(t-5)dt$

(c) $\displaystyle\left.\frac{\sin(5\omega)}{\omega/2}\right|_{\omega=0}$

(d) $\left\{e^{-5(t-1)}u(t-1)\right\} \delta(t-5)$

(e) $\left\{t^2\delta(t-5)\right\} * \delta(t-1)$

**PROBLEM s-11-Q.4.2:**

In each of the following cases, determine the (inverse or forward) Fourier transform. Give your answer as a plot, or a simple formula that is ***real-valued.***

**Explain** each answer (briefly) by stating which property and/or transform pair you used.

(a) Find $x(t)$ when $X(j\omega) = e^{-j\omega/1000}\{j\pi\delta(\omega - 400\pi) - j\pi\delta(\omega + 400\pi)\}$.

(b) Find $S(j\omega)$ when $s(t) = 1024\pi$.

(c) Find $r(t)$ when $R(j\omega) = \dfrac{12}{6 + j4\omega}$.

**PROBLEM s-11-Q.4.3:**

Convolution is often carried out graphically, or can be viewed graphically in a GUI like `cconvdemo`. Suppose that a signal $r(t)$ is known to be a rectangular pulse. In addition, the convolution of $r(t)$ with a unit-step signal is known to have the following form:

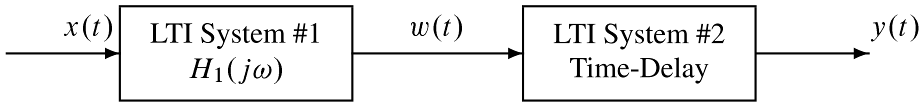
$$r(t) * u(t) = y(t) = 0.7(t-150)u(t-150) - 0.7(t-250)u(t-250)$$

(a) Make a plot of the signal $y(t)$ defined above.


(b) Determine the rectangular-pulse signal $r(t)$. ***Give your answer as a carefully labelled plot.*** *Justify your answer.*

**PROBLEM s-11-Q.4.4:**

A cascade of linear time-invariant systems is depicted by the following block diagram, where $x(t)$ is the input signal, and $y(t)$ is the output of the overall system.




$H_1(j\omega) = 12u(\omega+30) - 12u(\omega-30)$ $\qquad$ $y(t) = w(t-0.6)$

(a) Determine the impulse response of the ***overall system***. Express your answer in the ***simplest possible form.***

(b) If the input to the overall system is a sinusoid:

$$x(t) = 12\cos(4t)$$

Determine the output of the ***overall system***, $y(t)$. Give your answer in the ***simplest possible form.***

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**PROBLEM s-11-Q.4.1:**

For each of the following expressions, reduce the expression to the simplest possible form. *Provide justification or intermediate steps.* (The operator $*$ denotes convolution.)

(a) $\left\{e^{-5(t-1)}u(t-1)\right\} * \delta(t-5)$ **Answer** = $e^{-5(t-6)}u(t-6)$

(b) $\displaystyle\int_{-\infty}^{0} 5\delta(t-5)dt$ **Answer** = $0$

(c) $\left.\dfrac{\sin(5\omega)}{\omega/2}\right|_{\omega=0}$ **Answer** = $10$

(d) $\left\{e^{-5(t-1)}u(t-1)\right\}\delta(t-5)$ **Answer** = $e^{-5(4)}\delta(t-5)$

(e) $\left\{t^2\delta(t-5)\right\} * \delta(t-1)$ **Answer** = $25\delta(t-6)$

**PROBLEM s-11-Q.4.2:**

In each of the following cases, determine the (inverse or forward) Fourier transform. Give your answer as a plot, or a simple formula that is ***real-valued.***

**Explain** each answer (briefly) by stating which property and/or transform pair you used.

(a) Find $x(t)$ when $X(j\omega) = e^{-j\omega/1000}\{j\pi\delta(\omega-400\pi) - j\pi\delta(\omega+400\pi)\}$.

$$x(t) = -\sin(400\pi(t-1/1000)) = -\sin(400\pi t - 0.4\pi) = \cos(100\pi t + 0.3146)$$

(b) Find $S(j\omega)$ when $s(t) = 1024\pi$.

$$S(j\omega) = 2048\pi^2\delta(\omega)$$

(c) Find $r(t)$ when $R(j\omega) = \dfrac{12}{6+j4\omega}$.

$$r(t) = 3e^{-1.5t}u(t)$$

**PROBLEM s-11-Q.4.3:**

Convolution is often carried out graphically, or can be viewed graphically in a GUI like `cconvdemo`. Suppose that a signal $r(t)$ is known to be a rectangular pulse. In addition, the convolution of $r(t)$ with a unit-step signal is known to have the following form:

$$r(t) * u(t) = y(t) = 0.7(t-150)u(t-150) - 0.7(t-250)u(t-250)$$

(a) Make a plot of the signal $y(t)$ defined above.

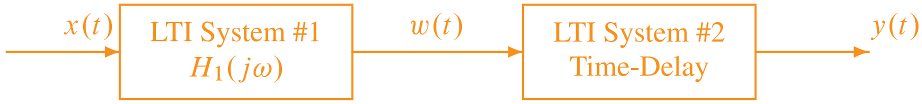
Plot is zero for $t \leq 150$, changes linearly to $y(250) = 70$ at $t = 250$, and then equals 70 for $t > 250$.


(b) Determine the rectangular-pulse signal $r(t)$. ***Give your answer as a carefully labelled plot.*** *Justify your answer.*

$r(t) = 0.7u(t-150) - 0.7u(t-250)$ from which you can make a plot.

**PROBLEM s-11-Q.4.4:**

A cascade of linear time-invariant systems is depicted by the following block diagram, where $x(t)$ is the input signal, and $y(t)$ is the output of the overall system.




$H_1(j\omega) = 12u(\omega+30) - 12u(\omega-30)$ $\qquad$ $y(t) = w(t-0.6)$

(a) Determine the impulse response of the ***overall system***. Express your answer in the ***simplest possible form.***

The inverse transform of $H_1(j\omega)$ is a "sinc" function; the second system delays the output of the first system:

$$h(t) = 12\,\frac{\sin(30(t-0.6))}{\pi(t-0.6)}$$

(b) If the input to the overall system is a sinusoid:

$$x(t) = 12\cos(4t)$$

Determine the output of the ***overall system***, $y(t)$. Give your answer in the ***simplest possible form.***

The first system is a magnitude change by a factor of 12; the second system is a time delay which causes a phase change. Multiply the magnitudes and add the phases:

$$y(t) = 144\cos(4t - 2.4)$$