GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3

DATE: 18-Mar-11 COURSE: ECE-2025

NAME: ______________________________ GT username: ______________

LAST, FIRST (ex: gpburdell8)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Stüber)		L06:Thur-Noon (Bhatti)
	L07:Tues-1:30pm (Stüber)		L08:Thur-1:30pm (Bhatti)
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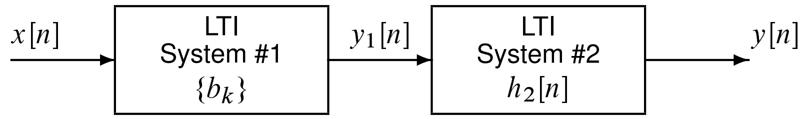
L03:M-4:30pm (Lee)	L11:Tues-4:30pm (Lee)	L04:W-4:30pm (Chang)	

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning CLEARLY to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	30	
2	30	
3	40	
No/Wrong Rec	−3	

PROBLEM Spring-11-Q.3.1:

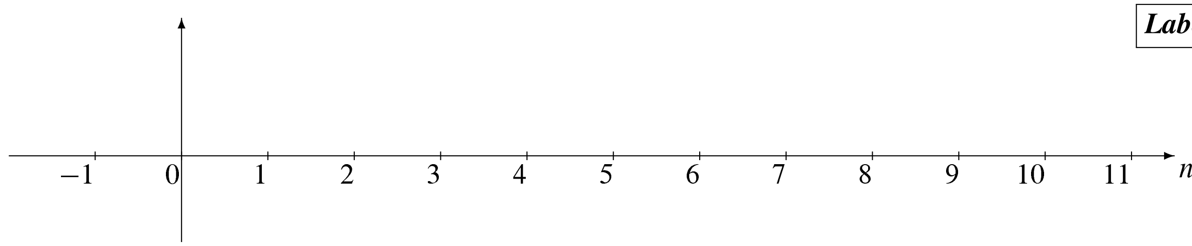
The diagram in the figure below depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.



Suppose that System #1 is an FIR filter whose filter coefficients are $\{b_k\} = \{1, -2, 0, 0, 4\}$ and System #2 is described by the impulse response $h_2[n] = \delta[n-1] + 2\delta[n-2]$.

(a) Determine the impulse response sequence, $h_1[n]$, of the first system. Give your answer as a ***stem plot.***

Label Carefully

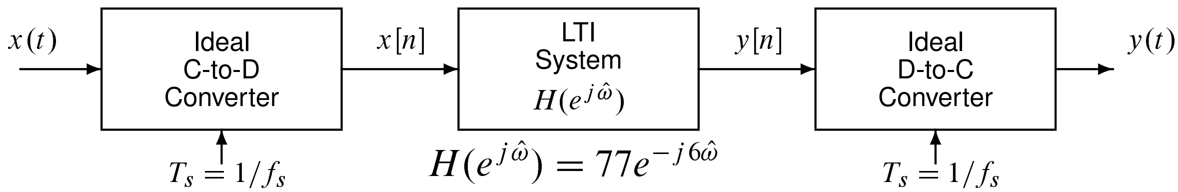


(b) Determine the impulse response of the overall system; give your answer as a ***sum of shifted deltas.***

(c) If the input signal is a DC signal, $x[n] = 100$ (for all n), determine both output signals, $y_1[n]$ and $y[n]$, which are also DC signals, i.e., constants. *Hint: use convolution.*

PROBLEM Spring-11-Q.3.2:

Consider the following system for discrete-time filtering of a continuous-time signal:



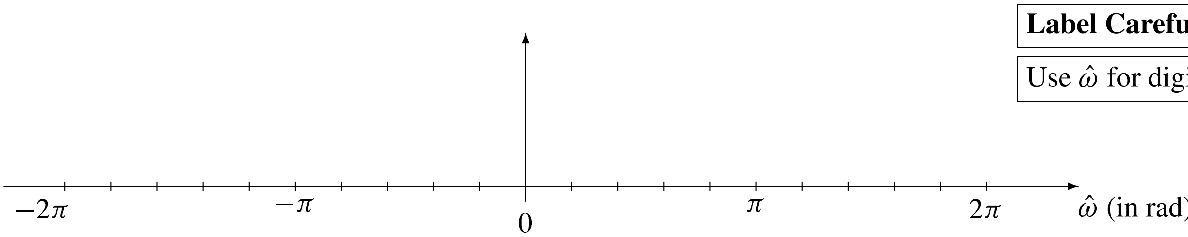
For all parts below, the input to the ideal C-to-D converter is $x(t) = 2\cos(6000\pi t)$.

(a) For the discrete-time LTI system, determine the difference equation relating $y[n]$ to $x[n]$.

(b) If the sampling rate of the C-to-D converter is $f_s = 10000$ samples/sec, make a plot of the spectrum of the discrete-time signal $x[n]$ over the interval $-2\pi \le \hat{\omega} \le 2\pi$, ***which is a larger range than normal.*** Make sure to show **all spectrum lines** and label the frequency, amplitude and phase of each spectral component.

Label Carefully

Use $\hat{\omega}$ for digital freq.



(c) If the sampling rate of the ideal D-to-C converter is $f_s = 10000$ samples/sec, determine the mathematical formula for the continuous-time output signal, $y(t)$. Use $x(t)$ defined above.

PROBLEM Spring-11-Q.3.3:

(a) The frequency response of an LTI system is: $H(e^{j\hat{\omega}}) = 100e^{-j\hat{\omega}} + 100e^{-j4\hat{\omega}}$.
When the input to this system is $x[n] = \cos(2.7n)$, the frequency response must be evaluated at $\hat{\omega} = 2.7$. Determine the numerical value for $H(e^{j2.7})$ as a complex number in *polar form.*

$H(e^{j2.7}) =$

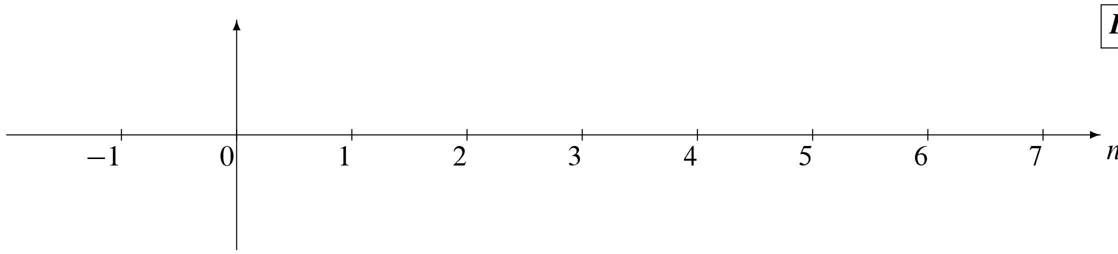
(b) The frequency response of an FIR filter is given by the following "Dirichlet-like" form:

$$H(e^{j\hat{\omega}}) = 30\pi \left(\frac{\sin(5\hat{\omega})}{\sin(0.5\hat{\omega})} \right) e^{-j4.5\hat{\omega}}$$

In a MATLAB implementation, the FIR filter would be implemented as **`yn = conv(bk,xn)`**, where **`xn`** and **`yn`** correspond to $x[n]$ and $y[n]$. Write a correct MATLAB statement to define the vector **`bk`**.

(c) Make a ***stem plot*** of the signal $s[n] = -40u[n-2] + 40u[n-6]$ for $-1 \leq n \leq 7$, where $u[n]$ is the unit-step signal.

Label all points



(d) Suppose that $\mathcal{S}$ is a linear, time-invariant system whose exact form is unknown. It can be tested by running some inputs into the system, and then observing the output signals. When the ***input signal*** is a ***unit-step*** signal, $x[n] = u[n]$, the observed ***output signal*** is $y[n] = 100\delta[n-2]$. Use this test case to determine the impulse response, $h[n]$, of the system. Write $h[n]$ as a sum of shifted deltas.

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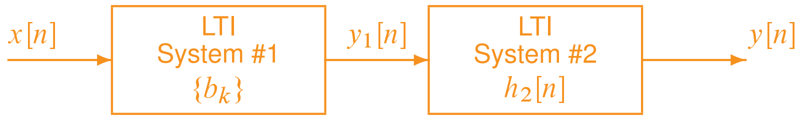
PROBLEM Spring-11-Q.3.1:

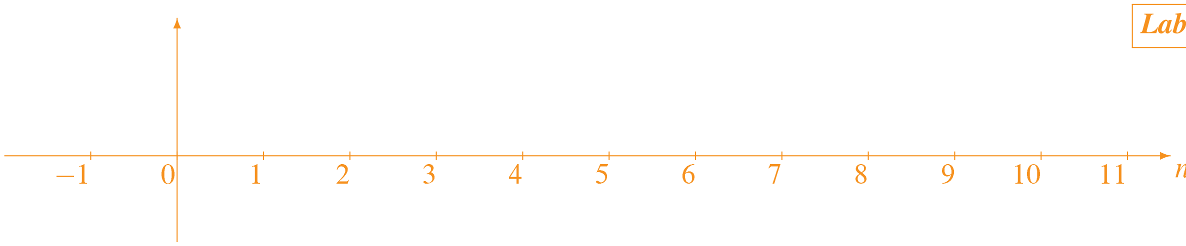
The diagram in the figure below depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.



Suppose that System #1 is an FIR filter whose filter coefficients are $\{b_k\} = \{1, -2, 0, 0, 4\}$ and System #2 is described by the impulse response $\delta[n-1] + 2\delta[n-2]$.

(a) Determine the impulse response sequence, $h_1[n]$, of the first system. Give your answer as a ***stem plot.***

Label Carefully



Make a plot of $h_1[n] = \delta[n] - 2\delta[n-1] + 4\delta[n-4]$.

(b) Determine the impulse response of the overall system; give your answer as a ***sum of shifted deltas.***

Convolve: $(\delta[n] - 2\delta[n-1] + 4\delta[n-4]) * (\delta[n-1] + 2\delta[n-2])$ to get

$$h[n] = \delta[n-1] - 4\delta[n-3] + 4\delta[n-5] + 8\delta[n-6]$$

(c) If the input signal is a DC signal, $x[n] = 100$ (for all n), determine both output signals, $y_1[n]$ and $y[n]$, which are also DC signals, i.e., constants. *Hint: use convolution.*

Note: when convolving a shifted impulse with a constant signal you get a constant, i.e., $A\delta[n-n_d] * (C) = AC$. Furthermore, the shifted unit impulse $\delta[n]$ is the impulse response of a delay filter, and delaying a constant gives that same constant.

Convolve: $(\delta[n] - 2\delta[n-1] + 4\delta[n-4]) * (100)$ to get

$$y_1[n] = 1(100) - 2(100) + 4(100) = 300 \qquad \text{for all } n$$

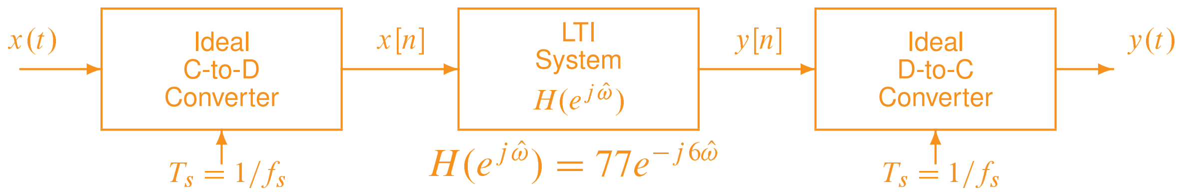
Note that the constant term (100) can be factored out when you convolve

$y[n] = (\delta[n-1] - 4\delta[n-3] + 4\delta[n-5] + 8\delta[n-6]) * (100)$, so

$$y[n] = (1 - 4 + 4 + 8)(100) = 900 \qquad \text{for all } n$$

PROBLEM Spring-11-Q.3.2:

Consider the following system for discrete-time filtering of a continuous-time signal:



For all parts below, the input to the ideal C-to-D converter is $x(t) = 2\cos(6000\pi t)$.

(a) For the discrete-time LTI system, determine the difference equation relating $y[n]$ to $x[n]$.

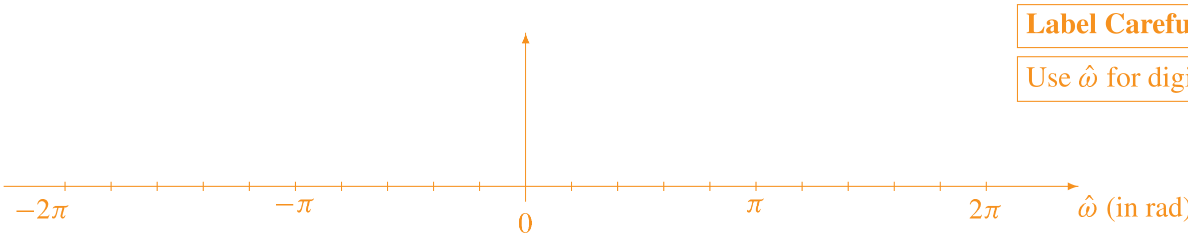
There is only one filter coefficient, because the frequency response has only one complex exponential.
The FIR filter is a pure delay: $y[n] = 77x[n-6]$.

(b) If the sampling rate of the C-to-D converter is $f_s = 10000$ samples/sec, make a plot of the spectrum of the discrete-time signal $x[n]$ over the interval $-2\pi \le \hat{\omega} \le 2\pi$, ***which is a larger range than normal.*** Make sure to show **all spectrum lines** and label the frequency, amplitude and phase of each spectral component.

The signal $x[n]$ has a discrete-time frequency of $\hat{\omega} = 6000\pi/10000 = 0.6\pi$, so there will be spectral lines at $\hat{\omega} = \pm 0.6\pi \pm 2\pi$.
In the figure, the spectrum lines will be at $\hat{\omega} = \{0.6\pi, -1.4\pi, -0.6\pi, 1.4\pi\}$. The complex amplitudes will all be $\frac{1}{2}(2) = 1e^{j0}$.

Label Carefully

Use $\hat{\omega}$ for digital freq.



(c) If the sampling rate of the ideal D-to-C converter is $f_s = 10000$ samples/sec, determine the mathematical formula for the continuous-time output signal, $y(t)$. Use $x(t)$ defined above.

The frequency response at $\hat{\omega} = 0.6\pi$ is

$$H(e^{j\hat{\omega}}) = 77e^{-j6\hat{\omega}}\Big|_{\hat{\omega}=0.6\pi} = 77e^{-j3.6\pi} = 3e^{+j0.4\pi}$$

Therefore, the output of the FIR filter is $y[n] = 154\cos(0.6\pi n + 0.4\pi)$, which is a discrete-time signal. Then, converting from $\hat{\omega}$ to the output frequency ($\omega_{\text{out}} = \hat{\omega} f_s$) gives

$$y(t) = 154\cos(6000\pi t + 0.4\pi)$$

PROBLEM Spring-11-Q.3.3:

(a) The frequency response of an LTI system is: $H(e^{j\hat{\omega}}) = 100e^{-j\hat{\omega}} + 100e^{-j4\hat{\omega}}$.
When the input to this system is $x[n] = \cos(2.7n)$, the frequency response must be evaluated at $\hat{\omega} = 2.7$. Determine the numerical value for $H(e^{j2.7})$ as a complex number in *polar form.*

$H(e^{j2.7}) = 123e^{j2.675} = 123e^{j0.851\pi}$ Evaluate $100e^{-j2.7} + 100e^{-j10.8}$

(b) The frequency response of an FIR filter is given by the following "Dirichlet-like" form:

$$H(e^{j\hat{\omega}}) = 30\pi \left(\frac{\sin(5\hat{\omega})}{\sin(0.5\hat{\omega})} \right) e^{-j4.5\hat{\omega}}$$

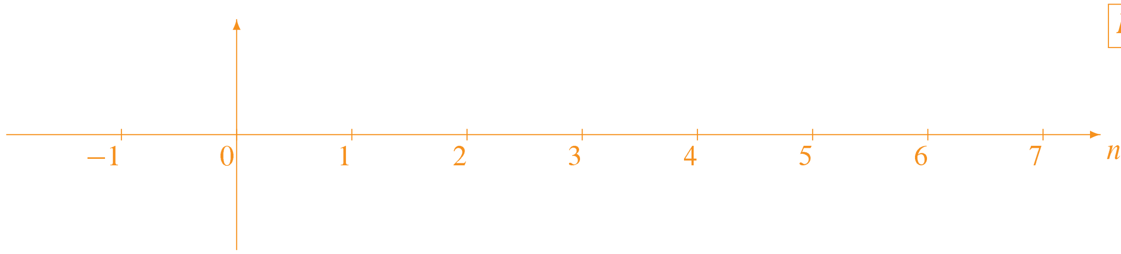
In a MATLAB implementation, the FIR filter would be implemented as **`yn = conv(bk,xn)`**, where **`xn`** and **`yn`** correspond to $x[n]$ and $y[n]$. Write a correct MATLAB statement to define the vector **`bk`**.

The Dirichlet form is the frequency response of a filter with L identical filter coefficients; in other words, a scaled running-sum filter. The length is $L = 10$. The scale is 30π.

```
bk = 30*pi*ones(1,10)
```

(c) Make a ***stem plot*** of the signal $s[n] = -40u[n-2] + 40u[n-6]$ for $-1 \leq n \leq 7$, where $u[n]$ is the unit-step signal.

Label all points



Make a plot of $s[n] = \begin{cases} 0 & n < 2 \\ -40 & 2 \leq n \leq 5 \\ 0 & n \geq 6 \end{cases}$

(d) Suppose that $\mathcal{S}$ is a linear, time-invariant system whose exact form is unknown. It can be tested by running some inputs into the system, and then observing the output signals. When the ***input signal*** is a ***unit-step*** signal, $x[n] = u[n]$, the observed ***output signal*** is $y[n] = 100\delta[n-2]$. Use this test case to determine the impulse response, $h[n]$, of the system. Write $h[n]$ as a sum of shifted deltas.

Exploit the properties of linearity and time-invariance to write the unit impulse in terms of shifts, adds and scales of the unit-step signal. For example, if a new input is formed as $x_c[n] = x[n] + cx[n-1]$, then the output will be $y_c[n] = y[n] + cy[n-1]$. In this problem, we can use $c = -1$ because it is true that $\delta[n] = u[n] - u[n-1]$.

Thus, the same operations would be performed on the output signal $y[n]$ to get the response from an impulse at the input: $h[n] = y[n] - y[n-1] = 100\delta[n-2] - 100\delta[n-3]$