

PROBLEM s-11-Q.1.1:

Evaluate the expressions below, where angles are given in radians and frequencies in rad/s. Give *numerical answers*; the magnitudes, r , or amplitudes, A , *must be nonnegative*; the angles, θ or φ , *must be in radians*, and lie between $-\pi$ and $+\pi$. Use a calculator; only the answers will be graded—no explanations necessary.

(a) Determine r and θ , such that $re^{j\theta} = (-1 + j2)e^{j3}$.

$r =$	$\theta =$
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(b) Determine r and θ , such that $re^{j\theta} = 777e^{j3} + 321e^{-j1.2}$.

$r =$	$\theta =$
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(c) Determine r and θ , such that $re^{j\theta} = \pi j$.

$r =$	$\theta =$
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(d) Determine r and θ , such that $re^{j\theta} = \frac{41}{-700 + j200}$.

$r =$	$\theta =$
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(e) Express this signal, $\Re\{(-209 - j234)e^{j250t}\}$, as a sinusoid, i.e., $A \cos(\omega_0 t + \varphi)$.

$A =$	$\varphi =$
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(f) Express this signal, $\Re\{\frac{d}{dt}e^{j111t}\}$, as a sinusoid, i.e., $A \cos(\omega_0 t + \varphi)$.

$A =$	$\varphi =$
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(g) Express this signal, $\Re\{240e^{j1.7}e^{j931t}\}$, as a sinusoid, i.e., $A \cos(\omega_0 t + \varphi)$.

$A =$	$\varphi =$
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(h) Express this signal, $222 \cos(\pi t + 1.4) + 888 \cos(\pi t + 2.8)$, as a sinusoid, i.e., $A \cos(\omega_0 t + \varphi)$.

$A =$	$\varphi =$	$\omega_0 =$
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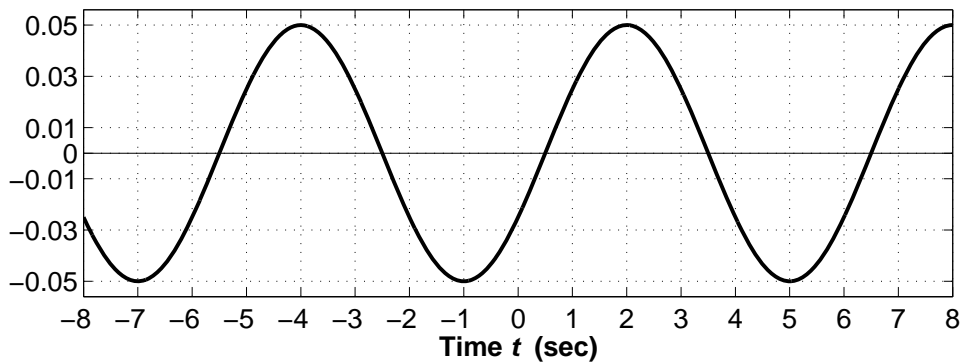
PROBLEM s-11-Q.1.2:

- (a) Evaluate this definite integral, and express the answer in polar form: $\int_{100}^{300} e^{j0.005\pi t} dt = r e^{j\theta}$
- $r =$ $\theta =$

- (b) Find a complex-valued signal $z_1(t) = (A e^{j\varphi}) e^{j\omega t}$ such that $\Re\{\frac{d}{dt} z_1(t)\} = 9 \cos(0.1\pi(t-2))$.
- $A =$ $\varphi =$ $\omega =$

- (c) Values of the sinusoid shown below can be generated via the following MATLAB statements:

```
tt = -8:0.01:8; XX = ??; ww = ??; xt = real( XX * exp(j*ww*tt) );
```



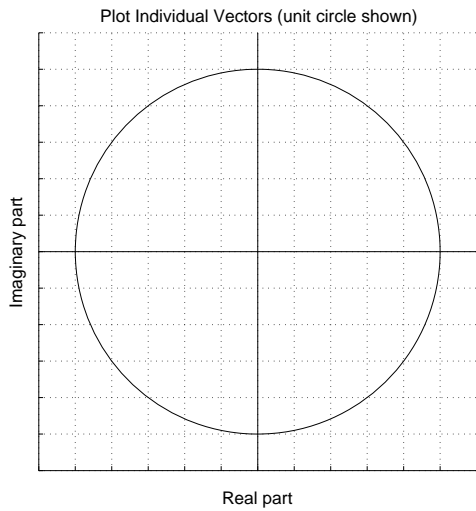
Write the appropriate MATLAB statements needed to define XX and ww.

XX= _____

ww= _____

PROBLEM s-11-Q.1.3:

- (a) For the following sum: $\sum_{k=0}^3 e^{j2\pi(k+3.5)/10}$ make a plot of the individual vectors that represent the complex exponentials being added together. Label each vector with the corresponding value of the index k . It is not necessary to actually find the sum.



- (b) Recall that adding N consecutive complex exponentials whose phases differ by $2\pi/N$ will give a sum equal to zero, e.g., $\sum_{k=1}^N e^{j2\pi k/N} = 0$. The MATLAB code below adds many sinusoids whose phases differ by $2\pi/N$. The plot made from the vector `xx` is a single sinusoid, i.e., $A \cos(\omega_0 t + \varphi)$.

```
tt = 0:0.001:1;
xx = 0*tt;
for kk=2:82
    xx = xx + 1.5*cos(120*pi*tt + 0.25*pi*kk);
end
plot(tt,xx), title('SECTION of a SINUSOID'), xlabel('TIME (sec)')
```

Determine the parameters for the sinusoid in the vector `xx`. Also, identify the value of N , as well as the number of sinusoids being added, N_s .

$N_s =$	$N =$	$A =$	$\varphi =$	$\omega_0 =$
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PROBLEM s-11-Q.1.1:

Evaluate the expressions below, where angles are given in radians and frequencies in rad/s. Give *numerical answers*; the magnitudes, r , or amplitudes, A , *must be nonnegative*; the angles, θ or φ , *must be in radians*, and lie between $-\pi$ and $+\pi$. Use a calculator; only the answers will be graded—no explanations necessary.

(a) Determine r and θ , such that $re^{j\theta} = (-1 + j2)e^{j3}$.

$r = 2.236$ $\theta = -1.249$ rads

(b) Determine r and θ , such that $re^{j\theta} = 777e^{j3} + 321e^{-j1.2}$.

$r = 679.9$ $\theta = -2.859$ rads

(c) Determine r and θ , such that $re^{j\theta} = \pi j$.

$r = 3.14159$ $\theta = \pi/2$ rads

(d) Determine r and θ , such that $re^{j\theta} = \frac{41}{-700 + j200}$.

$r = 0.05632$ $\theta = -2.863$ rads

(e) Express this signal, $\Re\{(-209 - j234)e^{j250t}\}$, as a sinusoid, i.e., $A \cos(\omega_0 t + \varphi)$.

$A = 313.7$ $\varphi = -2.300$ rads

(f) Express this signal, $\Re\{\frac{d}{dt}e^{j111t}\}$, as a sinusoid, i.e., $A \cos(\omega_0 t + \varphi)$.

$A = 111$ $\varphi = \pi/2$ rads

(g) Express this signal, $\Re\{240e^{j1.7}e^{j931t}\}$, as a sinusoid, i.e., $A \cos(\omega_0 t + \varphi)$.

$A = 240$ $\varphi = +1.7$ rads

(h) Express this signal, $222 \cos(\pi t + 1.4) + 888 \cos(\pi t + 2.8)$, as a sinusoid, i.e., $A \cos(\omega_0 t + \varphi)$.

$A = 951.2$ $\varphi = 2.568$ rads $\omega_0 = \pi$ rad/s

PROBLEM s-11-Q.1.2:

- (a) Evaluate this definite integral, and express the answer in polar form: $\int_{100}^{300} e^{j0.005\pi t} dt = r e^{j\theta}$
- $$r e^{j\theta} = \frac{2}{0.005\pi} e^{\pm j\pi} = 127.324 e^{\pm j3.14159}$$

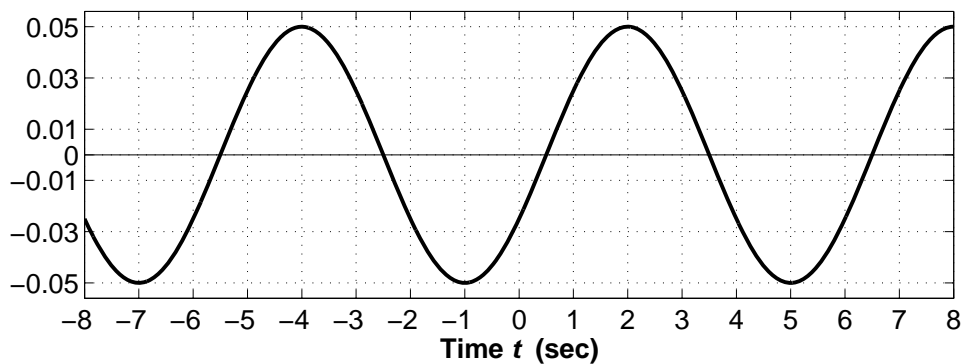
Approach: The integral of an exponential is an exponential, but you end up with a j in the denominator because the exponent contains a j . After evaluating at the limits of the definite integral, the numerator has a complex number. Finally, convert the complex numerator-denominator into polar form with a calculator.

- (b) Find a complex-valued signal $z_1(t) = (A e^{j\varphi}) e^{j\omega t}$ such that $\Re\{\frac{d}{dt} z_1(t)\} = 9 \cos(0.1\pi(t-2))$.
- $$z_1(t) = (A e^{j\varphi}) e^{j\omega t} = 28.648 e^{-j0.7\pi} e^{j0.1\pi t}$$

Approach: The derivative of $z_1(t)$ is the exponential multiplied by $j\omega$. Thus, we must match $(j\omega A e^{j\varphi}) e^{j\omega t} = (\omega A) e^{j(\varphi+\pi/2)} e^{j\omega t}$ with the parameters of the sinusoid. The amplitude is $A = 9/0.1\pi = 28.648$, and the phase $\varphi = 0.1\pi(-2) - \frac{1}{2}\pi = -0.7\pi$ rads.

- (c) Values of the sinusoid shown below can be generated via the following MATLAB statements:

```
tt = -8:0.01:8; XX = ??; ww = ??; xt = real( XX * exp(j*ww*tt) );
```



Write the appropriate MATLAB statements needed to define XX and ww.

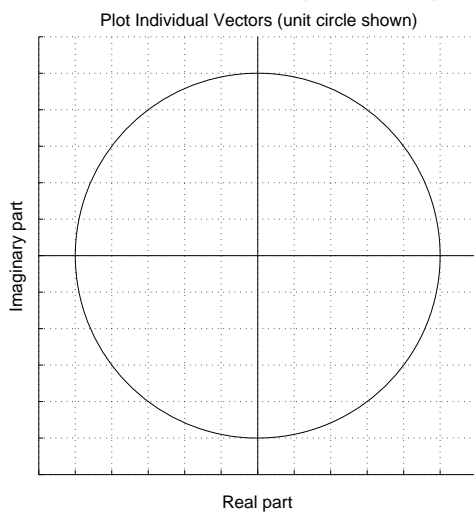
```
XX = 0.05*exp(-j*2*pi/3)
```

```
ww = 2*pi/6
```

Approach: Measure the period to obtain $T = 6$ s, and measure the location of a positive peak, $t_m = 2$ s. Measure the amplitude, A , from the height of a positive peak. Calculate the frequency (in rad/s) via $\omega = 2\pi/T = 2\pi/6$, and then the phase (in rads) via $\varphi = -\omega t_m = -2\pi(2)/6$. Finally, use A and φ to define XX from the complex amplitude $A e^{j\varphi}$.

PROBLEM s-11-Q.1.3:

- (a) For the following sum: $\sum_{k=0}^3 e^{j2\pi(k+3.5)/10}$ make a plot of the individual vectors that represent the complex exponentials being added together. Label each vector with the corresponding value of the index k . It is not necessary to actually find the sum.



Approach: The length of all vectors is one. The exponents are angles. Convert from radians to degrees to make the plotting easy.

The four vectors are at angles:

$$2\pi(3.5)/10 = 7\pi/10 = +126^\circ,$$

$$2\pi(4.5)/10 = 9\pi/10 = 162^\circ,$$

$$2\pi(5.5)/10 = 11\pi/10 = -162^\circ,$$

$$2\pi(6.5)/10 = 13\pi/10 = -126^\circ.$$

- (b) Recall that adding N consecutive complex exponentials whose phases differ by $2\pi/N$ will give a sum equal to zero, e.g., $\sum_{k=1}^N e^{j2\pi k/N} = 0$. The MATLAB code below adds many sinusoids whose phases differ by $2\pi/N$. The plot made from the vector `xx` is a single sinusoid, i.e., $A \cos(\omega_0 t + \varphi)$.

```
tt = 0:0.001:1;
xx = 0*tt;
for kk=2:82
    xx = xx + 1.5*cos(120*pi*tt + 0.25*pi*kk);
end
plot(tt,xx), title('SECTION of a SINUSOID'), xlabel('TIME (sec)')
```

Determine the parameters for the sinusoid in the vector `xx`. Also, identify the value of N , as well as the number of sinusoids being added, N_s .

$$N_s = 81$$

$$N = 8$$

$$A = 1.5$$

$$\varphi = 0.5\pi \text{ rads}$$

$$\omega_0 = 120\pi \text{ rad/s}$$

Approach: The `for` loop adds 81 sinusoids, which can be done as the phasor addition of 81 complex amplitudes. The phases of the sinusoids are $2\pi k/8$, i.e., the angular difference between successive complex amplitudes is $2\pi/8$. The identity tells us that adding 8 successive complex exponentials will give zero, and we are adding 10 groups of 8, but we have one left over. That “left over” one is the complex amplitude of the answer; you can choose the first one, or the last one. Since the range of k is `2:82`, the first one is at $k = 2$, so it is $1.5e^{j2\pi(2)/8}$ which will be the complex amplitude $Ae^{j\varphi}$, giving A and φ for the surviving sinusoid.