GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 4-Feb-11 COURSE: ECE-2025

NAME: ______________________________ GT username: ____________________

LAST, FIRST (ex: gtbuzz3)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Stüber)		L06:Thur-Noon (Bhatti)
	L07:Tues-1:30pm (Stüber)		L08:Thur-1:30pm (Bhatti)
L01:M-3pm (McClellan)	L09:Tues-3pm (Lee)	L02:W-3pm (Chang)	L10:Thur-3pm (Madisetti)
L03:M-4:30pm (Lee)	L11:Tues-4:30pm (Lee)	L04:W-4:30pm (Chang)	

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also REQUIRED to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	50	
2	30	
3	20	
No/Wrong Rec	−3	

PROBLEM Spring-11-Q.1.1:

Evaluate the expressions below, where angles are given in radians and frequencies in rad/s. Give ***numerical answers***; the magnitudes, r, or amplitudes, A, ***must be nonnegative***; the angles, θ or φ, ***must be in radians***, and lie between $-\pi$ and $+\pi$. Use a calculator; only the answers will be graded—no explanations necessary.

(a) Determine r and θ, such that $re^{j\theta} = 87e^{-j2} + 31e^{j3}$.

$r =$	$\theta =$

(b) Determine r and θ, such that $re^{j\theta} = \dfrac{23}{-30 - j25}$.

$r =$	$\theta =$

(c) Determine r and θ, such that $re^{j\theta} = (-0.4 + j0.1)e^{j2}$.

$r =$	$\theta =$

(d) Determine r and θ, such that $re^{j\theta} = 0.123j$.

$r =$	$\theta =$

(e) Express this signal, $\Re\{47e^{j2.5}e^{j0.71t}\}$, as a sinusoid, i.e., $A\cos(\omega_0 t + \varphi)$.

$A =$	$\varphi =$

(f) Express this signal, $\Re\{(-35 + j23)e^{j44t}\}$, as a sinusoid, i.e., $A\cos(\omega_0 t + \varphi)$.

$A =$	$\varphi =$

(g) Express this signal, $\Re\{\frac{d}{dt}e^{j20t}\}$, as a sinusoid, i.e., $A\cos(\omega_0 t + \varphi)$.

$A =$	$\varphi =$

(h) Express this signal, $0.5\cos(100t - 1.5) + 0.6\cos(100t - 1.8)$, as a sinusoid, i.e., $A\cos(\omega_0 t + \varphi)$.

$A =$	$\varphi =$	$\omega_0 =$

PROBLEM Spring-11-Q.1.2:

(a) Evaluate this definite integral, and express the answer in polar form: $\int_0^{0.1} e^{-j25\pi t}\,dt = re^{j\theta}$

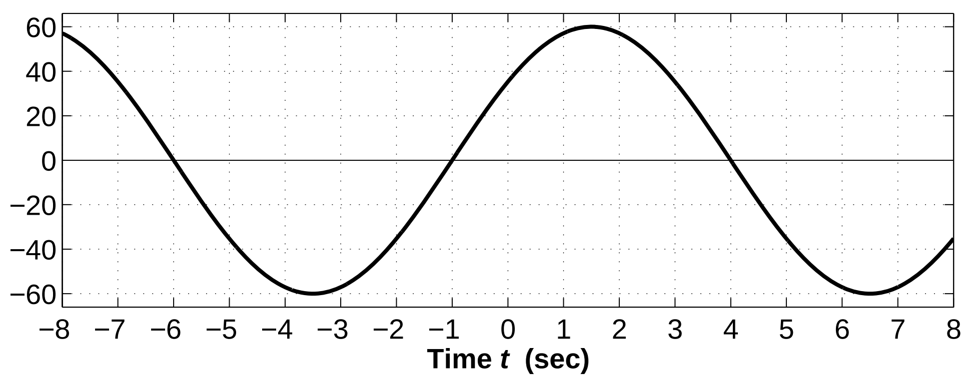
$r =$	$\theta =$

(b) Find a complex-valued signal $z_1(t) = (Ae^{j\varphi})e^{j\omega t}$ such that $\Re\{\frac{d}{dt}z_1(t)\} = 200\cos(40(t+0.05))$.

$A =$	$\varphi =$	$\omega =$

(c) Values of the sinusoid shown below can be generated via the following MATLAB statements:

```
tt = -8:0.01:8;  XX = ??;  ww = ??;  xt = real( XX * exp(j*ww*tt) );
```



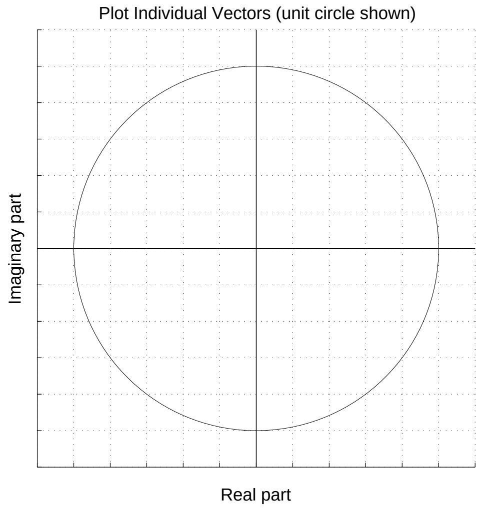
Write the appropriate MATLAB statements needed to define `XX` and `ww`.

`XX=` ____

`ww=` ____________________________

PROBLEM Spring-11-Q.1.3:

(a) For the following sum: $\sum_{k=0}^{3} e^{j2\pi(k+0.5)/6}$ make a plot of the individual vectors that represent the complex exponentials being added together. Label each vector with the corresponding value of the index k. It is not necessary to actually find the sum.



(b) Recall that adding N consecutive complex exponentials whose phases differ by $2\pi/N$ will give a sum equal to zero, e.g., $\sum_{k=1}^{N} e^{j2\pi k/N} = 0$. The MATLAB code below adds many sinusoids whose phases differ by $2\pi/N$. The plot made from the vector xx is a single sinusoid, i.e., $A\cos(\omega_0 t + \varphi)$.

```
tt = 0:0.001:1;
xx = 0*tt;
for kk=3:103
   xx = xx + 43*cos(55*pi*tt + 0.2*pi*kk);
end
plot(tt,xx), title('SECTION of a SINUSOID'), xlabel('TIME (sec)')
```

Determine the parameters for the sinusoid in the vector xx. Also, identify the value of N, as well as the number of sinusoids being added, N_s.

$N_s =$	$N =$	$A =$	$\varphi =$	$\omega_0 =$

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LAST, FIRST (ex: gtbuzz3)

3 points 3 points 3 points

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PROBLEM Spring-11-Q.1.1:

Evaluate the expressions below, where angles are given in radians and frequencies in rad/s. Give ***numerical answers***; the magnitudes, r, or amplitudes, A, ***must be nonnegative***; the angles, θ or φ, ***must be in radians***, and lie between $-\pi$ and $+\pi$. Use a calculator; only the answers will be graded—no explanations necessary.

(a) Determine r and θ, such that $re^{j\theta} = 87e^{-j2} + 31e^{j3}$.

$\boxed{r = 100.3}$ $\boxed{\theta = -2.301 \text{ rads}}$

(b) Determine r and θ, such that $re^{j\theta} = \dfrac{23}{-30 - j25}$.

$\boxed{r = 0.589}$ $\boxed{\theta = 2.447 \text{ rads}}$

(c) Determine r and θ, such that $re^{j\theta} = (-0.4 + j0.1)e^{j2}$.

$\boxed{r = 0.4123}$ $\boxed{\theta = -1.387 \text{ rads}}$

(d) Determine r and θ, such that $re^{j\theta} = 0.123j$.

$\boxed{r = 0.123}$ $\boxed{\theta = \pi/2 \text{ rads}}$

(e) Express this signal, $\Re\{47e^{j2.5}e^{j0.71t}\}$, as a sinusoid, i.e., $A\cos(\omega_0 t + \varphi)$.

$\boxed{A = 47}$ $\boxed{\varphi = +2.5 \text{ rads}}$

(f) Express this signal, $\Re\{(-35 + j23)e^{j44t}\}$, as a sinusoid, i.e., $A\cos(\omega_0 t + \varphi)$.

$\boxed{A = 41.88}$ $\boxed{\varphi = 2.560 \text{ rads}}$

(g) Express this signal, $\Re\{\frac{d}{dt}e^{j20t}\}$, as a sinusoid, i.e., $A\cos(\omega_0 t + \varphi)$.

$\boxed{A = 20}$ $\boxed{\varphi = \pi/2 \text{ rads}}$

(h) Express this signal, $0.5\cos(100t - 1.5) + 0.6\cos(100t - 1.8)$, as a sinusoid, i.e., $A\cos(\omega_0 t + \varphi)$.

$\boxed{A = 1.088}$ $\boxed{\varphi = -1.664 \text{ rads}}$ $\boxed{\omega_0 = 100 \text{ rad/s}}$

PROBLEM Spring-11-Q.1.2:

(a) Evaluate this definite integral, and express the answer in polar form: $\int_0^{0.1} e^{-j25\pi t}\,dt = re^{j\theta}$

$re^{j\theta} = \dfrac{\sqrt{2}}{25\pi}\, e^{-j\pi/4} = 0.0180\, e^{-j0.7854}$

Approach: The integral of an exponential is an exponential, but you end up with a j in the denominator because the exponent contains a j. After evaluating at the limits of the definite integral, the numerator has a complex number. Finally, convert the complex numerator-denominator into polar form with a calculator.

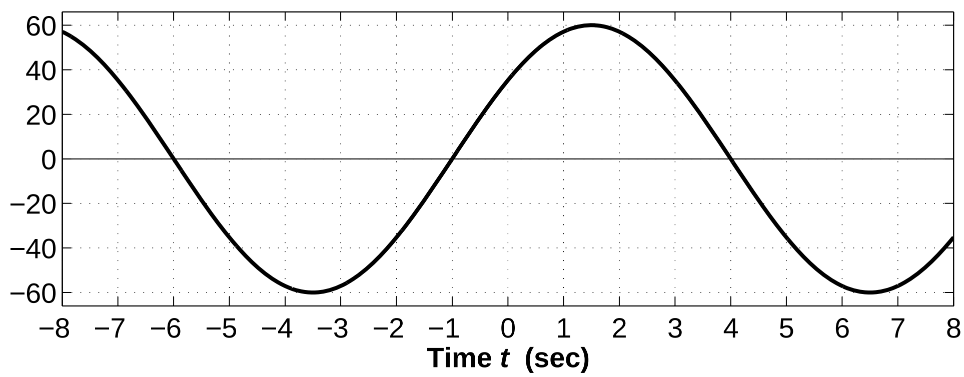
(b) Find a complex-valued signal $z_1(t) = (Ae^{j\varphi})e^{j\omega t}$ such that $\Re\{\frac{d}{dt}z_1(t)\} = 200\cos(40(t+0.05))$.

$z_1(t) = (Ae^{j\varphi})e^{j\omega t} = 5\, e^{j0.429}e^{j40t}$

Approach: The derivative of $z_1(t)$ is the exponential multiplied by $j\omega$. Thus, we must match $(j\omega Ae^{j\varphi})e^{j\omega t} = (\omega A)e^{j(\varphi+\pi/2)}e^{j\omega t}$ with the parameters of the sinusoid. The amplitude is $A = 200/40 = 5$, and the phase $\varphi = 40(+0.05) - \frac{1}{2}\pi = 2 - 0.5\pi = 0.429$ rads.

(c) Values of the sinusoid shown below can be generated via the following MATLAB statements:

```
tt = -8:0.01:8;   XX = ??;   ww = ??;   xt = real( XX * exp(j*ww*tt) );
```



Write the appropriate MATLAB statements needed to define `XX` and `ww`.

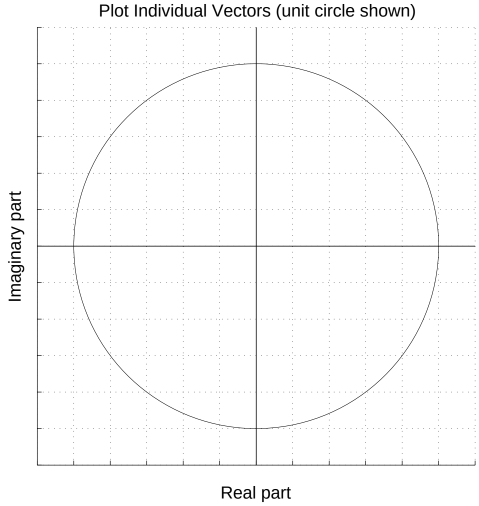
```
XX = 60*exp(-j*0.3*pi)

ww = 2*pi/10
```

Approach: Measure the period to obtain $T = 10$ s, and measure the location of a positive peak, $t_m = +1.5$ s. Measure the amplitude, A, from the height of a positive peak.
Calculate the frequency (in rad/s) via $\omega = 2\pi/T = 2\pi/10$, and then the phase (in rads) via $\varphi = -\omega t_m = -2\pi(+1.5)/10$.
Finally, use A and φ to define `XX` from the complex amplitude $Ae^{j\varphi}$.

PROBLEM Spring-11-Q.1.3:

(a) For the following sum: $\sum_{k=0}^{3} e^{j2\pi(k+0.5)/6}$ make a plot of the individual vectors that represent the complex exponentials being added together. Label each vector with the corresponding value of the index k. It is not necessary to actually find the sum.

Plot Individual Vectors (unit circle shown)



Approach: The length of all vectors is one. The exponents are angles. Convert from radians to degrees to make the plotting easy. The four vectors are at angles:

$$2\pi(0.5)/6 = \pi/6 = +30°,$$

$$2\pi(1.5)/6 = 3\pi/6 = 90°,$$

$$2\pi(2.5)/6 = 5\pi/6 = 150°,$$

$$2\pi(3.5)/6 = 7\pi/6 = -150°.$$

(b) Recall that adding N consecutive complex exponentials whose phases differ by $2\pi/N$ will give a sum equal to zero, e.g., $\sum_{k=1}^{N} e^{j2\pi k/N} = 0$. The MATLAB code below adds many sinusoids whose phases differ by $2\pi/N$. The plot made from the vector xx is a single sinusoid, i.e., $A\cos(\omega_0 t + \varphi)$.

```
tt = 0:0.001:1;
xx = 0*tt;
for kk=3:103
   xx = xx + 43*cos(55*pi*tt + 0.2*pi*kk);
end
plot(tt,xx), title('SECTION of a SINUSOID'), xlabel('TIME (sec)')
```

Determine the parameters for the sinusoid in the vector xx. Also, identify the value of N, as well as the number of sinusoids being added, N_s.

$N_s = 101$	$N = 10$	$A = 43$	$\varphi = 0.6\pi$ rads	$\omega_0 = 55\pi$ rad/s

Approach: The `for` loop adds 101 sinusoids, which can be done as the phasor addition of 101 complex amplitudes. The phases of the sinusoids are $2\pi k/10$, i.e., the angular difference between successive complex amplitudes is $2\pi/10$. The identity tells us that adding 10 successive complex exponentials will give zero, and we are adding 10 groups of 10, but we have one left over. That “left over” one is the complex amplitude of the answer; you can choose the first one, or the last one. Since the range of k is `3:103`, the first one is at $k = 3$, so it is $43e^{j2\pi(3)/10}$ which will be the complex amplitude $Ae^{j\varphi}$, giving A and φ for the surviving sinusoid.