GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #4

DATE: 23-April-10 COURSE: ECE-2025

NAME: ______________________________ GT username: ____________________

LAST, FIRST (ex: gpburdell8)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Michaels)	L06:Thur-Noon (Bhatti)
	L07:Tues-1:30pm (Michaels)	L08:Thur-1:30pm (Bhatti)
L01:M-3pm (Lee)	L09:Tues-3pm (Fekri)	
L03:M-4:30pm (Lee)	L11:Tues-4:30pm (Fekri)	

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	30	
2	30	
3	20	
4	20	
No/Wrong Rec	−3	

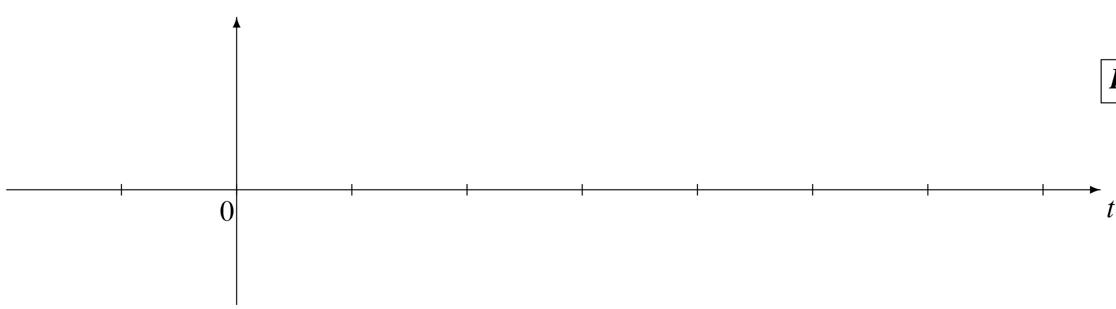
PROBLEM sp-10-Q.4.1:

In each of the following cases, use properties of the unit-impulse signal $\delta(t)$ to simplify the expression ***as much as possible.*** Provide some **explanation** or intermediate steps for each answer. *Note:* Star $*$ is the convolution operator.

(a) Simplify $x(t) = \left(\frac{d}{dt}\{t^2 u(t-\frac{1}{2})\}\right) * \delta(t-3)$

(b) Simplify $y(t) = \frac{\sin(8(t-10))}{\pi(t-10)}\,\delta(t-10)$

(c) Simplify $z(t) = \int_{1}^{5} 2\delta(\lambda - t)\,d\lambda$. Give your answer as a plot vs. t.



Label all points

PROBLEM sp-10-Q.4.2:

In each of the following cases, determine the (inverse or forward) Fourier transform. Give your answer as a plot, or a simple formula. ***Explain*** each answer (briefly) by stating which property and/or transform pair was used.

(a) Find $A(j\omega)$ when $a(t) = 2\pi u(t-3) - 2\pi u(t-6)$.

(b) Find $b(t)$ when $B(j\omega) = \left[\omega^2 + j\omega - 4\right] \delta(\omega)$. Simplify to get a real-valued answer.

(c) Find $c(t)$ when $C(j\omega) = \dfrac{8}{6 + j2\omega}$.

PROBLEM sp-10-Q.4.3:

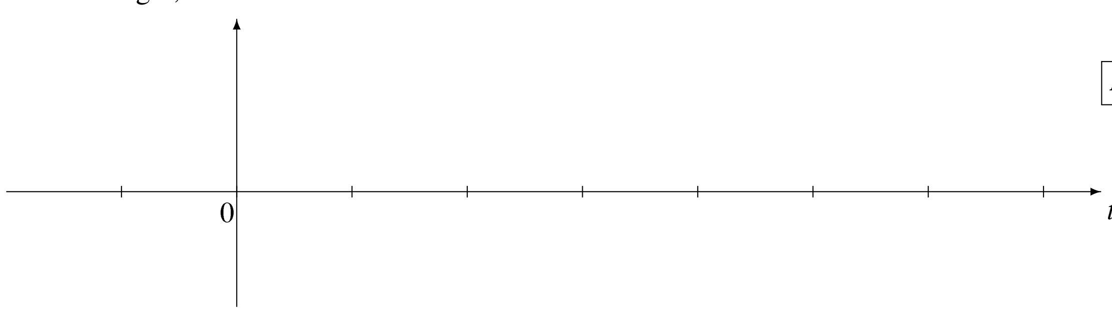
Two questions about convolution (denoted by the $*$ operator):

(a) In the following convolution the result $y(t)$ is a single sinusoid, $y(t) = A\cos(\omega t + \varphi)$. Determine A, φ and ω.

$$y(t) = \frac{9\sin(7\pi(t-0.05))}{\pi(t-0.05)} * [\cos(4\pi t) + \cos(8\pi t)]$$

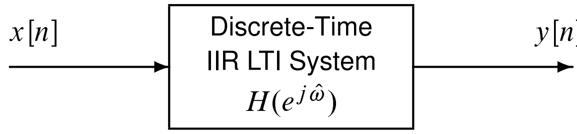
$A =$
$\varphi =$
$\omega =$

(b) Find the output of a LTI system whose impulse response is $h(t) = 3.3[u(t) - u(t-1)]$ when the input is a rectangular pulse signal, $x(t) = -2[u(t-1) - u(t-3)]$. Give the answer as a **plot**. Label the vertical height, as well as the horizontal axis.



Label all points

PROBLEM sp-10-Q.4.4:



(a) Lab #11 dealt with the design of two-pole IIR bandpass filters. The frequency axis for $H(e^{j\hat{\omega}})$ is $-\pi < \hat{\omega} \le \pi$. Suppose that the relationship between the 3-dB passband width and the pole radius is

$$\Delta\hat{\omega} = 2(1-r)$$

where $\Delta\hat{\omega}$ is the 3-dB passband width and r is the radius of the pole pair used to create the passband. Determine the poles and zeros of a second-order IIR filter whose frequency response meets the following specs: zero at $\hat{\omega} = \pi$ and at DC; passband over the region $0.5\pi \le \hat{\omega} \le 0.58\pi$.

POLES: $z =$

ZEROS: $z =$

(b) The system function of an IIR notch filter has the form:

$$H(z) = \frac{(1-e^{j1.5}z^{-1})(1-e^{-j1.5}z^{-1})}{(1-0.8e^{j1.5}z^{-1})(1-0.8e^{-j1.5}z^{-1})}$$

Write the appropriate MATLAB code to use the system defined by this $H(z)$ to filter a discrete-time signal $x[n]$ to get the output $y[n]$. Assume that the input signal $x[n]$ is contained in the MATLAB vector `xn`, and call the output vector `yn`. Numerical values will needed in all vectors that are used to set up the filtering in MATLAB. You can either give these numerical values explicitly, or write MATLAB statements that would generate them.

```
yn =
```

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #4

DATE: 23-April-10 COURSE: ECE-2025

NAME: **ANSWER KEY** GT username: **VERSION #1**

LAST, FIRST (ex: gpburdell8)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Michaels)	L06:Thur-Noon (Bhatti)
	L07:Tues-1:30pm (Michaels)	L08:Thur-1:30pm (Bhatti)
L01:M-3pm (Lee)	L09:Tues-3pm (Fekri)	
L03:M-4:30pm (Lee)	L11:Tues-4:30pm (Fekri)	

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page $(8\frac{1}{2}'' \times 11'')$ of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	30	
2	30	
3	20	
4	20	
No/Wrong Rec	−3	

PROBLEM sp-10-Q.4.1:

In each of the following cases, use properties of the unit-impulse signal $\delta(t)$ to simplify the expression ***as much as possible.*** Provide some **explanation** or intermediate steps for each answer. *Note:* Star $*$ is the convolution operator.

(a) Simplify $x(t) = \left(\frac{d}{dt}\left\{t^2 u(t-\tfrac{1}{2})\right\}\right) * \delta(t-3)$

Take the derivative and evaluate at the location of the impulse ($t = \frac{1}{2}$):

$$\frac{d}{dt}\left\{t^2 u(t-\tfrac{1}{2})\right\} = 2t\,u(t-\tfrac{1}{2}) + t^2\,\delta(t-\tfrac{1}{2})$$
$$= 2t\,u(t-\tfrac{1}{2}) + \frac{1}{4}\delta(t-\tfrac{1}{2})$$

Then convolve with the shifted delta which shifts the derivative result by 3.

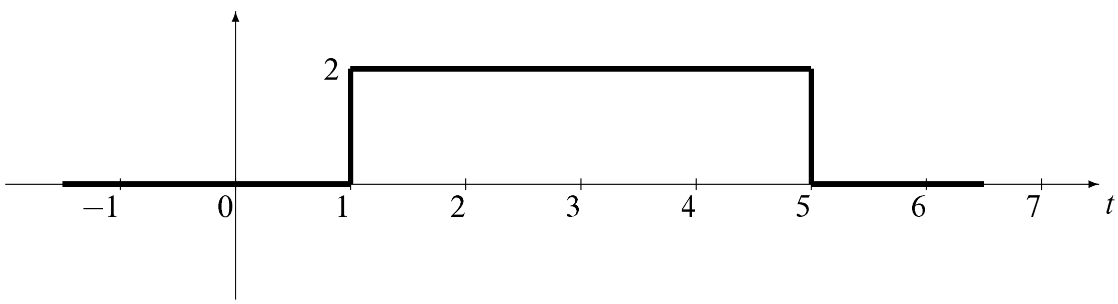
$$x(t) = 2(t-3)\,u(t-3.5) + \frac{1}{4}\delta(t-3.5)$$

(b) Simplify $y(t) = \frac{\sin(8(t-10))}{\pi(t-10)}\,\delta(t-10)$

Evaluate the sinc at $t = 10$

$$\left.\frac{\sin(8(t-10))}{\pi(t-10)}\right|_{t=10} = \frac{8}{\pi} \qquad \Longrightarrow y(t) = \frac{8}{\pi}\delta(t-10)$$

(c) Simplify $z(t) = \int_1^5 2\delta(\lambda - t)\,d\lambda$. Give your answer as a plot vs. t.



$$z(t) = \int_1^5 2\,\delta(\lambda - t)\,d\lambda = 2[u(5-t) - u(1-t)] = \begin{cases} 0 & t < 1 \\ 2 & 1 \le t \le 5 \\ 0 & t > 5 \end{cases}$$

PROBLEM sp-10-Q.4.2:

In each of the following cases, determine the (inverse or forward) Fourier transform. Give your answer as a plot, or a simple formula. ***Explain*** each answer (briefly) by stating which property and/or transform pair was used.

(a) Find $A(j\omega)$ when $a(t) = 2\pi u(t-3) - 2\pi u(t-6)$.

This is a shifted rectangle. The rectangle transforms to a sinc. The amount of shift is 4.5 which gives a complex exponential in the frequency domain.

$$A(j\omega) = 2\pi \frac{\sin(3\omega/2)}{\omega/2} e^{-j4.5\omega}$$

(b) Find $b(t)$ when $B(j\omega) = \left[\omega^2 + j\omega - 4\right] \delta(\omega)$. Simplify to get a real-valued answer.

Multiplication by the delta in the frequency domain evaluates at $\omega = 0$.

$$B(j\omega) = -4\delta(\omega) \qquad \Longrightarrow \qquad b(t) = \frac{-4}{2\pi} = \frac{-2}{\pi}$$

(c) Find $c(t)$ when $C(j\omega) = \dfrac{8}{6 + j2\omega}$.

Use the transform pair: $e^{-at}u(t)$ transforms to $\dfrac{1}{a + j\omega}$. So, we write $C(j\omega)$ as:

$$C(j\omega) = \frac{8}{6 + j2\omega} = \frac{4}{3 + j\omega}$$

Then we get the inverse transform:

$$c(t) = 4e^{-3t}u(t)$$

PROBLEM sp-10-Q.4.3:

Two questions about convolution (denoted by the $*$ operator):

(a) In the following convolution the result $y(t)$ is a single sinusoid, $y(t) = A\cos(\omega t + \varphi)$. Determine A, φ and ω.

$A =$

$\varphi =$

$\omega =$

$$y(t) = \frac{9\sin(7\pi(t-0.05))}{\pi(t-0.05)} * [\cos(4\pi t) + \cos(8\pi t)]$$

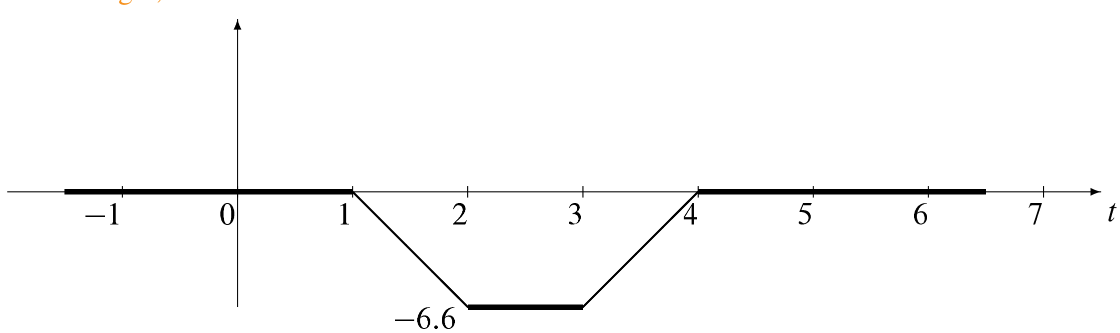
The Fourier Transform of the shifted sinc is a rectangle multiplied by a complex exponential. If we called the shifted sinc the impulse response, $h(t)$, then we get the following frequency response:

$$H(j\omega) = 9[u(\omega + 7\pi) - u(\omega - 7\pi)]e^{-j0.05\omega}$$

This filter is an ideal lowpass. The cosine at $\omega = 4\pi$ rad/s lies within the passband, the other cosine is in the stopband. As a result, the value of the frequency response $H(j4\pi)$ determines the amplitude and phase of the output sinusoid:

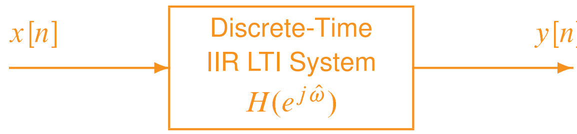
$$y(t) = 9\cos(4\pi t - 0.2\pi)$$

(b) Find the output of a LTI system whose impulse response is $h(t) = 3.3[u(t) - u(t-1)]$ when the input is a rectangular pulse signal, $x(t) = -2[u(t-1) - u(t-3)]$. Give the answer as a **plot**. Label the vertical height, as well as the horizontal axis.



The convolution of two rectangular pulses will give an output that is shaped like a trapezoid. The output starts at $t = 1$ and ends at $t = 4$. The flat top extends from $t = 2$ to $t = 3$, and has a value of -6.6 which is the area of the shorter pulse times the amplitude of the longer one.

PROBLEM sp-10-Q.4.4:



(a) Lab #11 dealt with the design of two-pole IIR bandpass filters. The frequency axis for $H(e^{j\hat{\omega}})$ is $\pi < \hat{\omega} \leq \pi$. Suppose that the relationship between the 3-dB passband width and the pole radius is

$$\Delta\hat{\omega} = 2(1-r)$$

where $\Delta\hat{\omega}$ is the 3-dB passband width and r is the radius of the pole pair used to create the passband. Determine the poles and zeros of a second-order IIR filter whose frequency response meets the following specs: zero at $\hat{\omega} = \pi$ and at DC; passband over the region $0.5\pi \leq \hat{\omega} \leq 0.58\pi$.

From the given passband width, the pole radius can be computed via:

$$r = 1 - \frac{\Delta\hat{\omega}}{2} = 1 - \frac{(0.58\pi - 0.5\pi)}{2} = 0.874$$

The angle of the two poles is determined by the center frequency of the passband.

$$p_1 = 0.874e^{j0.54\pi} \qquad \text{and} \qquad p_2 = 0.874e^{-j0.54\pi}$$

For zeros at DC, i.e., $\hat{\omega} = 0$, and $\hat{\omega} = \pi$, we get

$$z_1 = e^{j0} = 1 \qquad \text{and} \qquad z_2 = e^{j\pi} = -1$$

(b) The system function of an IIR notch filter has the form:

$$H(z) = \frac{(1-e^{j1.5}z^{-1})(1-e^{-j1.5}z^{-1})}{(1-0.8e^{j1.5}z^{-1})(1-0.8e^{-j1.5}z^{-1})}$$

Write the appropriate MATLAB code to use the system defined by this $H(z)$ to filter a discrete-time signal $x[n]$ to get the output $y[n]$. Assume that the input signal $x[n]$ is contained in the MATLAB vector `xn`, and call the output vector `yn`. Numerical values will needed in all vectors that are used to set up the filtering in MATLAB. You can either give these numerical values explicitly, or write MATLAB statements that would generate them.

We need the filter coefficients for the numerator and denominator, so the first-order factors must be multiplied out. This can be done with MATLAB's `poly` function, so one possible MATLAB program is

```
bb = poly([exp(j*1.5), exp(-j*1.5)]);
aa = poly([0.8*exp(j*1.5), 0.8*exp(-j*1.5)]);
yn = filter(bb, aa, xn);
```

If you compute the numerical values via algebra, then the system function is

$$H(z) = \frac{1 - 0.1415z^{-1} + z^{-2}}{1 - 0.1132z^{-1} + 0.64z^{-2}}$$

and the MATLAB program would be

```
yn = filter([1, -0.1415, 1], [1, -0.1132, 0.64], xn);
```