GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 22-Feb-10 COURSE: ECE-2025

NAME: ______________________________ GT username: ______________
LAST, FIRST (ex: gpburdell3)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Michaels)	L06:Thur-Noon (Bhatti)
	L07:Tues-1:30pm (Michaels)	L08:Thur-1:30pm (Bhatti)
L01:M-3pm (Lee)	L09:Tues-3pm (Fekri)	
L03:M-4:30pm (Lee)	L11:Tues-4:30pm (Fekri)	

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also REQUIRED to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	30	
2	30	
3	40	
No/Wrong Rec	-3	

PROBLEM sp-10-Q.2.1:

Shown below are spectrograms (labeled as **S1** – **S6**) for six signals. The (vertical) frequency axis for each plot has units of Hz; the horizontal axis is time, $0 \le t \le 14$ s. For each signal description below, identify the corresponding spectrogram. ***Write each answer in the box provided.***

(a) ☐ $x(t) = \cos(50\pi t + 2\cos(10\pi t))$

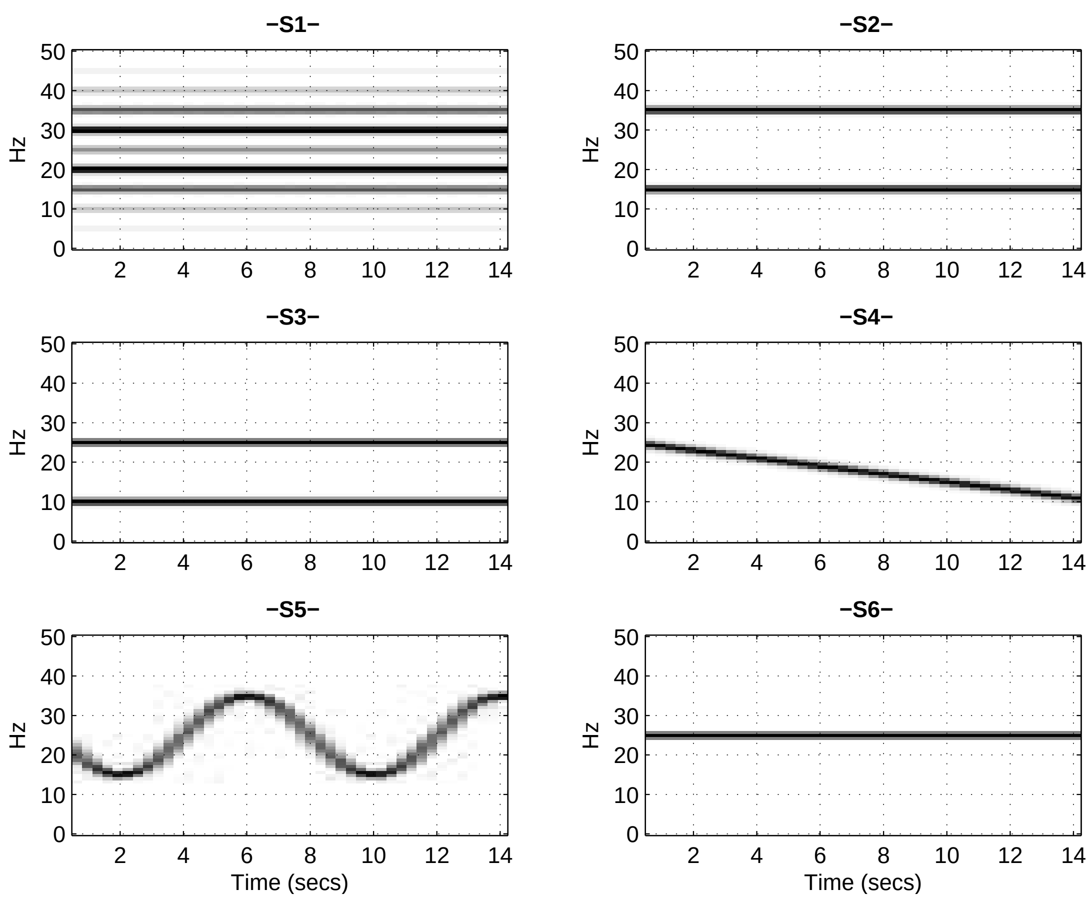
(b) ☐ $x(t) = \cos(50\pi t - 1) + \cos(50\pi t - 2)$

(c) ☐ $x(t) = \cos(50\pi t - \pi t^2)$

(d) ☐ $x(t) = \cos(50\pi t + 80\cos(0.25\pi t))$

(e) ☐ $x(t) = \cos(20\pi t) + \cos(50\pi t)$

(f) ☐ $x(t) = \cos(20\pi t)\cos(50\pi t)$



PROBLEM sp-10-Q.2.2:

The following MATLAB code defines vectors xt and zt, which correspond to the signals $x(t)$ and $z(t)$.

```
tt = -1000:0.00001:1000;  %- in seconds
xt = zeros(size(tt));
for k = [-3,-1,0,1,3]    %- loop on k for only these indices
   xt = xt + (j*k - 3)*exp(j*8*pi*k*tt);
end
zt = xt - 5 + 6*cos(24*pi*tt - pi/2);
```

(a) Determine the fundamental **period** of the signal $x(t)$ defined in the MATLAB code above.

$T_0 =$ secs.

(b) The signal $x(t)$ has a Fourier Series with coefficients $\{a_k\}$, defined by the MATLAB code. Fill in the table below with the numerical values of the $\{a_k\}$ coefficients in *polar form*.

(c) The new signal defined for $z(t)$ is also periodic with a Fourier Series, so it can be expressed in the following Fourier Series with new coefficients $\{b_k\}$:

$$z(t) = \sum_{k=-3}^{3} b_k e^{j8\pi kt}$$

In the following tables, enter the *numerical values* for each $\{a_k\}$ and $\{b_k\}$ in polar form.
Note: A magnitude value must be nonnegative; angles must be in radians.

Signal: $x(t)$

a_k	Mag≥ 0	Phase, $-\pi < \varphi_k \leq \pi$
a_3		
a_2		
a_1		
a_0		
a_{-1}		
a_{-2}		
a_{-3}		

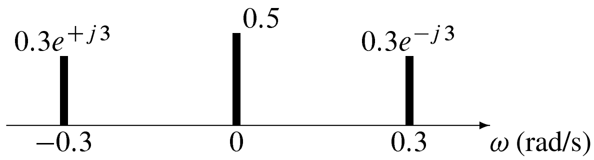
Signal: $z(t)$

b_k	Mag≥ 0	Phase, $-\pi < \psi_k \leq \pi$
b_3		
b_2		
b_1		
b_0		
b_{-1}		
b_{-2}		
b_{-3}		

Note: Whenever a b_k coefficient is equal the corresponding a_k coefficient, just write ***SAME*** in the b_k table.

PROBLEM sp-10-Q.2.3:

(a) A real signal $x(t)$ has the two-sided spectrum shown below. *The frequency axis has units of* rad/s. Write the formula for $x(t)$ as a sum of cosines.



(b) Determine the fundamental frequency f_0 (in Hz) of a signal $x(t)$ whose spectrum has lines at the six frequencies in the set $\{\pm 1000, \pm 1600, \pm 2200\}$ Hz.

$f_0 =$ Hz

(c) In AM radio, the transmitted signal is voice (or music) modulating a *carrier signal.* A typical transmitted signal is:

$$s_{\mathrm{AM}}(t) = (v(t) + A)\cos(2\pi(680 \times 10^3)t)$$

where A is a constant. Assume that $A = 9$ and $v(t) = 9\cos(2\pi(8000)t)$. Sketch the spectrum for $s_{\mathrm{AM}}(t)$ over the *positive frequency region only*. Label the frequency axis with f (in kiloHz).

(d) Suppose that the Fourier series coefficients of a periodic signal $q(t)$ are defined via the integral:

$$a_k = \frac{1}{9}\int_0^9 q(t)\, e^{-j(2\pi/9)kt}\, dt \qquad \text{where} \quad q(t) = \begin{cases} 0 & 0 \le t < 6 \\ 10 & 6 \le t < 9 \end{cases}$$

Evaluate a_k for $k = 5$; *express your answer as a complex number in polar form,* i.e., $r_k e^{j\theta_k}$.

$r_k =$	$\theta_k =$

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QUIZ #2

DATE: 22-Feb-10 COURSE: ECE-2025

NAME: **ANSWER KEY** GT username: **VERSION #1**

LAST, FIRST (ex: gpburdell3)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

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PROBLEM sp-10-Q.2.1:

Shown below are spectrograms (labeled as **S1 – S6**) for six signals. The (vertical) frequency axis for each plot has units of Hz; the horizontal axis is time, $0 \le t \le 14$ s. For each signal description below, identify the corresponding spectrogram. ***Write each answer in the box provided.***

(a) **S1** $x(t) = \cos(50\pi t + 2\cos(10\pi t))$

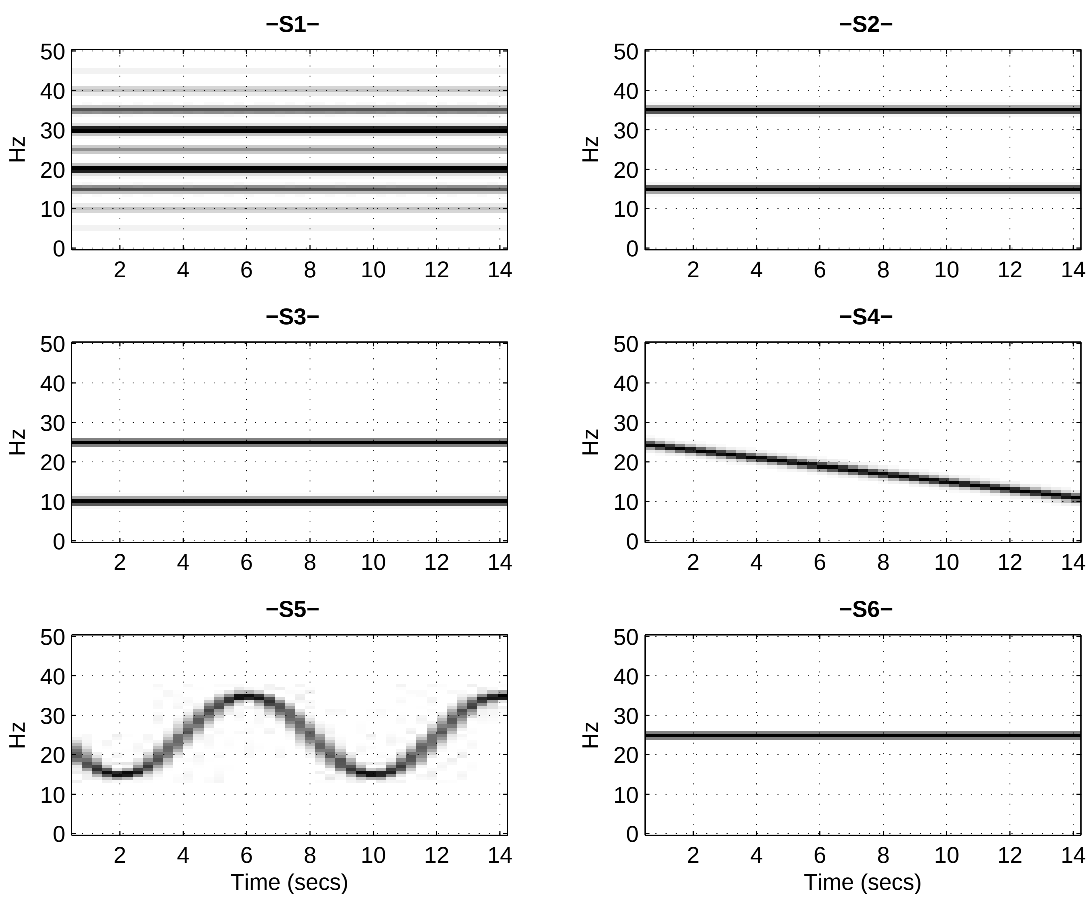
(b) **S6** $x(t) = \cos(50\pi t - 1) + \cos(50\pi t - 2)$

(c) **S4** $x(t) = \cos(50\pi t - \pi t^2)$

(d) **S5** $x(t) = \cos(50\pi t + 80\cos(0.25\pi t))$

(e) **S3** $x(t) = \cos(20\pi t) + \cos(50\pi t)$

(f) **S2** $x(t) = \cos(20\pi t)\cos(50\pi t)$



PROBLEM sp-10-Q.2.2:

The following MATLAB code defines vectors `xt` and `zt`, which correspond to the signals $x(t)$ and $z(t)$.

```
tt = -1000:0.00001:1000;  %- in seconds
xt = zeros(size(tt));
for k = [-3,-1,0,1,3]    %- loop on k for only these indices
   xt = xt + (j*k - 3)*exp(j*8*pi*k*tt);
end
zt = xt - 5 + 6*cos(24*pi*tt - pi/2);
```

(a) Determine the fundamental **period** of the signal $x(t)$ defined in the MATLAB code above.

$T_0 = 0.25$ s

(b) The signal $x(t)$ has a Fourier Series with coefficients $\{a_k\}$, defined by the MATLAB code. Fill in the table below with the numerical values of the $\{a_k\}$ coefficients in *polar form*.

(c) The new signal defined for $z(t)$ is also periodic with a Fourier Series, so it can be expressed in the following Fourier Series with new coefficients $\{b_k\}$:

$$z(t) = \sum_{k=-3}^{3} b_k e^{j8\pi kt}$$

In the following tables, enter the *numerical values* for each $\{a_k\}$ and $\{b_k\}$ in polar form.
Note: A magnitude value must be nonnegative; angles must be in radians.

Signal: $x(t)$

a_k	Mag≥ 0	Phase, $-\pi < \varphi_k \leq \pi$
a_3	$3\sqrt{2} = 4.243$	$3\pi/4 = 2.356$
a_2	0	0
a_1	$\sqrt{10} = 3.162$	2.82
a_0	3	π
a_{-1}	$\sqrt{10} = 3.162$	$-\varphi_1$
a_{-2}	0	$-\varphi_2$
a_{-3}	$3\sqrt{2} = 4.243$	$-\varphi_3$

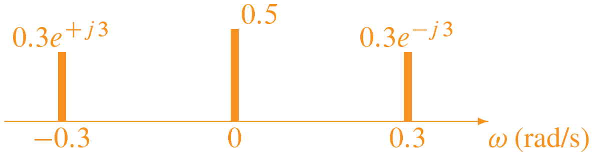
Signal: $z(t)$

b_k	Mag≥ 0	Phase, $-\pi < \psi_k \leq \pi$
b_3	3	π
b_2	0	0
b_1	$\sqrt{10} = 3.162$	2.82
b_0	8	π
b_{-1}	$\sqrt{10} = 3.162$	$-\psi_1$
b_{-2}	0	$-\psi_2$
b_{-3}	3	$-\psi_3$

Note: Whenever a b_k coefficient is equal the corresponding a_k coefficient, just write ***SAME*** in the b_k table.

PROBLEM sp-10-Q.2.3:

(a) A real signal $x(t)$ has the two-sided spectrum shown below. *The frequency axis has units of* rad/s. Write the formula for $x(t)$ as a sum of cosines.



$$x(t) = 0.5 + 0.6\cos(0.3t - 3)$$

(b) Determine the fundamental frequency f_0 (in Hz) of a signal $x(t)$ whose spectrum has lines at the six frequencies in the set $\{\pm 1000, \pm 1600, \pm 2200\}$ Hz.

$f_0 = 200\,\text{Hz}$

(c) In AM radio, the transmitted signal is voice (or music) modulating a *carrier signal.* A typical transmitted signal is:

$$s_{\text{AM}}(t) = (v(t) + A)\cos(2\pi(680 \times 10^3)t)$$

where A is a constant. Assume that $A = 9$ and $v(t) = 9\cos(2\pi(8000)t)$. Sketch the spectrum for $s_{\text{AM}}(t)$ over the *positive frequency region only*. Label the frequency axis with f (in kiloHz).

Need one spectrum line for each complex exponential below:

$$4.5e^{j2\pi(680\times 10^3)t} + 2.25e^{j2\pi(688\times 10^3)t} + 2.25e^{j2\pi(672\times 10^3)t}$$

(d) Suppose that the Fourier series coefficients of a periodic signal $q(t)$ are defined via the integral:

$$a_k = \frac{1}{9}\int_0^9 q(t)\,e^{-j(2\pi/9)kt}\,dt \qquad \text{where} \quad q(t) = \begin{cases} 0 & 0 \le t < 6 \\ 10 & 6 \le t < 9 \end{cases}$$

Evaluate a_k for $k = 5$; *express your answer as a complex number in polar form,* i.e., $r_k e^{j\theta_k}$.

$r_k = 0.5513$ | $\theta_k = 2.094 = 2\pi/3$ rads