



**PROBLEM Spring-06-Q.3.1:**

In each of the following cases, use properties of the unit-impulse function to simplify the expression *as much as possible*. Provide some **explanation** or intermediate steps for each answer. *Note:* Star \* is the convolution operator.

(a) Simplify  $H(j\omega) = \delta(\omega - 8\pi) * \sum_{\ell=0}^2 \sin(\omega/12) \delta(\omega - 3\pi\ell)$

(b) Simplify  $q(t) = \int_{-\infty}^{t-9} \delta(\tau - 7) \sqrt{4(\tau - t)} d\tau$

(c) Simplify  $x(t) = \frac{d}{dt} \left\{ \sqrt{3t} u(t - 12) \right\}$

**PROBLEM Spring-06-Q.3.2:**

In each of the following cases, determine the (inverse or forward) Fourier transform. Give your answer as a plot, or a simple formula (two of the answers will be *real-valued*.)

*Explain* each answer (briefly) by stating which property and/or transform pair you used.

(a) Find  $s(t)$  when  $S(j\omega) = \frac{\sin(\omega/4)}{\omega/8} e^{-j\omega/2}$ .

(b) Find  $h(t)$  when  $H(j\omega) = \frac{1000j\omega}{1000 + j50\omega}$ .

(c) Find  $X(j\omega)$  when  $x(t) = \sqrt{e} \cos(377t + 0.3\pi)$ .

**PROBLEM Spring-06-Q.3.3:**

Two questions about convolution:

(a) Find  $y(t) = e^{-8(t-3)}u(t-3) * 5u(t-9)$ . Give the answer as a simple formula.

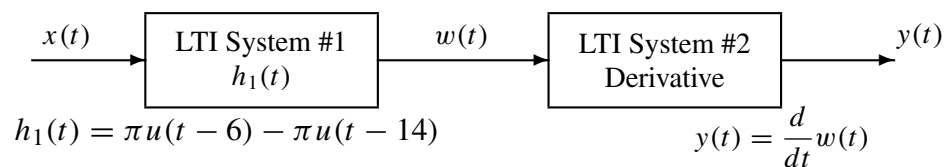
(b) If the signal  $r(t)$  is a rectangular pulse, then  $r(t) * r(t)$  is a triangle. Suppose that

$$r(t) * r(t) = y(t) = \begin{cases} 100t & \text{for } 0 \leq t \leq 40 \\ 4000 - 100(t - 40) & \text{for } 40 \leq t \leq 80 \\ 0 & \text{elsewhere} \end{cases}$$

Determine the rectangular signal  $r(t)$ .

**PROBLEM Spring-06-Q.3.4:**

A cascade of linear time-invariant systems is depicted by the following block diagram:



- (a) Determine the overall impulse response for this cascade of two systems. Give your answer in the *simplest possible form*.

- (b) The overall frequency response of this system,  $H(j\omega)$ , is zero for infinitely many values of  $\omega$ . Derive a general formula that gives **all** the zeros of  $H(j\omega)$ . **Explain**.