



**PROBLEM Spring-06-Q.1.1:**

The sum of two sinusoids is another sinusoid:

$$A \cos(\omega t + \varphi) = 8 \cos\left(\frac{1}{4}\pi(t + 21)\right) + 10 \cos\left(\frac{1}{4}\pi t + 9\pi/2\right)$$

- (a) Determine the numerical values of  $A$  and  $\varphi$ , as well as  $\omega$  (give the correct units).

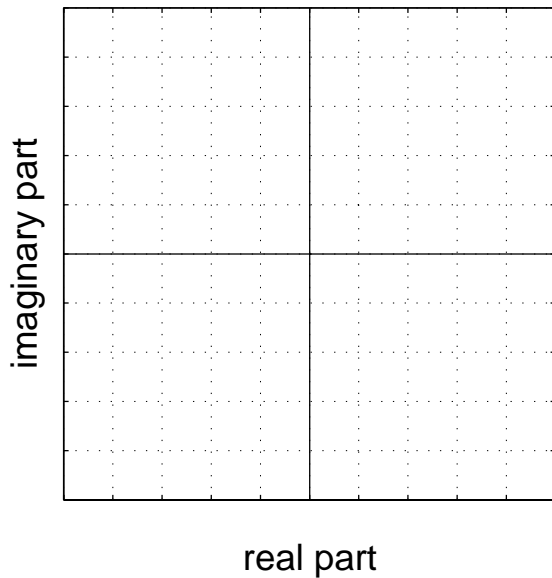
$A =$  \_\_\_\_\_

$\varphi =$  \_\_\_\_\_

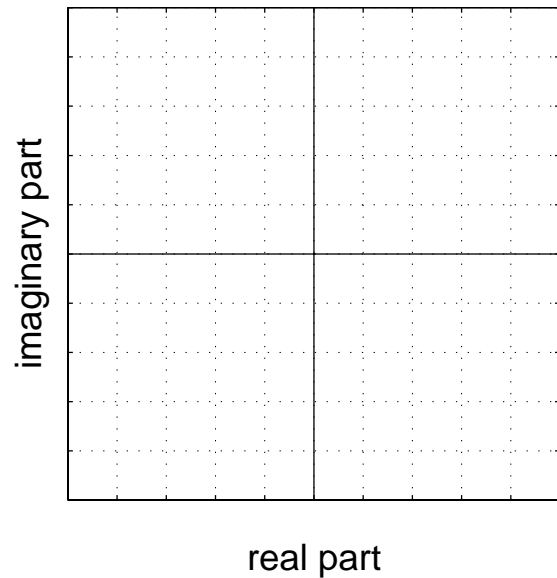
$\omega =$  \_\_\_\_\_

- (b) Make two complex plane plots to illustrate how complex amplitudes (phasors) were combined to solve part (a). On the first plot, show a vector plot of the two complex amplitudes whose values are given by the sinusoids on the **right** hand side of the equal sign; on the second plot, show a “head-to-tail” vector plot of those same two complex amplitudes plus the resultant vector that gives the solution. *Use an appropriate scale on the grids below.*

Two vectors here.



Head-to-tail plot here.



**PROBLEM Spring-06-Q.1.2:**

The signal  $x(t)$  is defined by complex exponentials and complex amplitudes:

$$x(t) = 2e^{j2\pi/5}e^{-j17t} + 2e^{-j2\pi/5}e^{j17t} + 9e^{j\pi}$$

(a) Write the formula for  $x(t)$  as a sum of real-valued sinusoids.

(b) Define a new signal  $y(t)$  to be the derivative of  $x(t)$ , i.e.,  $y(t) = \frac{dx(t)}{dt}$ .

Make a (well-labeled) sketch of the spectrum of the signal  $y(t)$ . Simplify the numerical values for the complex amplitudes, so that the values of magnitude and phase are obvious.

### PROBLEM Spring-06-Q.1.3:

Two questions about sinusoids,  $A \cos(\omega t + \varphi)$ .

(a) The following MATLAB code makes a plot of a sinusoid:

```
tt = 0:0.0001:1;  
znum = 5*j*exp(j*9*pi*tt);  
zden = exp(-j*9*pi*tt)-j*exp(-j*9*pi*tt);  
xx = real(znum./zden);  
plot(tt,xx), grid on, shg
```

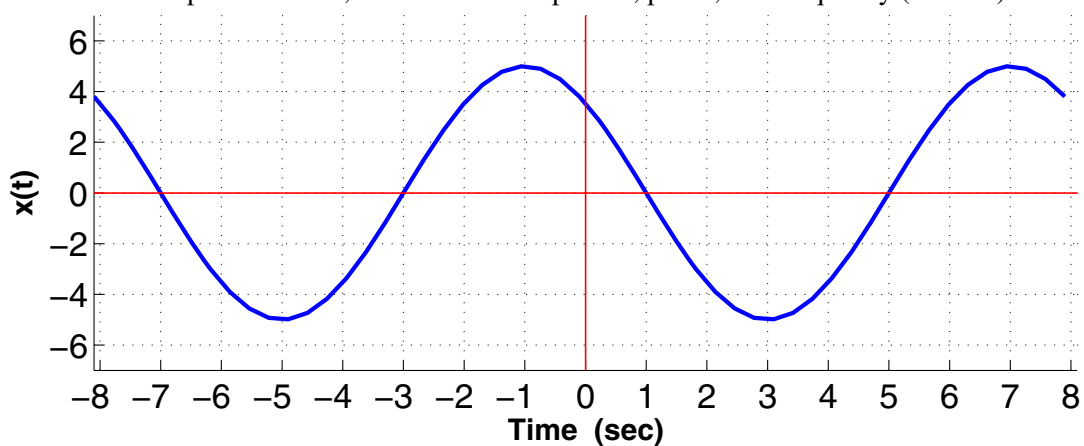
Determine the mathematical formula by giving numerical values for  $A$ ,  $\varphi$ , and  $\omega$  (in rad/s).

$A =$  \_\_\_\_\_

$\varphi =$  \_\_\_\_\_

$\omega =$  \_\_\_\_\_

(b) For the sinusoid plotted below, determine its amplitude, phase, and frequency (in rad/s).



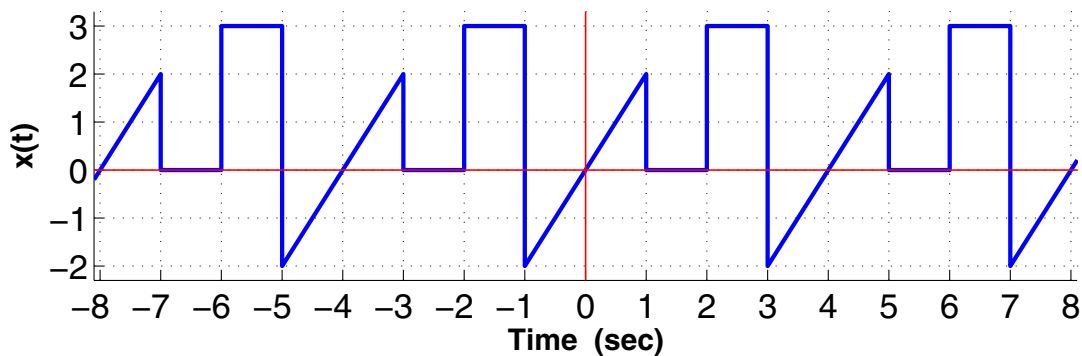
$A =$  \_\_\_\_\_

$\varphi =$  \_\_\_\_\_

$\omega =$  \_\_\_\_\_

**PROBLEM Spring-06-Q.1.4:**

Suppose that a periodic signal  $x(t)$  is defined by the plot below (only the section  $-8 \leq t \leq 8$  is shown):



(a) Determine the fundamental frequency of  $x(t)$  in Hz.

(b) Determine the DC value of  $x(t)$ .

(c) Write the Fourier integral expression for the coefficient  $a_5$  in terms of the specific signal  $x(t)$  defined above. *Set up all the specifics of the integral (e.g., limits of integration), but do not evaluate the integral. All parameters in the integral(s) should have numeric values. NOTE: more than one integral might be needed.*

**PROBLEM Spring-06-Q.1.5:**

For the FM signal  $x(t)$  defined as:

$$x(t) = \Re \{ \exp\{jC e^{\lambda t}\} \}$$

we will denote its instantaneous frequency (in Hz) as  $f_x(t)$ .

- (a) For  $C = 60\pi$  and  $\lambda = 0.8$ , make a *carefully labeled* plot of the instantaneous frequency  $f_x(t)$  over the time interval  $0 \leq t \leq 3$  secs. *Note: the frequency should be in hertz (Hz).*

- (b) Evaluate  $\int_{-3}^3 |-10j \exp\{j\pi e^{0.8t}\}|^2 dt$ . Since the integral is a definite integral, give a numerical answer.