GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

FINAL EXAM

DATE: 5-May-06 COURSE: ECE-2025

NAME: ______________________________ GT #: ____________________

LAST, FIRST (ex: gtz127a)

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

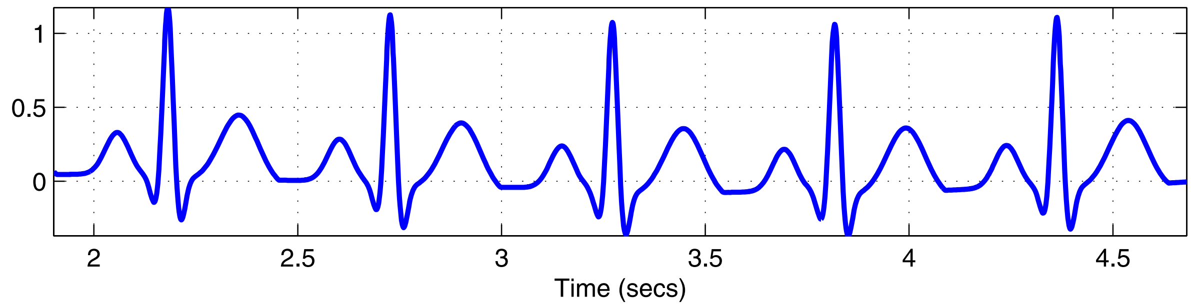
	L05:Tues-Noon (Juang)		L06:Thur-Noon (Verriest)
	L07:Tues-1:30pm (Juang)		
L01:M-3pm (McClellan)	L09:Tues-3pm (Chang)	L02:W-3pm (Zhou)	L10:Thur-3pm (Taylor)
L03:M-4:30pm (Fekri)	L11:Tues-4:30pm (Chang)	L04:W-4:30pm (Zhou)	

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required. to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	30	
2	30	
3	30	
4	30	
5	30	
6	30	
7	30	

PROBLEM Spring-06-F.1:

(a) An ECG signal is (more or less) periodic. If we ***ignore small variations and assume*** that the ECG signal is exactly periodic, then it could be represented as a Fourier Series. For the ECG signal below, determine the ***fundamental frequency(in Hz)*** that would be used in the Fourier Series. Make ***accurate estimates*** from the plot.



$f_0 =$ Hz

(b) Suppose that a signal $s(t)$ is periodic and is represented by the following Fourier Series:

$$s(t) = \sum_{k=-3}^{3} a_k e^{j100\pi kt}$$

i.e., the sum contains a *finite* number of terms. Determine the ***Nyquist rate (in Hz)*** for sampling $s(t)$.

Nyquist rate = Hz

(c) Suppose that the signal $x(t)$ is an FM signal defined via:

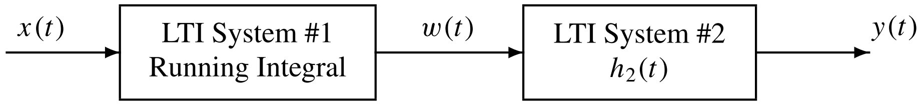
$$x(t) = \Re e\left\{\exp\{jCe^{\lambda t}\}\right\}$$

over the time interval $0 \leq t \leq 1.5$ secs. Let the parameters be $C = 500\pi$ and $\lambda = 0.8$. Use the instantaneous frequency, $f_x(t)$ (in Hz), to determine the ***Nyquist rate (in Hz)*** for sampling $x(t)$.

Nyquist rate = Hz

PROBLEM Spring-06-F.2:

A cascade of linear time-invariant systems is depicted by the following block diagram:



$$w(t) = \int_{-\infty}^{t-7} x(\tau)d\tau \qquad h_2(t) = \pi\delta(t-6) - \pi\delta(t-14)$$

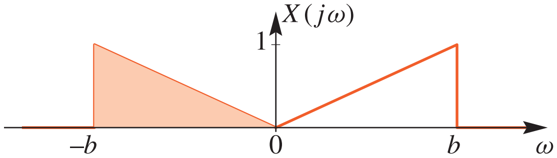
(a) Determine the impulse response of the first system.

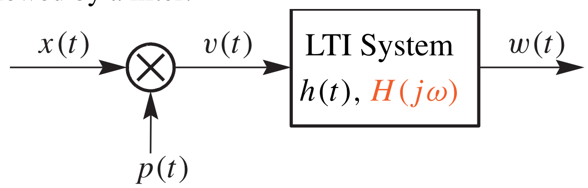
(b) Determine the overall impulse response for this cascade of two systems. Give your answer in the ***simplest possible form.***

(c) The overall frequency response of this system, $H(j\omega)$, is zero for infinitely many values of ω. Derive a general formula that gives **<u>all</u>** the zeros of $H(j\omega)$. ***Explain.***

PROBLEM Spring-06-F.3:

The transmitter system below involves a multiplier followed by a filter:

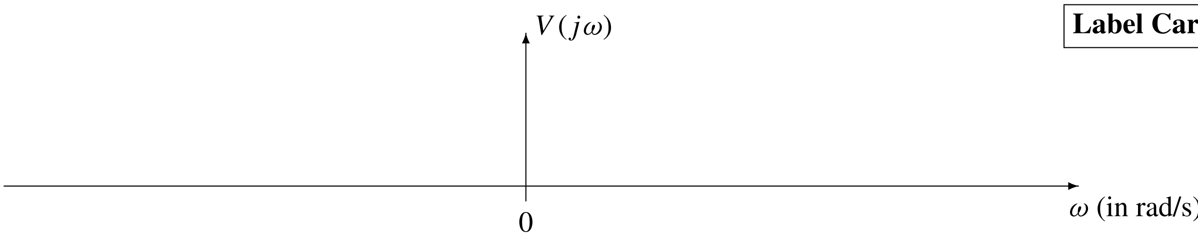




The Fourier transform of the input is $X(j\omega)$. In all parts, assume that $p(t) = \cos(50\pi t)$, and $b = 10\pi$.

(a) Make a sketch of $V(j\omega)$, the Fourier transform of $v(t)$, when the input is $X(j\omega)$ shown above.

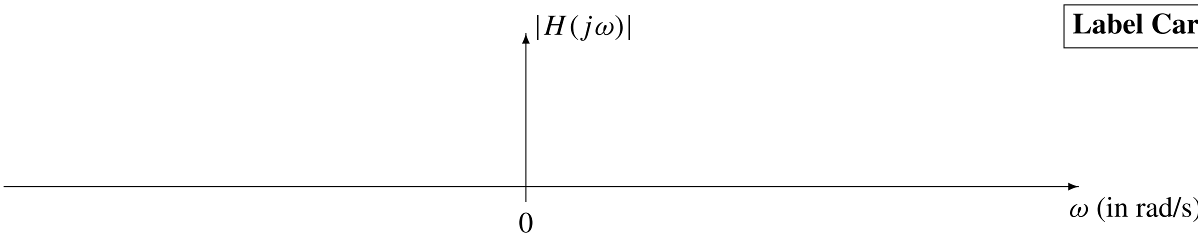
Label Carefully



(b) If the filter is an ideal filter defined by $H(j\omega) = \begin{cases} 0 & |\omega| < 30\pi \\ e^{-j\omega/150} & 30\pi \le |\omega| \le 50\pi \\ 0 & |\omega| \ge 50\pi \end{cases}$

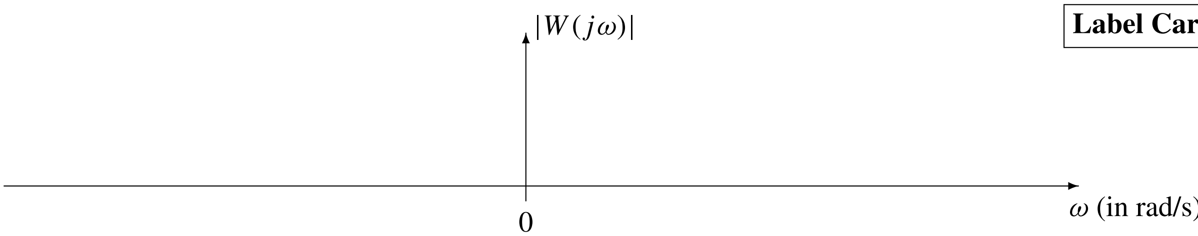
Make a sketch of $|H(j\omega)|$, the magnitude of the Fourier transform of the filter.

Label Carefully



(c) Using the filter from part (b), make a sketch of $|W(j\omega)|$, the magnitude of the Fourier transform of the output $w(t)$, when the input is $X(j\omega)$ shown above.

Label Carefully



PROBLEM Spring-06-F.4:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

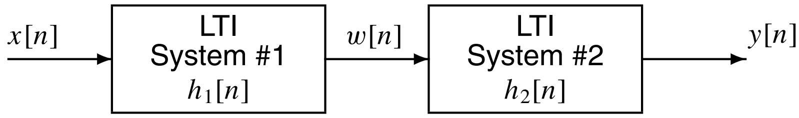


Figure 1: Cascade connection of two discrete-time LTI systems.

In all parts, assume that System #1 is an IIR filter described by the system function: $H_1(z) = \dfrac{2z^{-2} - 2z^{-3}}{1 + 0.5z^{-1}}$

(a) When the input signal $x[n]$ is a ***unit-step*** signal, determine the output of the first system, $w[n]$.

(b) If the difference equation of the second system is

$$y[n] = y[n-1] + 4w[n]$$

Determine the overall system function, $H(z)$, for the cascade. ***Simplify the expression for*** $H(z)$ ***to have the lowest degree polynomials in the numerator and denominator.***

(c) When the input signal $x[n]$ is a ***unit-impulse*** signal, the output $y[n]$ of the overall cascaded system is:

$$y[n] = \delta[n-3]$$

From this information, determine the system function $H_2(z)$ for the second system. Simplify your answer. *Note:* $H_2(z)$ obtained in this part will be different from part (b).

PROBLEM Spring-06-F.5:

Pick the correct frequency response characteristic and enter the number in the answer box:

Difference Equation or Impulse Response

(a) $h[n] = \sum_{k=0}^{3} \delta[n-k]$

ANS =

(b) $y[n] = x[n-1] + 2x[n-3] + x[n-5]$

ANS =

(c) `filter(1,[1,-0.5],xn)`

ANS =

(d) $h[n] = (-\frac{1}{2})^n u[n]$

ANS =

(e) $y[n] = -\frac{1}{2}y[n-1] + 2x[n] + x[n-1]$

ANS =

(f) `conv(ones(1,3),xn)`

ANS =

Frequency Response

1. $H(e^{j\hat{\omega}}) = \dfrac{\sin\hat{\omega}}{\sin(\frac{1}{2}\hat{\omega})}$

2. $H(e^{j\hat{\omega}}) = \dfrac{1}{1 + \frac{1}{2}e^{-j\hat{\omega}}}$

3. $H(e^{j\hat{\omega}}) = 1 + \frac{1}{2}e^{-j\hat{\omega}}$

4. $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(1 + 2\cos(\hat{\omega}))$

5. $H(e^{j\hat{\omega}}) = \dfrac{\sin 2\hat{\omega}}{\sin(\frac{1}{2}\hat{\omega})}\, e^{-j1.5\hat{\omega}}$

6. $|H(e^{j\hat{\omega}})| = 2$

7. $\angle H(e^{j\hat{\omega}}) = -3\hat{\omega}$

8. $H(e^{j\hat{\omega}}) = \dfrac{1}{1 - \frac{1}{2}e^{-j\hat{\omega}}}$

PROBLEM Spring-06-F.6:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write your answers in the boxes next to the questions.*** (The operator $*$ denotes convolution.)

(a) ☐ $x(t) = \int_{-\infty}^{t} e^{-t+\tau}\delta(\tau - 4)d\tau$

(b) ☐ $x(t) = \cos(\pi t)\delta(t - 4)$

(c) ☐ $x(t) = \delta(t) - \delta(t - 8)$

(d) ☐ $x(t) = -e^{-t}u(t) + \delta(t)$

(e) ☐ $x(t) = u(t - 3) - u(t - 5)$

(f) ☐ $x(t) = \delta(t + 2) * e^{-t+1}u(t - 1) * \delta(t - 1)$

(g) ☐ $x(t) = e^{-(t-4)} \int_{-\infty}^{0} \delta(\tau - 4)d\tau$

Each of the time signals above has a Fourier transform that can be found in the list below.

[1] $X(j\omega) = e^{-j4\omega}\left[\pi\delta(\omega - \pi) + \pi\delta(\omega + \pi)\right]$

[2] $X(j\omega) = e^{-j4\omega}\left[u(\omega) - u(\omega - 8)\right]$

[3] $X(j\omega) = \dfrac{j\omega}{1 + j\omega}$

[4] $X(j\omega) = \dfrac{1}{1 + j\omega}$

[5] $X(j\omega) = \dfrac{e^{-j4\omega}}{1 + j\omega}$

[6] $X(j\omega) = 0$

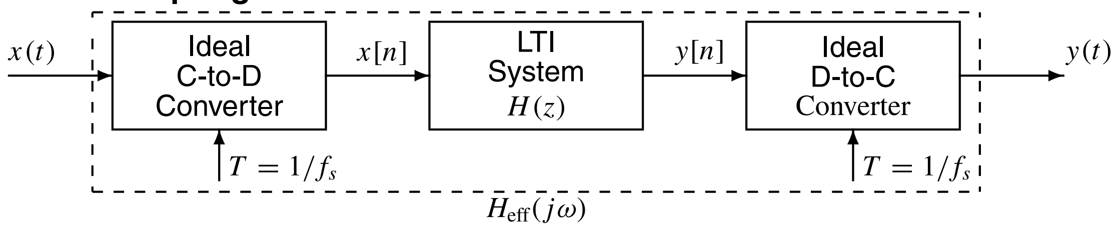
[7] $X(j\omega) = j2e^{-j4\omega}\sin(4\omega)$

[8] $X(j\omega) = e^{-j4\omega}$

[9] $X(j\omega) = 2e^{-j4\omega}\dfrac{\sin(\omega)}{\omega}$

[10] $X(j\omega) = \dfrac{\sin(4\omega)}{\omega/2}$

PROBLEM Spring-06-F.7:



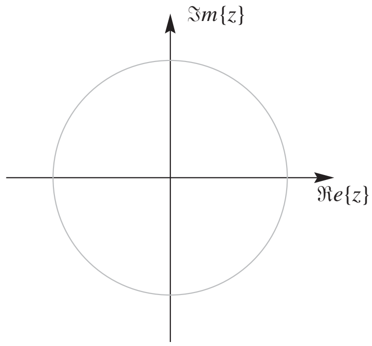
In all parts below, the sampling rates of both converters is **equal to** $f_s = 60$ **samples/sec,** the LTI system is an IIR notch filter described by the difference equation:

$$y[n] = -0.8y[n-1] - 0.64y[n-2] + 5x[n] + 5x[n-1] + 5x[n-2]$$

and the input signal $x(t)$ is a periodic signal whose Fourier Series is

$$x(t) = \sum_{k=-6}^{6} a_k e^{j8\pi kt}, \quad \text{where} \quad a_k = \begin{cases} \dfrac{1}{\pi(1+|k|)} & k \neq 0 \\ \frac{1}{2} & k = 0 \end{cases}$$

(a) Determine the poles and zeros of the LTI system, and give your answer as a plot in the z-plane.



(b) Using the periodic input signal given above, determine the DC value of the output signal, $y(t)$.

DC value =

(c) The filter, $H_{\text{eff}}(j\omega)$, defined above is a notch filter—like the one used to remove sinusoidal interference from an ECG signal. For the periodic input signal $x(t)$ given above via its Fourier Series, determine which terms in the Fourier Series will be removed completely, i.e., nulled, by the notch filter. ***Explain.***

Indices of terms removed, $k =$