GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2003
Problem Set #13

Assigned: 4-Apr-03
Due Date: Week of 14-April-03

Quiz #3 will be given on 11-April. One page ($8\frac{1}{2} \times 11$ in.) of **handwritten** notes allowed.

Reading: In *SP First*, all of Chapter 10: *Frequency Response*,
and Chapter 11: *Continuous-Time Fourier Transform*, Sects. 11-1 to 11-4.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 13.1*:

Use the **`CLTIdemo`** GUI in MATLAB to solve the following problems:

(a) Find the output $y(t)$ when the input signal is $x(t) = 2 + 3\cos(60\pi t)$, and the filter is a "First-order Highpass" with a cutoff frequency of 30 Hz.

(b) Determine the cutoff frequency of a "First-order Lowpass" filter that will have an output equal to $y(t) = 2 + 1.5\cos(60\pi t + \phi)$ when the input signal is $x(t) = 2 + 3\cos(60\pi t)$. In addition, find the value of ϕ in the output signal.

PROBLEM 13.2*:

A continuous-time system is defined by the impulse response:

$$h(t) = \frac{d}{dt}\left\{e^{-bt}u(t)\right\}$$

(a) Determine a simple expression for the frequency response of this system.

(b) Make a plot of the frequency response (magnitude only) when $b = 60\pi$.

(c) Describe the type of filter in the plot of part (b), e.g., LPF, HPF, BPF, or something else.

(d) Find the output $y(t)$ when the input signal is $x(t) = 2 + 3\cos(60\pi t)$, and the parameter b is $b = 60\pi$.

PROBLEM 13.3*:
The impulse response of an LTI system is given by

$$h(t) = \frac{\sin(10t)}{t}$$

(a) First, make a detailed and accurately labeled sketch of $h(t)$ over the time interval $-3 \le t \le 3$. Mark the important amplitudes and time locations of peaks and zero crossings.

(b) Now determine the Fourier transform $H(j\omega)$ of this impulse response, which is equivalent to finding the frequency response of the system. Make a detailed plot of $|H(j\omega)|$ versus ω. Label your plots carefully.

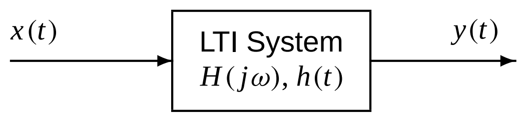
PROBLEM 13.4*:
An LTI system has impulse response given by

$$h(t) = u(t-2) - u(t-10)$$

(a) Determine the Fourier transform $H(j\omega)$ of this impulse response. Remember that $H(j\omega)$ is also the frequency response of the system.

(b) Make detailed plots of $|H(j\omega)|$ and $\angle H(j\omega)$ versus ω. Label your plots carefully. Mark the important amplitudes and time locations of peaks and zero crossings.

PROBLEM 13.5*:



The impulse response of the system (above) is

$$h(t) = \frac{4\sin(\omega_{\text{co}}(t-\frac{1}{2}))}{\pi(t-\frac{1}{2})}$$

and the input to this system is a periodic signal (with period $T_0 = 1/20$ sec.) given by a Fourier series:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi f_0 k t}$$

where the Fourier coefficients are $a_k = \dfrac{1}{1+k^2}, \quad k = 0, \pm1, \pm2, \ldots$

(a) Recall how the spectrum is related to the Fourier series, and then plot the spectrum of the input signal, $x(t)$, over the frequency range $-100\pi < \omega < 100\pi$ in rad/s.

(b) Determine the frequency response $H(j\omega)$ of the system as a general formula. Exploit the fact that $h(t)$ and $H(j\omega)$ are a "Fourier Transform pair." Then, for the case $\omega_{\text{co}} = 50\pi$ rad/s, plot the magnitude $|H(j\omega)|$ vs. ω, and the phase $\angle H(j\omega)$ vs. ω. Use the frequency range $-100\pi < \omega < 100\pi$ rad/s.

(c) Determine the spectrum of the output signal, $y(t)$. Make a plot versus ω. This will be easy to do if you overlay the plots from parts (a) and (b) on the same frequency axis.

(d) Use your spectrum plot in (c) to determine an equation for $y(t)$, the output of the LTI system for the given input $x(t)$ when the cutoff frequency is $\omega_{\text{co}} = 50\pi$ rad/s.