GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2003
Problem Set #10

Assigned: 14-Mar-03
Due Date: Week of 24-March-03

Quiz #3 will be given on 11-April.

Reading: In *SP First*, Chapter 8: *IIR Filters*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 10.1*:

Given a feedback filter defined via the recursion:

$$y[n] = y[n-3] + x[n] \qquad \text{(DIFFERENCE EQUATION)}$$

(a) Determine the impulse response $h[n]$, assuming the "at rest" initial condition.

(b) Prove that the impulse response signal is periodic for $n > 0$, and determine the period.

(c) When the input to the system is the signal:

$$x[n] = -\delta[n-1] + 2\delta[n-4] - \delta[n-7]$$

determine the output signal $y[n]$, assuming the "at rest" initial condition (i.e., the output signal is zero for $n < 0$). Present your final answer as a plot of all of $y[n]$.

PROBLEM 10.2*:

For the following system:

$$H(z) = \frac{1 - z^{-1}}{1 + 0.9z^{-1}}$$

determine various aspects of the time-domain (n) behavior:

(a) The inverse z-transform of $H(z)$ is the impulse response $h[n]$. Determine the inverse z-transform for $H(z)$ as a mathematical formula, and sketch the first five values of the impulse response, $h[n]$.

(b) When the input is the unit-step signal, $u[n]$, determine the output signal.

(c) Find an input signal for which the output is *finite-duration*, i.e., $y[n] = 0$ for $n > n_0$ where n_0 is a fixed number.

PROBLEM 10.3*:

Determine the z-transforms of the following. *Express your answer as the ratio of polynomials in z^{-1} by placing all terms over a common denominator.*

(a) $x_a[n] = \left(-\frac{1}{2}\right)^n u[n-2]$

(b) $x_b[n] = 10(0.8)^n u[n] + 10(-0.8)^n u[n]$

(c) $x_c[n] = \delta[n] - u[n]$

PROBLEM 10.4*:

Suppose that three systems are hooked together in "cascade." In other words, the output of $\mathcal{S}_1$ is the input to $\mathcal{S}_2$, and the output of $\mathcal{S}_2$ is the input to $\mathcal{S}_3$. The three systems are specified as follows:

$$\mathcal{S}_1 : \quad y_1[n] = 4x_1[n] + 4x_1[n-2]$$

$$\mathcal{S}_2 : \quad y_2[n] = -3x_2[n] - 3x_2[n-1]$$

$$\mathcal{S}_3 : \quad y_3[n] = 5x_3[n-1] - 5x_3[n-2]$$

Note: the output of $\mathcal{S}_i$ is $y_i[n]$ and the input is $x_i[n]$.

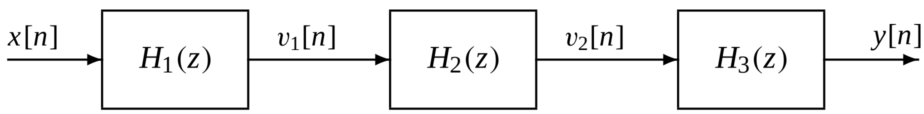
Determine the equivalent system that is a single operation from the input $x[n]$ (into $\mathcal{S}_1$) to the output $y[n]$ which is the output of $\mathcal{S}_3$. Thus $x[n]$ is $x_1[n]$ and $y[n]$ is $y_3[n]$.

(a) Determine the z-transform system function $H_i(z)$ for each system.

(b) Write *one difference equation* that defines the overall system in terms of $x[n]$ and $y[n]$ only.

PROBLEM 10.5*:

In the following cascade of systems, all of the individual system functions, $H_i(z)$, are known.



$$H_1(z) = z^{-2} + z^{-3} \qquad H_2(z) = 6 - 3z^{-1} \qquad H_3(z) = \frac{2}{8 - 10z^{-1} + 3z^{-2}}$$

(a) Determine $H(z)$ the z-transform of the cascaded system. Simplify $H(z)$ by cancelling common factors in the numerator and denominator.

(b) Consider the impulse response of the cascaded system, i.e., the response $y[n]$ when the input is $x[n] = \delta[n]$. Prove that the impulse response has the form $h[n] = G\,\alpha^n$ for $n \geq 3$. Find values for α and G.