GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2003
Problem Set #8

Assigned: 21-Feb-03
Due Date: Week of 10-March-03

Quiz #2 will be given on 14-March

Reading: In *SP First*, Chapter 6: *Frequency Response of FIR Filters*

⟹ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 8.1*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

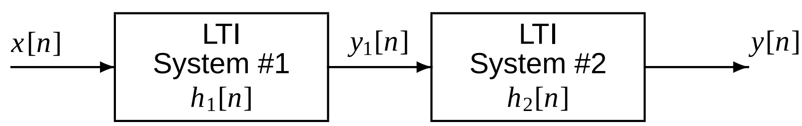


Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 is a filter described by the difference equation

$$y_1[n] = \beta x[n] - x[n-2]$$

and System #2 is described by the impulse response

$$h_2[n] = \delta[n-1] - \beta\delta[n-3],$$

(a) Determine the frequency response sequence, $H_1(e^{j\hat{\omega}})$, of the first system.

(b) Determine the frequency response, $H(e^{j\hat{\omega}})$, of the overall cascade system.

(c) For the case where $\beta = 0.8$, plot the magnitude and phase of the frequency response of the overall frequency response of the cascaded system.

(d) When $\beta = 0.8$ and the input to this system is

$$x[n] = 7\cos(0.3\pi n - 0.4\pi) + (-1)^n$$

Use the frequency response to compute the values of $y[n]$, over the range $-\infty \le n \le \infty$.

PROBLEM 8.2*:

A discrete-time system is defined by the input/output relation

$$y[n] = -3Gx[n-1] + 6Gx[n-2] - 3Gx[n-3]$$

where G is a constant to be determined.

(a) When the input is the signal, $x_1[n] = 1 + (-1)^n$, the output is $y_1[n] = 60(-1)^{n+1}$. Determine the value of G, and then determine the output when the input is

$$x_2[n] = \begin{cases} 5 & \text{for } n \text{ even} \\ 25 & \text{for } n \text{ odd} \end{cases}$$

Use linearity and time invariance to simplify your work.

(b) Obtain an expression for the frequency response of this system, using G from part (a).

(c) Make a sketch of the frequency response (magnitude and phase) as a function of frequency.
Hint: Use symmetry to simplify your expression before determining the magnitude and phase.

(d) For the system above, determine the output $y_1[n]$ when the input is

$$x_1[n] = 4 + 8\cos(0.5\pi n + \pi/2)$$

Hint: Use the frequency response and superposition to solve this problem.

PROBLEM 8.3*:

The frequency response of a linear time-invariant filter is given by the formula

$$H(e^{j\hat{\omega}}) = (1 - e^{-j\hat{\omega}})(1 + e^{-j\pi/4}e^{-j\hat{\omega}})(1 + e^{j\pi/4}e^{-j\hat{\omega}}) \quad (1)$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$.
Hint: Multiply out the factors to obtain a sum of powers of $e^{-j\hat{\omega}}$.

(b) What is the impulse response of this system?

(c) If the input is a complex exponential of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for which values of $-\pi \le \hat{\omega} \le \pi$ will $y[n] = 0$ for all n?
Hint: In this part, the answer is easy to obtain if you use the factored form of Eq. (1).

(d) Use superposition to determine the output of this system when the input is

$$x[n] = 3 + \delta[n-2] + \cos(0.5\pi n + \pi/4) \qquad \text{for } -\infty < n < \infty$$

Hint: Divide the input into three parts and find the outputs separately each by the easiest method and then add the results. This is what it means to apply the principle of *Superposition*.

PROBLEM 8.4*:
Consider the linear time-invariant system given by the difference equation

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4] + x[n-5] + x[n-6] + x[n-7] = \sum_{k=0}^{7} x[n-k]$$

(a) Find an expression for the frequency response $H(e^{j\hat{\omega}})$ of the system.

(b) Show that your answer in (a) can be expressed in the form

$$H(e^{j\hat{\omega}}) = \frac{\sin(8\hat{\omega}/2)}{\sin(\hat{\omega}/2)} e^{-j3.5\hat{\omega}}$$

(c) Sketch the frequency response (magnitude and phase) as a function of frequency from the formula above. You might want to check your plot by doing it in MATLAB with `freekz( )` or `freqz( )`.

(d) Suppose that the input is

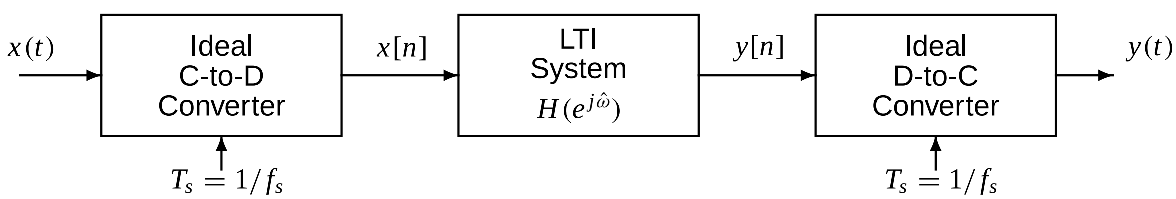
$$x[n] = 3 + 3\cos(\hat{\omega}_0 n) \qquad \text{for} \quad -\infty < n < \infty$$

Find all possible non-zero frequencies $0 < \hat{\omega}_0 < \pi$ for which the output $y[n]$ is a constant for all n, i.e.,

$$y[n] = c \qquad \text{for} \quad -\infty < n < \infty$$

and find the value for c. (In other words, the sinusoid is removed by the filter.)

PROBLEM 8.5*:
Consider the following system for discrete-time filtering of a continuous-time signal:



In this problem, assume that the frequency response of the discrete-time system is

$$H(e^{j\hat{\omega}}) = 1 + e^{-j\hat{\omega}}$$

(a) Make a plot of the frequency response magnitude for $H(e^{j\hat{\omega}})$ over the frequency range $-\pi < \hat{\omega} \leq \pi$.

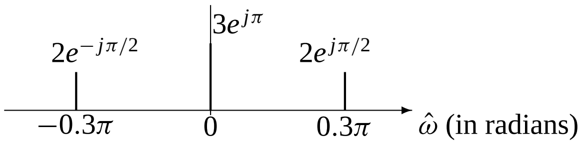
(b) For a sampling rate of $f_s = 300$ samples/sec, determine the frequency of an input sinusoid of the form $x(t) = \cos(\omega t)$ such that the resulting output will be zero.

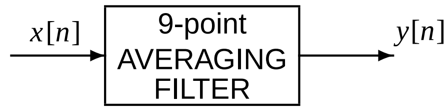
(c) In this part, assume that the input is

$$x(t) = 10 + 20\cos(100\pi t) \quad \text{for} \quad -\infty < t < \infty$$

For a sampling rate of $f_s = 300$ samples/sec, determine the output $y(t)$ for $-\infty < t < \infty$.

PROBLEM 8.6:
A discrete-time signal $x[n]$ has the two-sided spectrum representation shown below.





(a) Write an equation for $x[n]$. Make sure to express $x[n]$ as a real-valued signal.

(b) Determine the formula for the output signal $y[n]$.

See Problem 6.1 of Spring 1999 for solution to this problem.

PROBLEM 8.7:
A discrete-time system is defined by the input/output relation

$$y[n] = \begin{cases} 1 & \text{if } |x[n]| \geq 0.5 \\ 0 & \text{if } |x[n]| < 0.5 \end{cases}$$

(a) For the system above, determine the output $y_1[n]$ when the input is

$$x_1[n] = \cos(0.5\pi n)$$

(b) Explain why the result from part (a) proves that the system is not an LTI system.

(c) Is the system linear? or time-invariant? or neither?