GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2003
Problem Set #7

Assigned: 14-Feb-03
Due Date: Week of 24-Feb-03

Reading: In *SP First*, Chapter 5: *FIR Filters*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 7.1*:

For each of the following systems, determine if they are (1) linear; (2) time-invariant; (3) causal.

(a) $y[n] = x[n+1] - x[n]$ (Forward Difference)

(b) $y[n] = |x[n]|^2$ (Magnitude Squared)

(c) $y[n] = x[-n]$ (Flip)

PROBLEM 7.2*:

A linear time-invariant system is described by the difference equation

$$y[n] = x[n-1] - \beta x[n-2]$$

(a) When the input to this system is

$$x[n] = \begin{cases} 0 & n < 1 \\ \beta^n & n = 1, 2, 3, 4, 5, 6, 7 \\ 0 & n > 7 \end{cases}$$

Use convolution to compute the values of $y[n]$, over the range $0 \le n \le 10$. Give a general formula in terms of β, and also show that most of the output values are equal to zero.

(b) Use the results from the previous part and plot both $x[n]$ and $y[n]$ for the case where $\beta = 0.8$.

PROBLEM 7.3*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

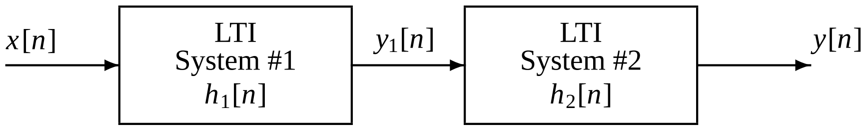


Figure 1: Cascade connection of two LTI systems.

(a) Suppose that System #1 is a "blurring" filter described by the difference equation

$$y_1[n] = \sum_{k=1}^{7} \beta^k x[n-k],$$

and System #2 is described by the impulse response

$$h_2[n] = \delta[n-1] - \beta\delta[n-2],$$

where β is a real number. Determine the impulse response sequence, $h_1[n]$, of the first system.

(b) Determine the impulse response sequence, $h[n] = h_1[n] * h_2[n]$, of the overall cascade system.

(c) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Fig. 1. Give numerical values of the filter coefficients for the specific case where $\beta = 0.8$.

PROBLEM 7.4*:

A linear time-invariant discrete-time system is described by the difference equation

$$y[n] = x[n] - 3x[n-1] + 4x[n-2] - 3x[n-3] + x[n-4].$$

(a) Determine the impulse response $h[n]$ for this system.

(b) Make a plot of the shift unit-step signal $s[n] = -u[n-7]$; plot enough to show its essential behavior.

(c) Use convolution to determine the output due to the input

$$x[n] = u[n-1] - u[n-7] = \begin{cases} 1 & n = 1, 2, 3, 4, 5, 6 \\ 0 & \text{otherwise} \end{cases}$$

Plot the output sequence $y[n]$ for $-3 \leq n \leq 12$.

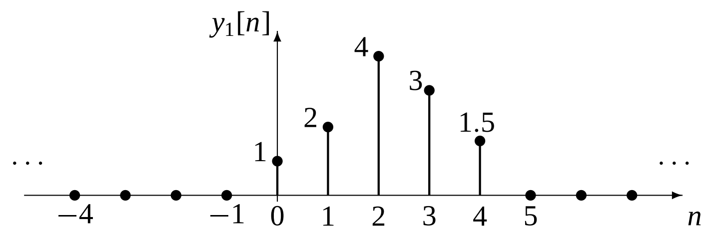
PROBLEM 7.5*:

Answer the following questions about the time-domain response of FIR digital filters:

$$y[n] = \sum_{k=0}^{M} b_k \, x[n-k]$$

(a) When tested with an input signal that is a shifted impulse, $x_1[n] = \delta[n+2]$, the observed output from the filter is the signal $h[n]$ shown below:

$$\delta[n+2] = \begin{cases} 1 & \text{for } n = -2 \\ 0 & \text{for } n \neq -2 \end{cases} \longrightarrow$$



Use linearity and time-invariance to solve the following problem. Determine the output when the input to the LTI system is $x_2[n] = \delta[n+1] - \delta[n-1]$. Give your answer as a plot of $y_2[n]$ versus n, or a list of values for $-\infty < n < \infty$.

(b) State the property of *causality* found in the text. Is this system *causal?*

PROBLEM 7.6:

Consider a system defined by $\quad y[n] = \sum_{k=0}^{M} b_k x[n-k]$

(a) What is the filter length?

(b) Suppose that the input $x[n]$ is non-zero only for $10 \leq n \leq 20$ and $M = 9$. Where will the output $y[n]$ first become non-zero? What is the index of the last non-zero value in the output sequence $y[n]$? What is the total length of the input sequence (in samples).

(c) Suppose that the input $x[n]$ is non-zero only for $N_1 \leq n \leq N_2$. What is the length of the input sequence (in samples).

(d) For the input in (b) and the above system, show that $y[n]$ is non-zero at most over a finite interval of the form $N_3 \leq n \leq N_4$ and determine N_3 and N_4.

(e) What is the length of the output sequence (in samples)?

Hint: Draw a sketch similar to Fig. 5.5 to illustrate the zero regions of the output signal.