GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2003
Problem Set #2

Assigned: 13-Jan-03
Due Date: Week of 20-Jan-03

Reading: In *SP First*, all of Ch. 2, and start reading in Chapter 3: *Spectrum Representation*, Section 3-1.

Chapters from *SP First* have been posted on WebCT, and have also been handed out in Recitation.

The *SP First* Toolbox for MATLAB has been posted on WebCT under the "Lab Assignments" link. You can install it to get some useful functions and GUIs for manipulating complex numbers.

⟹ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 2.1*:

Each of the following signals may be simplified, and expressed as one or two sinusoids of the form: $A\cos(\omega t + \phi)$. For each signal, draw a vector diagram of the complex amplitudes (phasors), and use vector addition to estimate the amplitude(s) A and phase(s) ϕ of the resultant sinusoid(s). Then use the phasor addition theorem to find the exact values for A and ϕ.

(a) $x_a(t) = 3\cos(22\pi t - 4\pi/3) + \cos(22\pi t + 3\pi/4) - 3\cos(55\pi t - 4\pi/3) - \cos(55\pi t + 3\pi/4)$

(b) $x_b(t) = \sqrt{2}\cos(89\pi t + 101\pi) + 5\cos(89\pi t - 101.25\pi) + \sqrt{2}\cos(89\pi t + 101.5\pi)$

(c) $x_c(t) = 7\cos(\pi t + \pi/12) + 7\cos(\pi t + 7\pi/12) + 7\cos(\pi t - 5\pi/12) + 7\cos(\pi t - 11\pi/12)$

PROBLEM 2.2*:

Define $x(t)$ as

$$x(t) = \sqrt{3}\cos(\omega_0 t - 2\pi/3) + 3\cos(\omega_0 t + 3\pi/2)$$

(a) Find a complex-valued signal $z_1(t)$ such that $\Re e\{z_1(t)\} = 3\cos(\omega_0 t + 3\pi/2)$.

(b) Find a complex-valued signal $z(t)$ such that $x(t) = \Re e\{z(t)\}$. Simplify $z(t)$ as much as possible, so that you can identify its complex amplitude.

(c) Assume that $\omega_0 = 0.1\pi$ rad/sec. Make a plot of $\Re e\{(-2 - j2)e^{j\omega_0 t}\}$ over the range $-10 \le t \le 10$ secs. How many periods are included in the plot?

PROBLEM 2.3*:

Complex exponentials obey the expected rules of algebra when doing integrals and derivatives. Consider the complex signal $z(t) = Ze^{j5\pi t}$ where $Z = 3e^{j\pi/4}$.

(a) Evaluate the definite integral of $z(t)$ over the range $0 \le t \le 0.1$: $\int_{0}^{0.1} z^2(t)dt = ?$

Note that integrating a complex quantity follows the expected rules of algebra: you could integrate the real and imaginary parts separately, but you can also *use the integration formula for an exponential* directly on $z(t)$.

(b) Evaluate the definite integral of $z(t)$ over the range $-0.1 \le t \le 0.1$: $\int_{-0.1}^{0.1} z^2(t)dt = ?$

(c) Recall that the magnitude magnitude squared $|z|^2$ of a complex number z is equal to $(z^*)z$ where z^* is the conjugate of z. Evaluate the following definite integral: $\int_{0}^{2} z^*(t)z(t)dt = ?$

PROBLEM 2.4*:

Solve the following simultaneous equations by using complex amplitudes. Show how to convert the sinusoidal equations into complex-number equations. If we assume that the amplitudes are positive, will the answers for A_1 and A_2 be unique? How about ϕ_1 and ϕ_2; are there other answers for the phases?

$$\begin{aligned} 2\cos(\omega_0 t + 2\pi/3) &= A_1\cos(\omega_0 t + \phi_1) + A_2\cos(\omega_0 t + \phi_2) \\ 2\cos(\omega_0 t + \pi) &= A_1\cos(\omega_0 t + \phi_1) - A_2\cos(\omega_0 t + \phi_2) \end{aligned}$$

PROBLEM 2.5*:

A real signal $x(t)$ can be represented with a two-sided spectrum by using the inverse Euler formula

$$\cos(\omega t + \phi) = \tfrac{1}{2}e^{j\phi}e^{j\omega t} + \tfrac{1}{2}e^{-j\phi}e^{-j\omega t}$$

(a) For the signal $x(t) = 9 + 10\cos(20\pi t - \pi/3) + 7\cos(50\pi t + \pi/4)$, determine a formula for $x(t)$ in terms of complex exponentials.

(b) Plot the spectrum representation for $x(t)$ in a form like that shown on the axis below.