

GEORGIA INSTITUTE OF TECHNOLOGY
 SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
QUIZ #1

DATE: Jan. 31, 2003

COURSE: ECE-2025

NAME: _____
 LAST, FIRST

GT #: _____

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L01:Tues-9:30am (McLaughlin)		L02:Thur-9:30am (Barry)
	L03:Tues-Noon (McLaughlin)		L04:Thur-Noon (Barry)
	L05:Tues-1:30pm (Li)		
L11:M-3pm (McClellan)	L07:Tues-3pm (Li)	L12:W-3pm (Hayes)	L08:Thur-3pm (Williams)
	L09:Tues-4:30pm (Zhou)	L14:W-4:30pm (Hayes)	
	L10:Tues-6pm (Zhou)		RPK:Thur-Late (Tugcu)

- Write your name on the front page **ONLY**. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- Justify your reasoning **CLEARLY** to receive partial credit.
 Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
 Only these answers will be graded. Circle your answers, or write them in the boxes provided.
 If space is needed for scratch work, use the backs of previous pages.

<i>Problem</i>	<i>Value</i>	<i>Score</i>
1	25	
2	25	
3	25	
4	25	

PROBLEM Spring-02-Q.1.1:

For each of the following signals, pick one of the representations below that defines *exactly* the same signal. Write your answer ((a), (b), (c), (d), (e), or (f)) in the box next to each signal.

ANS =	$-\cos(27\pi t - 3\pi/5)$
-------	---------------------------

ANS =	$\Re \{ j e^{j2\pi/5} e^{j27\pi t} \}$
-------	--

ANS =	$\frac{1}{2} e^{-j\pi/5} e^{j27\pi t} + \frac{1}{2} e^{j\pi/5} e^{-j27\pi t}$
-------	---

ANS =	$\frac{1}{2} e^{j\pi/2} e^{j27\pi t} + \frac{1}{2} e^{-j\pi/2} e^{-j27\pi t}$
-------	---

ANS =	$\cos(27\pi t + 6\pi/5)$
-------	--------------------------

POSSIBLE ANSWERS:

Your answer will be one of the following choices.

Please note: some of the following signals could be used more than one time to match the above signals.

(a) $x_a(t) = \Re \{ e^{-j4\pi/5} e^{j27\pi t} \}$

(b) $x_b(t) = -\sin(27\pi t)$

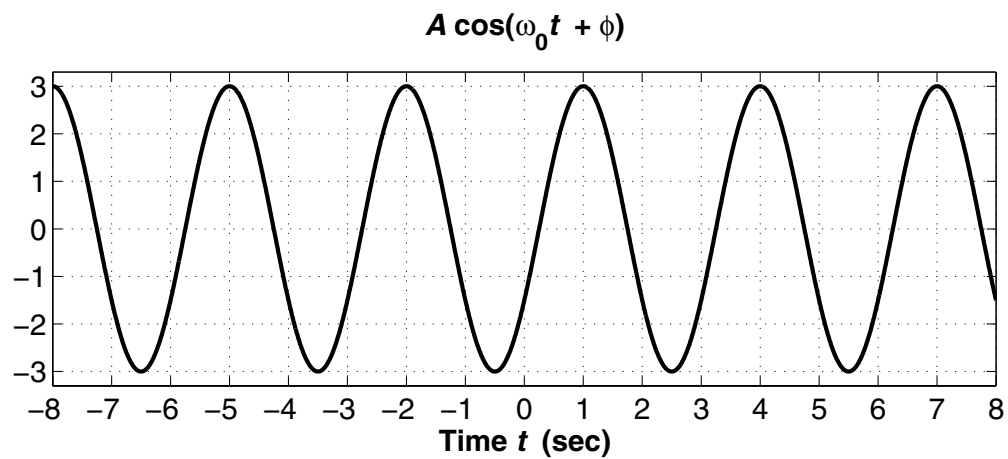
(c) $x_c(t) = \cos(27\pi t + 9\pi/5)$

(d) $x_d(t) = 1$

(e) $x_e(t) = \cos(27\pi t + 2\pi/5)$

(f) $x_f(t) = 0$

PROBLEM Spring-02-Q.1.2:



The above graph is a plot of a sinusoidal signal $x(t) = A \cos(\omega_0 t + \phi)$.

- (a) Determine numerical values for A , ω_0 and ϕ with $-\pi < \phi \leq \pi$. Include units for ω_0 and ϕ .

$$A = \underline{\hspace{2cm}}$$

$$\omega_0 = \underline{\hspace{2cm}}$$

$$\phi = \underline{\hspace{2cm}}$$

- (b) Using $x(t)$ from above, define a new signal as $y(t) = 4x(t - 1) - 2$. Make a plot of $y(t)$ over the same time interval as above. *Label everything carefully.*

PROBLEM Spring-02-Q.1.3:

The following MATLAB code defines several signals that are then multiplied and summed:

```
tt = -10:0.001:10;  
xxe = exp(-9*abs(tt));  
xx1 = 20*cos( 7*pi*(tt + 1/3) );  
xx2 = 15*cos( 7*pi*tt - pi/6 );  
xx = xxe.*xx1 + xxe.*xx2;
```

(a) If the signal $x_1(t)$ corresponds to the MATLAB vector `xx1`, determine the complex amplitude of $x_1(t)$.

(b) If the signal $x(t)$ corresponds to the MATLAB vector `xx`, then it is possible to express $x(t)$ in the form

$$x(t) = Ae^{-\beta|t|} \cos(\omega t + \phi)$$

Determine the numerical values of A , β , ω , and ϕ . *Hint:* Use phasor addition.

$A =$ _____

$\beta =$ _____

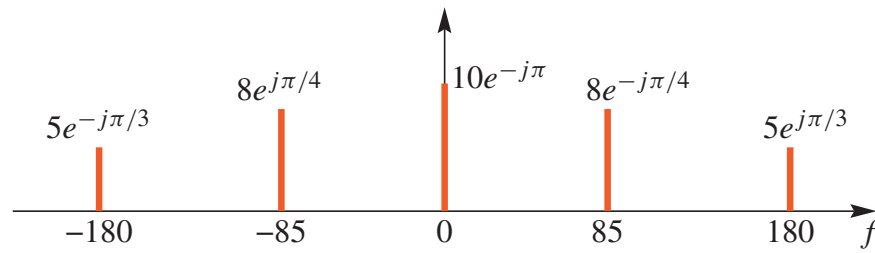
$\omega =$ _____

$\phi =$ _____

PROBLEM Spring-02-Q.1.4:

This problem involves the relationship between the time domain $x(t)$ and the frequency domain (spectrum).

- (a) The spectrum of a signal $x(t)$ is shown in the following figure:



Note: the frequency axis is cyclic frequency (f) in Hz.

Write an equation for $x(t)$ in terms of cosine functions.

- (b) Make a plot of the spectrum of the signal $y(t)$ defined as follows:

$$y(t) = (-2 + 4 \cos(30\pi t))^2$$

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
QUIZ #1

DATE: Jan. 31, 2003

COURSE: ECE-2025

NAME: Answer Key
LAST, FIRST

STUDENT #: C

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L01:Tues-9:30am (McLaughlin) L02:Thur-9:30am (Barry)
L03:Tues-Noon (McLaughlin) L04:Thur-Noon (Barry)
L05:Tues-1:30pm (Li)
L11:M-3pm (McClellan) L07:Tues-3pm (Li) L12:W-3pm (Hayes) L08:Thur-3pm (Williams)
L09:Tues-4:30pm (Zhou) L14:W-4:30pm (Hayes)
L10:Tues-6pm (Zhou) RPK:Thur-Late (Tugcu)

- Write your name on the front page **ONLY**. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2} \times 11$ ") of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- Justify your reasoning **CLEARLY** to receive partial credit.
Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
Only these answers will be graded. Circle your answers, or write them in the boxes provided.
If space is needed for scratch work, use the backs of previous pages.

<i>Problem</i>	<i>Value</i>	<i>Score</i>
1	25	
2	25	
3	25	
4	25	

PROBLEM Spring-02-Q.1.1:

For each of the following signals, pick one of the representations below that defines *exactly* the same signal. Write your answer ((a), (b), (c), (d), (e), or (f)) in the box next to each signal.

$$\text{ANS} = \boxed{e} \quad -\cos(27\pi t - 3\pi/5) = \cos(27\pi t - 3\pi/5 + \pi) = \cos(27\pi t + 2\pi/5)$$

$$\text{ANS} = \boxed{d} \quad \Re\{j e^{j2\pi/5} e^{j27\pi t}\} = \Re\{|e^{j0}\}| = \Re\{1\} = 1$$

$$\text{ANS} = \boxed{c} \quad \frac{1}{2} e^{-j\pi/5} e^{j27\pi t} + \frac{1}{2} e^{j\pi/5} e^{-j27\pi t} = 2 \Re\left\{ \frac{1}{2} e^{-j\pi/5} e^{j27\pi t} \right\} = \cos(27\pi t - \pi/5)$$

$$\text{ANS} = \boxed{b} \quad \frac{1}{2} e^{j\pi/2} e^{j27\pi t} + \frac{1}{2} e^{-j\pi/2} e^{-j27\pi t} = \cos(27\pi t + \pi/2)$$

$$\text{ANS} = \boxed{a} \quad \cos(27\pi t + 6\pi/5) = \cos(27\pi t + 6\pi/5 - 2\pi) = \cos(27\pi t - 4\pi/5)$$

POSSIBLE ANSWERS:

Your answer will be one of the following choices.

Please note: some of the following signals could be used more than one time to match the above signals.

$$(a) \quad x_a(t) = \Re\{e^{-j4\pi/5} e^{j27\pi t}\} = \cos(27\pi t - 4\pi/5)$$

$$(b) \quad x_b(t) = -\sin(27\pi t) = -\cos(27\pi t - \pi/2) = \cos(27\pi t - \pi/2 + \pi) \\ = \cos(27\pi t + \pi/2)$$

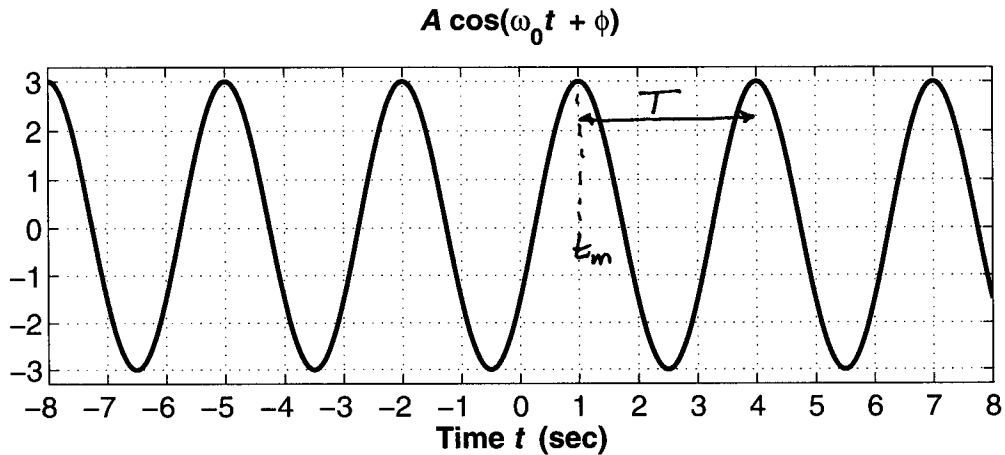
$$(c) \quad x_c(t) = \cos(27\pi t + 9\pi/5) = \cos(27\pi t + 9\pi/5 - 2\pi) = \cos(27\pi t - \pi/5)$$

$$(d) \quad x_d(t) = 1$$

$$(e) \quad x_e(t) = \cos(27\pi t + 2\pi/5)$$

$$(f) \quad x_f(t) = 0$$

PROBLEM Spring-02-Q.1.2:



The above graph is a plot of a sinusoidal signal $x(t) = A \cos(\omega_0 t + \phi)$.

- (a) Determine numerical values for A , ω_0 and ϕ with $-\pi < \phi \leq \pi$. Include units for ω_0 and ϕ .

$$\underline{A = 3}$$

$$T = 4 - 1 = 3 \text{ sec.}$$

$$\underline{\omega_0 = 2\pi/3 \text{ rad/s}}$$

$$f = 1/T = 1/3 \Rightarrow \omega_0 = 2\pi f = 2\pi/3$$

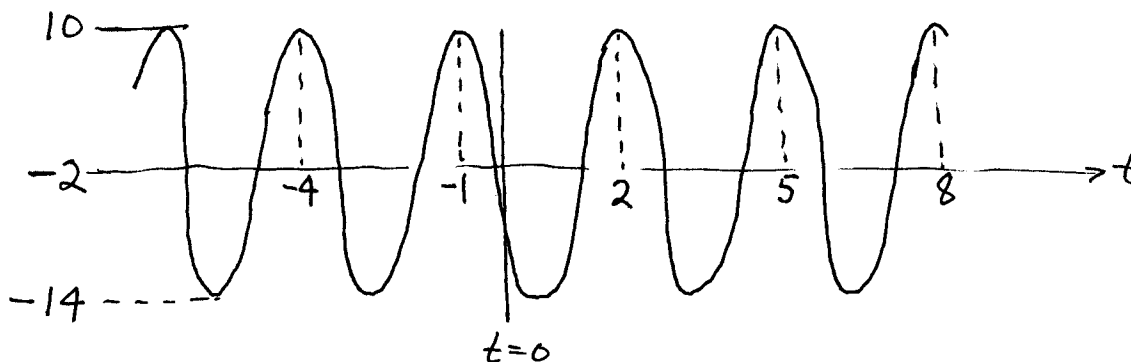
$$\underline{\phi = -2\pi/3 \text{ rad.}}$$

$$t_m = 1 \text{ sec.}$$

$$\phi = -\omega_0 t_m = -\left(\frac{2\pi}{3}\right)(1) = -2\pi/3$$

- (b) Using $x(t)$ from above, define a new signal as $y(t) = 4x(t - 1) - 2$. Make a plot of $y(t)$ over the same range as above. Label everything carefully.

change amplitude.
 DC becomes -2
 time shift to the right by 1 sec.



PROBLEM Spring-02-Q.1.3:

The following MATLAB code defines several signals that are then multiplied and summed:

```
tt = -10:0.001:10;
xxe = exp(-9*abs(tt));
xx1 = 20*cos( 7*pi*(tt + 1/3) );
xx2 = 15*cos( 7*pi*tt - pi/6 );
xx = xxe.*xx1 + xxe.*xx2;
```

- (a) If the signal $x_1(t)$ corresponds to the MATLAB vector $xx1$, determine the complex amplitude of $x_1(t)$.

$$X_1 = A_1 e^{j\phi_1} = 20 e^{j7\pi/3}$$

- (b) If the signal $x(t)$ corresponds to the MATLAB vector xx , then it is possible to express $x(t)$ in the form

$$x(t) = A e^{-\beta|t|} \cos(\omega t + \phi)$$

Determine the numerical values of A , β , ω , and ϕ . *Hint: Use phasor addition.*

$$A = 25$$

$$\beta = 9$$

$$\omega = 7\pi \text{ rad/s}$$

$$\phi = 0.404 \text{ rad.}$$

$$x_1(t) = 20 \cos(7\pi t + 7\pi/3) \quad \text{subtract } 2\pi \text{ from the phase}$$

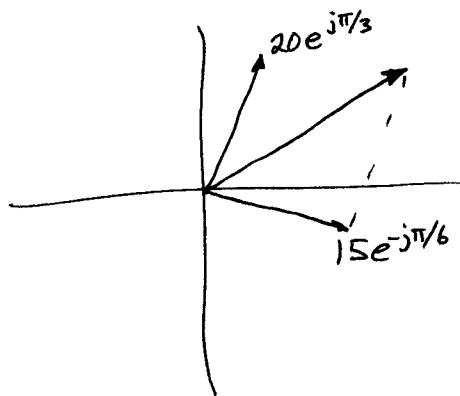
$$x_2(t) = 15 \cos(7\pi t - \pi/6)$$

$$x_e(t) = e^{-9|t|}$$

$$x(t) = e^{-9|t|} \left(20 \cos(7\pi t + \pi/3) + 15 \cos(7\pi t - \pi/6) \right) \quad \text{phasor add.}$$

$$\begin{aligned} & 20e^{j\pi/3} + 15e^{-j\pi/6} \\ &= 25e^{j0.404} \\ &= 25e^{j0.129\pi} \end{aligned}$$

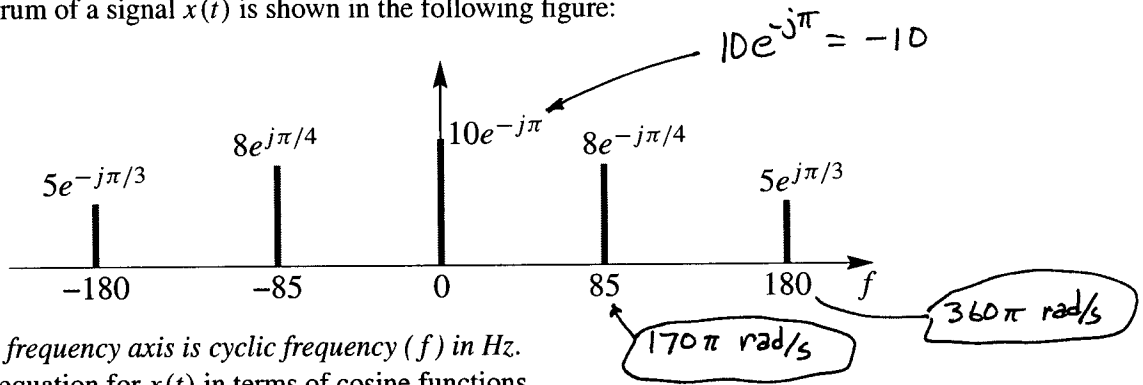
$\nearrow 23.13^\circ$



PROBLEM Spring-02-Q.1.4:

This problem involves the relationship between the time domain $x(t)$ and the frequency domain (spectrum).

(a) The spectrum of a signal $x(t)$ is shown in the following figure:



Note: the frequency axis is cyclic frequency (f) in Hz.
Write an equation for $x(t)$ in terms of cosine functions.

Read values from the spectrum:

$$x(t) = -10 + 16 \cos(170\pi t - \pi/4) + 10 \cos(360\pi t + \pi/3)$$

(b) Make a plot of the spectrum of the signal $y(t)$ defined as follows:

$$y(t) = (-2 + 4 \cos(30\pi t))^2$$

$$y(t) = (-2 + 2e^{j30\pi t} + 2e^{-j30\pi t})^2$$

$$= 4 + 4e^{j60\pi t} + 4e^{-j60\pi t} - 8e^{j30\pi t} + 8 - 8e^{-j30\pi t}$$

$-8 = 8e^{j\pi}$
 \downarrow
 15 Hz

