

Lecture 20
Amplitude Modulation (AM)
12-Nov-00

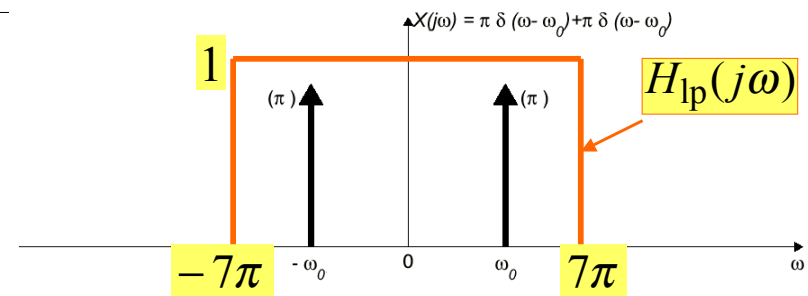
Info: Web-CT, Lab, HW

- Calendar:
 - Quiz #3 is Monday, 19-Nov
 - One page hand-written notes
 - Calculator is OK
 - Covers CT signals/sys, Impulses, Convolution and Fourier Transform, AND z-Transform
- Quiz Review: Sunday (18-Nov) @ 6:30 pm
- Prob Set #10 is due this week
- Labs will be held on 19 & 20 Nov

Info: Final Exam, etc.

- Finals Week:
 - 11am Lecture: Thurs, 13-Dec @ 11:30am
 - Noon Lecture: Mon, 10-Dec @ 2:50pm
 - Must take the Final with your assigned section.
- Reading Assignment: Ch 13 of Notes.
- Prob Set #11 - due after Holiday
- Lab #11 due week of 26-Nov
 - Lab #12 due last week (3 — 6 Dec)
 - ALL Labs must be turned in by 7-Dec-01 (Friday)

Pop Quiz



$x(t) = \cos(3\pi t)$

$\omega_0 = ?$
 $y(t) = ???$

LECTURE OBJECTIVES

- Review of FT properties
 - Convolution \leftrightarrow multiplication
 - Frequency shifting
- Sinewave Amplitude Modulation
 - AM radio
- Frequency-division multiplexing
 - FDM

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Table of Easy FT Properties

Linearity Property

$$ax_1(t) + bx_2(t) \Leftrightarrow aX_1(j\omega) + bX_2(j\omega)$$

Delay Property

$$x(t - t_d) \Leftrightarrow e^{-j\omega t_d} X(j\omega)$$

Frequency Shifting

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

Scaling

$$x(at) \Leftrightarrow \frac{1}{|a|} X(j(\frac{\omega}{a}))$$

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Table of FT Properties

$$x(t) * h(t) \Leftrightarrow H(j\omega)X(j\omega)$$

$$x(t)p(t) \Leftrightarrow \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

Differentiation Property

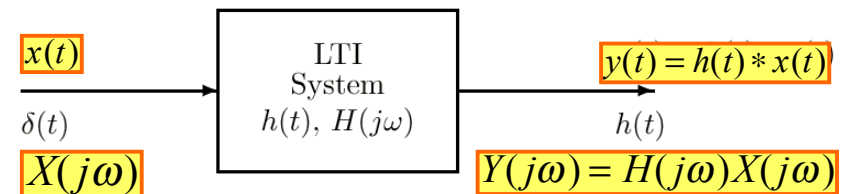
$$\frac{dx(t)}{dt} \Leftrightarrow (j\omega)X(j\omega)$$

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Convolution Property



- Convolution in the time-domain

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau$$

corresponds to **MULTIPLICATION** in the frequency-domain

$$Y(j\omega) = H(j\omega)X(j\omega)$$

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Cosine Input to LTI System

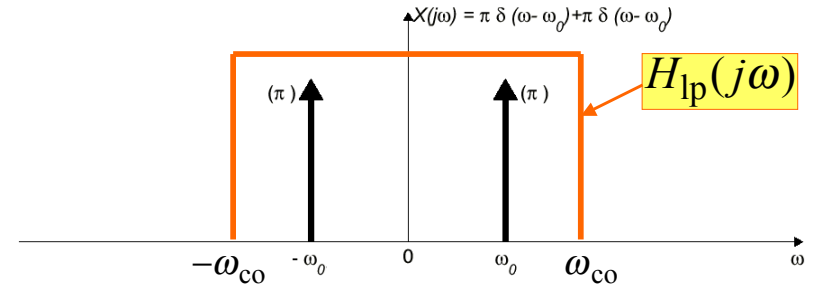
$$\begin{aligned}
 Y(j\omega) &= H(j\omega)X(j\omega) \\
 &= H(j\omega)[\pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)] \\
 &= H(j\omega_0)\pi\delta(\omega - \omega_0) + H(-j\omega_0)\pi\delta(\omega + \omega_0) \\
 y(t) &= H(j\omega_0)\frac{1}{2}e^{j\omega_0 t} + H(-j\omega_0)\frac{1}{2}e^{-j\omega_0 t} \\
 &= H(j\omega_0)\frac{1}{2}e^{j\omega_0 t} + H^*(j\omega_0)\frac{1}{2}e^{-j\omega_0 t} \\
 &= |H(j\omega_0)|\cos(\omega_0 t + \angle H(j\omega_0))
 \end{aligned}$$

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Ideal Lowpass Filter

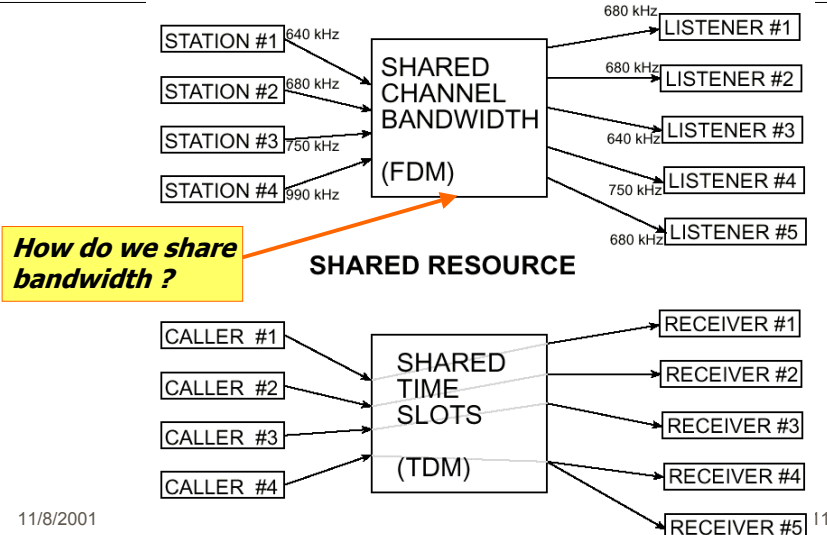


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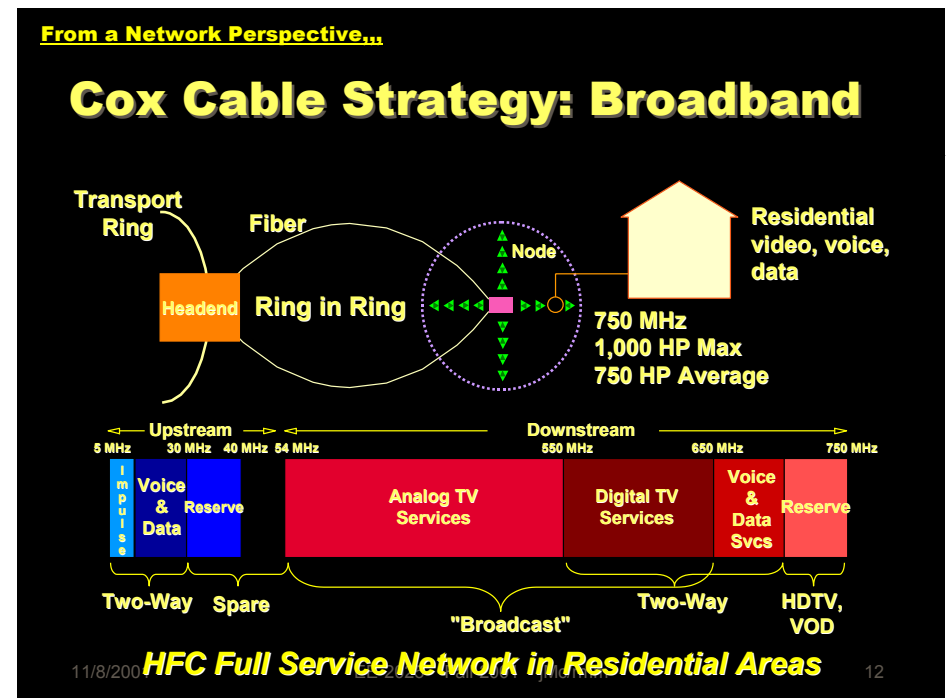
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The way communication systems work



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Table of FT Properties

$$x(t) * h(t) \Leftrightarrow H(j\omega)X(j\omega)$$

$$x(t)p(t) \Leftrightarrow \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

Differentiation Property

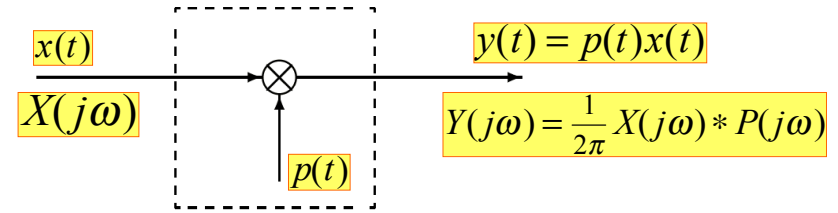
$$\frac{dx(t)}{dt} \Leftrightarrow (j\omega)X(j\omega)$$

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Signal Multiplier (Modulator)



- Multiplication in the time-domain corresponds to convolution in the frequency-domain.

$$Y(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\theta)P(j(\omega - \theta))d\theta$$

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$$y(t) = x(t)p(t) \Leftrightarrow Y(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$x(t) = \begin{cases} 1 & |t| < T/2 \\ 0 & |t| > T/2 \end{cases} \Leftrightarrow X(j\omega) = \frac{\sin(\omega T/2)}{(\omega/2)}$$

$$p(t) = \cos(\omega_0 t) \Leftrightarrow P(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$$

$$y(t) = x(t)\cos(\omega_0 t) \Leftrightarrow$$

$$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * [\pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)]$$

$$Y(j\omega) = \frac{1}{2} X(j(\omega - \omega_0)) + \frac{1}{2} X(j(\omega + \omega_0))$$

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Frequency Shifting Property

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

$$\begin{aligned} \int_{-\infty}^{\infty} e^{-j\omega_0 t} x(t) e^{-j\omega t} dt &= \int_{-\infty}^{\infty} x(t) e^{-j(\omega - \omega_0)t} dt \\ &= X(j(\omega - \omega_0)) \end{aligned}$$

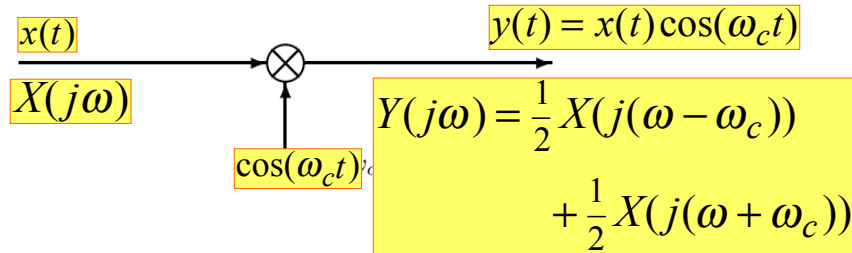
$$y(t) = \frac{\sin 7t}{\pi t} e^{j\omega_0 t} \Leftrightarrow Y(j\omega) = \begin{cases} 1 & \omega_0 - 7 < \omega < \omega_0 + 7 \\ 0 & \text{elsewhere} \end{cases}$$

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Amplitude Modulator



- $x(t)$ **modulates** the amplitude of the cosine wave. The result in the frequency-domain is two shifted copies of $X(j\omega)$.

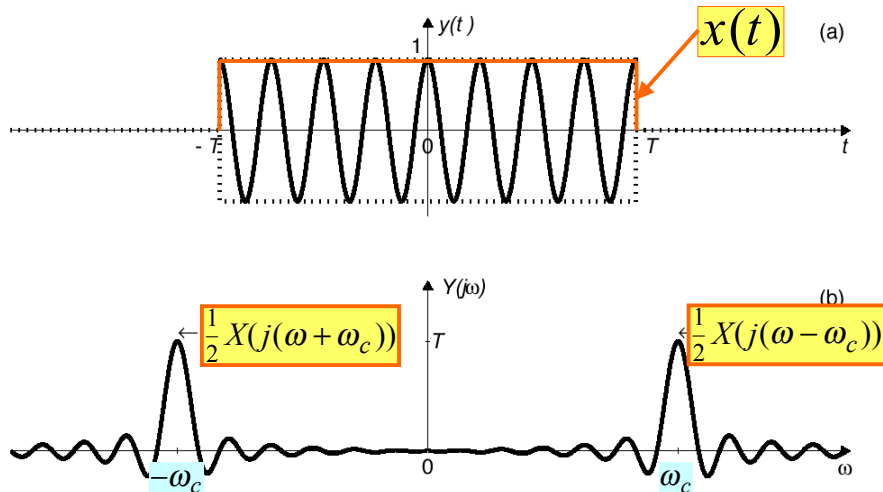
$$y(t) = x(t) \cos(\omega_c t) \Leftrightarrow Y(j\omega) = \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c))$$

$$x(t) = \begin{cases} 1 & |t| < T \\ 0 & |t| > T \end{cases} \Leftrightarrow X(j\omega) = 2 \frac{\sin(\omega T)}{\omega}$$

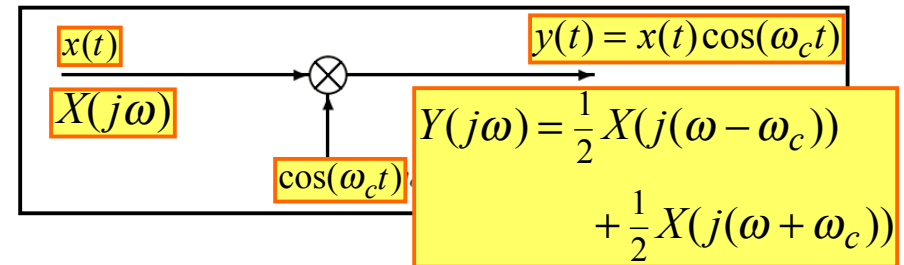
$$y(t) = x(t) \cos(\omega_c t) \Leftrightarrow$$

$$Y(j\omega) = \frac{\sin((\omega - \omega_c))}{(\omega - \omega_c)} + \frac{\sin((\omega + \omega_c))}{(\omega + \omega_c)}$$

$$y(t) = x(t) \cos(\omega_c t) \Leftrightarrow Y(j\omega) = \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c))$$



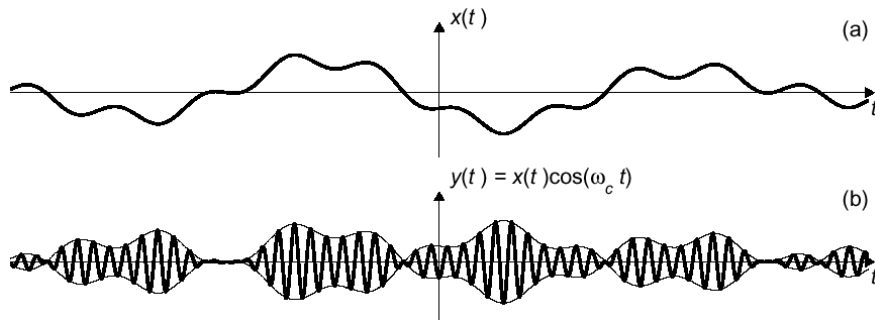
DSBAM Modulator



- If $X(j\omega)=0$ for $|\omega| > \omega_b$ and $\omega_c > \omega_b$, the result in the frequency-domain is two shifted and scaled **exact copies** of $X(j\omega)$.

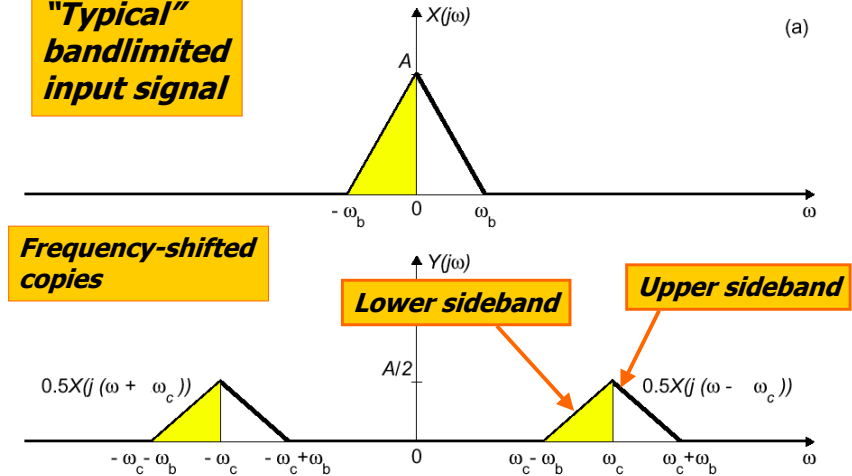
DSBAM Waveform

In the time-domain, the "envelope" of sine-wave peaks follows $|x(t)|$

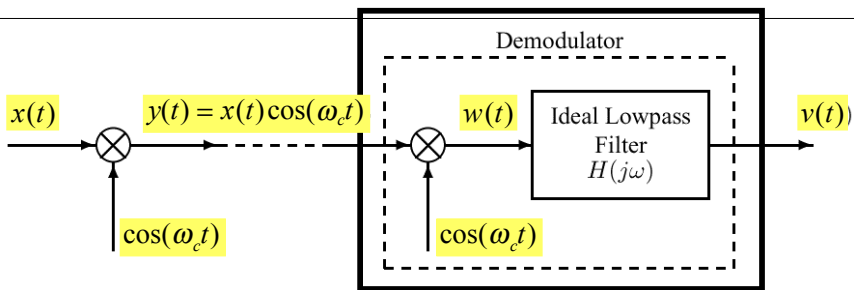


Double Sideband AM (DSBAM)

"Typical" bandlimited input signal



DSBAM Demodulator

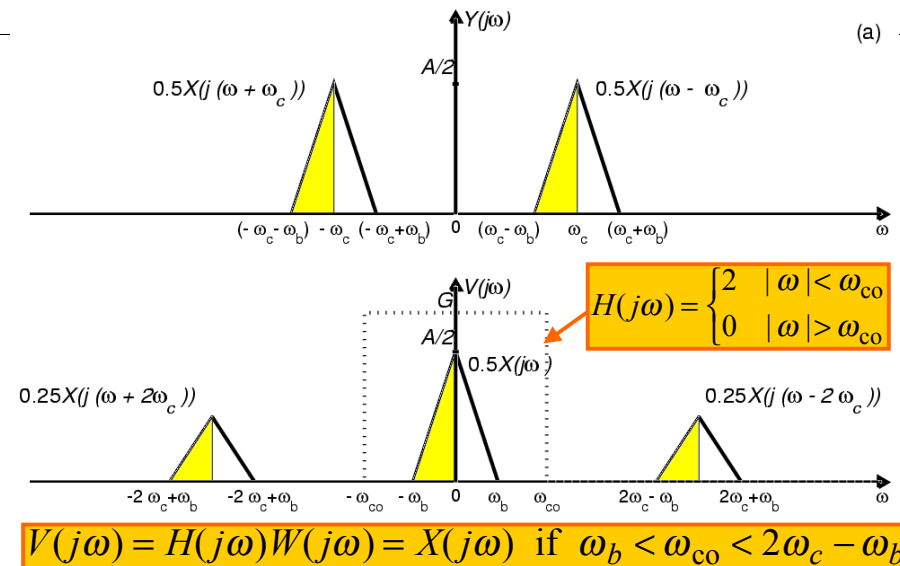


$$w(t) = x(t)[\cos(\omega_c t)]^2 = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2\omega_c t)$$

$$W(j\omega) = \frac{1}{2}X(j\omega) + \frac{1}{4}X(j(\omega - 2\omega_c)) + \frac{1}{4}X(j(\omega + 2\omega_c))$$

$$V(j\omega) = H(j\omega)W(j\omega)$$

DSBAM Demodulation



$V(j\omega) = H(j\omega)W(j\omega) = X(j\omega)$ if $\omega_b < \omega_{co} < 2\omega_c - \omega_b$

Frequency-Division Multiplexing (FDM)

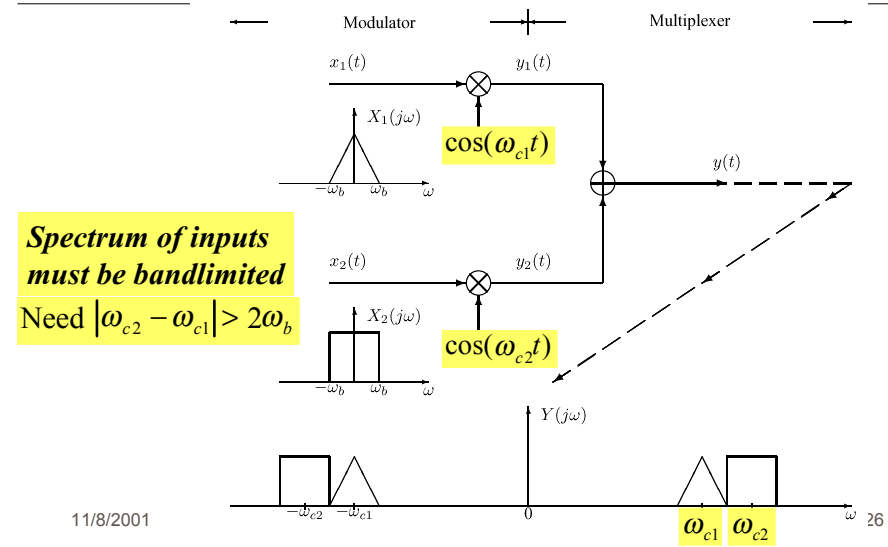
- Shifting spectrum of signal to higher frequency:
 - Permits transmission of low-frequency signals with high-frequency EM waves
 - By allocating a frequency band to each signal multiple **bandlimited** signals can share the same channel
 - AM radio: 530-1620 kHz (10 kHz bands)
 - FM radio: 88.1-107.9 MHz (200 kHz bands)

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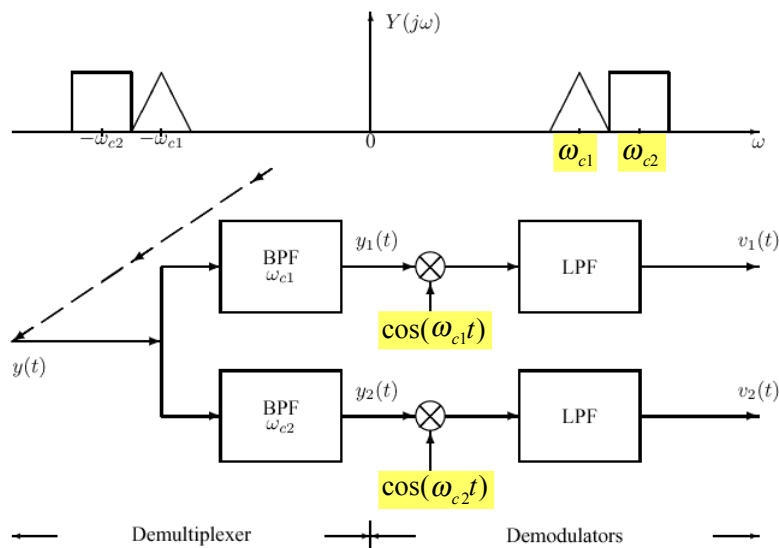
FDM Block Diagram



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Frequency-Division De-Mux



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Bandpass Filters for De-Mux

