

**Lecture 14**  
**Zeros of H(z)**  
**Frequency Domain**  
**19-Oct-01**

**Info: Quiz #2**

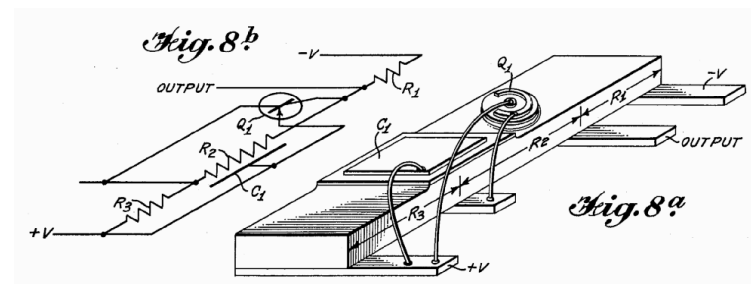
- Quiz #2 is 22-Oct (Monday)
  - Coverage: HW #3 -- #7
  - One page of notes (8.5 by 11, two sided)
  - Calculator OK
- Review Session 6:30pm on Sunday
  - ECE Auditorium on 21-Oct
- Prob Set #7 was due this past week
  - Solution will be posted Friday nite @ 6:00 pm

**Info: Web-CT, Lab, HW**

- NEW CHAPTERS will be available
  - Chapters 10 - 13
  - Continuous-Time Signals & Systems
  - PDF (Web-CT) or Bookstore
- Lab #8 on Touch-Tone Decoding
  - Bandpass Filters detect tones

**NOBEL PRIZE WINNER**

- Integrated Circuit: 1959 by Jack Kilby
  - <http://www.eepatents.com/feature/>



## READING ASSIGNMENTS

- This Lecture:
  - Chapter 7, pp. 220-230
- Other Reading:
  - Recitation & Lab: Ch. 7, pp. 220-239
    - ZEROS (and POLES)
  - Next Lecture: Notes on Continuous-Time

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## LECTURE OBJECTIVES

- ZEROS and POLES
- Relate  $H(z)$  to FREQUENCY RESPONSE

$$H(\hat{\omega}) = H(z) \Big|_{z=e^{j\hat{\omega}}}$$

- THREE DOMAINS:
  - Show Relationship for FIR:

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

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## DESIGN PROBLEM

- Example:
  - Design a Lowpass FIR filter (Find  $b_k$ )
  - Reject completely  $0.7\pi$ ,  $0.8\pi$ , and  $0.9\pi$
  - Estimate the filter length needed to accomplish this task. How many  $b_k$ ?
- Z POLYNOMIALS provide the TOOLS

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## Z-Transform DEFINITION

- POLYNOMIAL Representation of LTI SYSTEM:

$$H(z) = \sum_n h[n]z^{-n}$$

- EXAMPLE:

$$\{h[n]\} = \{2, 0, -3, 0, 2\}$$

$$H(z) = 2z^{-0} + 0z^{-1} - 3z^{-2} + 0z^{-3} + 2z^{-4}$$

$$= 2 - 3z^{-2} + 2z^{-4}$$

$$= 2 - 3(z^{-1})^2 + 2(z^{-1})^4$$

APPLIES to  
Any SIGNAL

POLYNOMIAL in  $z^{-1}$

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# CONVOLUTION PROPERTY

- Convolution in the **n**-domain  
 | SAME AS  
 Multiplication in the **z**-domain

$$y[n] = h[n] * x[n] \Leftrightarrow Y(z) = H(z)X(z)$$

$$y[n] = h[n] * x[n] = \sum_{k=0}^M h[k]x[n-k]$$

FIR Filter

MULTIPLY Z-TRANSFORMS

# CONVOLUTION EXAMPLE



$$x[n] = \delta[n-1] + 2\delta[n-2] \quad h[n] = \delta[n] - \delta[n-1]$$

$$y[n] = x[n] * h[n]$$

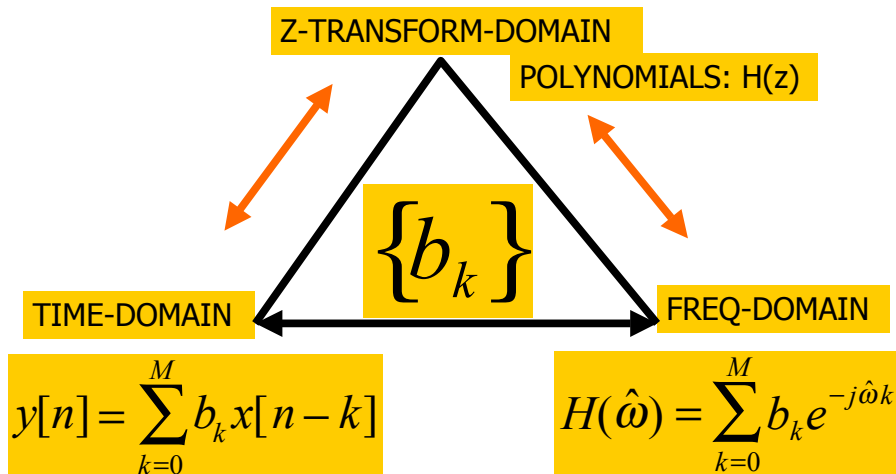
$$X(z) = z^{-1} + 2z^{-2}$$

$$H(z) = 1 - z^{-1}$$

$$Y(z) = (z^{-1} + 2z^{-2})(1 - z^{-1}) = z^{-1} + z^{-2} - 2z^{-3}$$

$$y[n] = \delta[n-1] + \delta[n-2] - 2\delta[n-3]$$

# THREE DOMAINS



# FREQUENCY RESPONSE ?

- Same Form:

$\hat{\omega}$  - Domain

$$H(\hat{\omega}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k}$$

$$H(\hat{\omega}) = \sum_{k=0}^M b_k (e^{j\hat{\omega}})^{-k}$$

$$z = e^{j\hat{\omega}}$$

z - Domain

$$H(z) = \sum_{k=0}^M b_k z^{-k}$$

SAME COEFFICIENTS

## CHANGE in NOTATION

- Relate  $H(z)$  to FREQUENCY RESPONSE

$$H(\hat{\omega}) = H(z) \Big|_{z=e^{j\hat{\omega}}}$$

- NEW NOTATION for FREQUENCY RESPONSE

$$H(\hat{\omega}) \leftrightarrow H(e^{j\hat{\omega}})$$

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## ANOTHER ANALYSIS TOOL

- z-Transform POLYNOMIALS are EASY !

┆ ROOTS, FACTORS, etc.

- **ZEROS and POLES: where is  $H(z) = 0$  ?**

- The z-domain is **COMPLEX**

┆  $H(z)$  is a **COMPLEX-VALUED** function of a **COMPLEX VARIABLE**  $z$ .

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## ZEROS of $H(z)$

- Find  $z$ , where  $H(z)=0$

$$H(z) = 1 - \frac{1}{2}z^{-1}$$

$$1 - \frac{1}{2}z^{-1} = 0 ?$$

$$z - \frac{1}{2} = 0$$

$$\text{Zero at: } z = \frac{1}{2}$$

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## ZEROS of $H(z)$

- Find  $z$ , where  $H(z)=0$

┆ Interesting when  $z$  is ON the unit circle.

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

$$H(z) = (1 - z^{-1})(1 - z^{-1} + z^{-2})$$

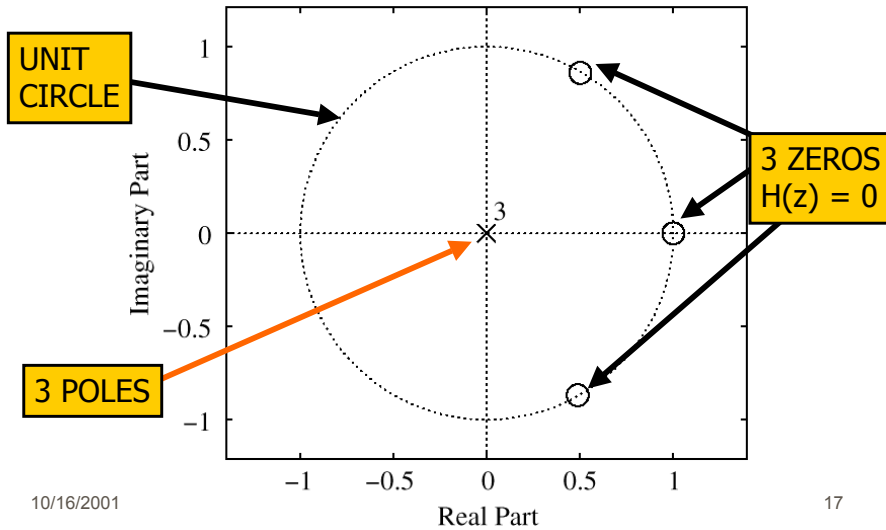
$$\text{Roots: } z = 1, \frac{1}{2} \pm j\frac{\sqrt{3}}{2} \quad \boxed{e^{\pm j\pi/3}}$$

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## PLOT ZEROS in z-DOMAIN



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## POLES of $H(z)$

- Find  $z$ , where  $H(z) \rightarrow \infty$ 
  - Not very interesting for the FIR case

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

$$H(z) = \frac{z^3 - 2z^2 + 2z - 1}{z^3}$$

Three Poles at:  $z = 0$

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## FREQ. RESPONSE from ZEROS

$$H(e^{j\hat{\omega}}) = H(z) \Big|_{z=e^{j\hat{\omega}}}$$

- Relate  $H(z)$  to FREQUENCY RESPONSE
- EVALUATE  $H(z)$  on the **UNIT CIRCLE**
  - ANGLE is same as FREQUENCY

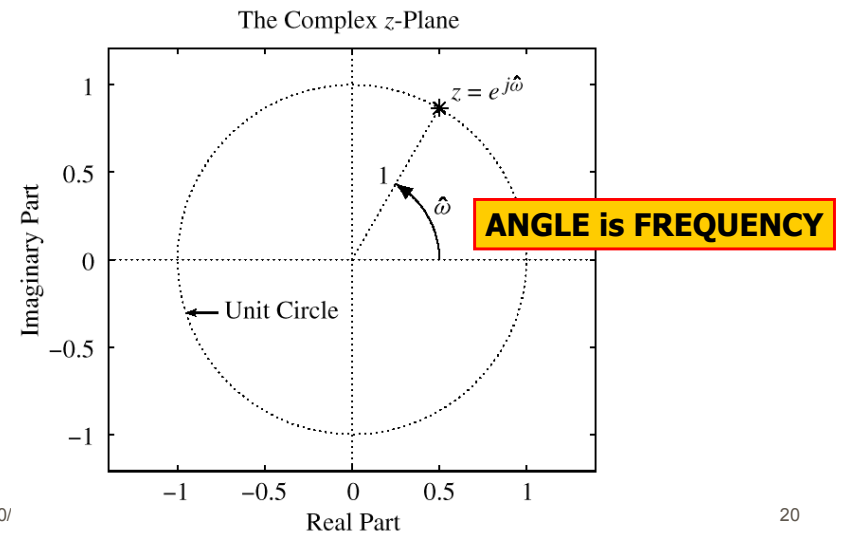
$z = e^{j\hat{\omega}}$  (as  $\hat{\omega}$  varies)  
defines a CIRCLE, radius = 1

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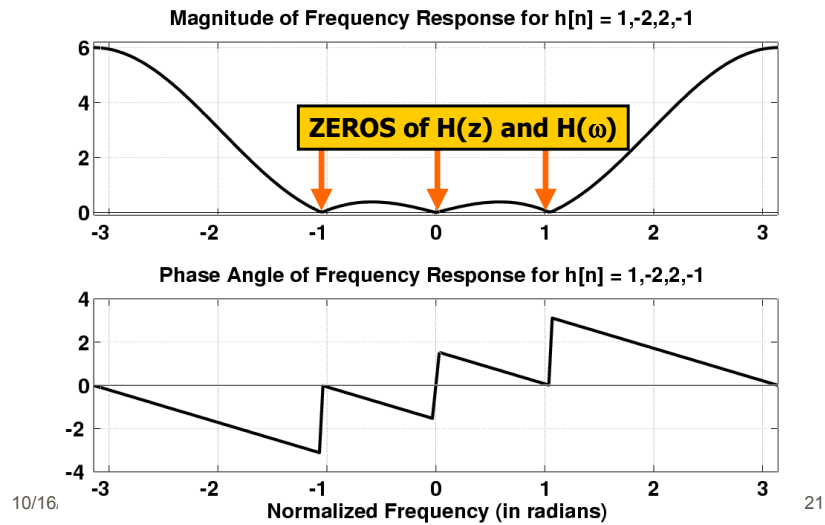
$$\mathcal{H}(\hat{\omega}) = H(e^{j\hat{\omega}}) = H(z) \Big|_{z=e^{j\hat{\omega}}}$$



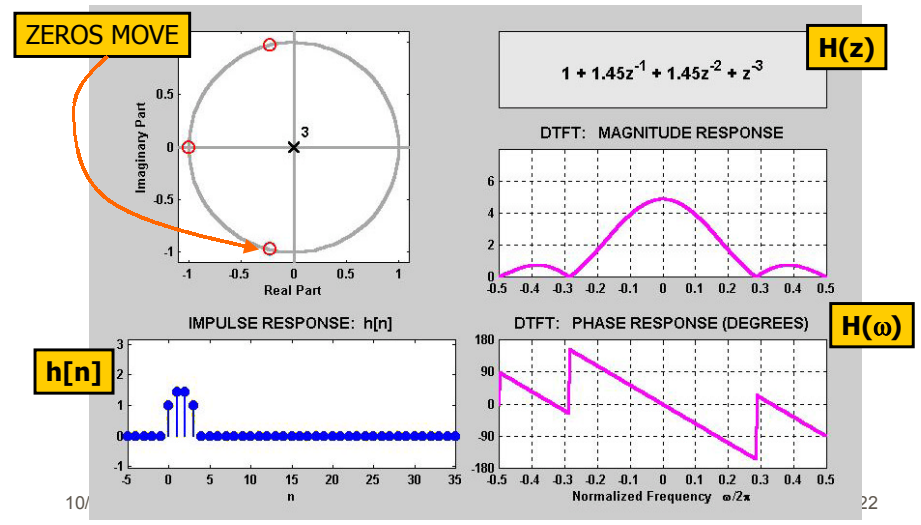
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# FIR Frequency Response



# 3 DOMAINS MOVIE: FIR



# NULLING PROPERTY of $H(z)$

- When  $H(z)=0$  on the unit circle.
- Find inputs  $x[n]$  that give zero output

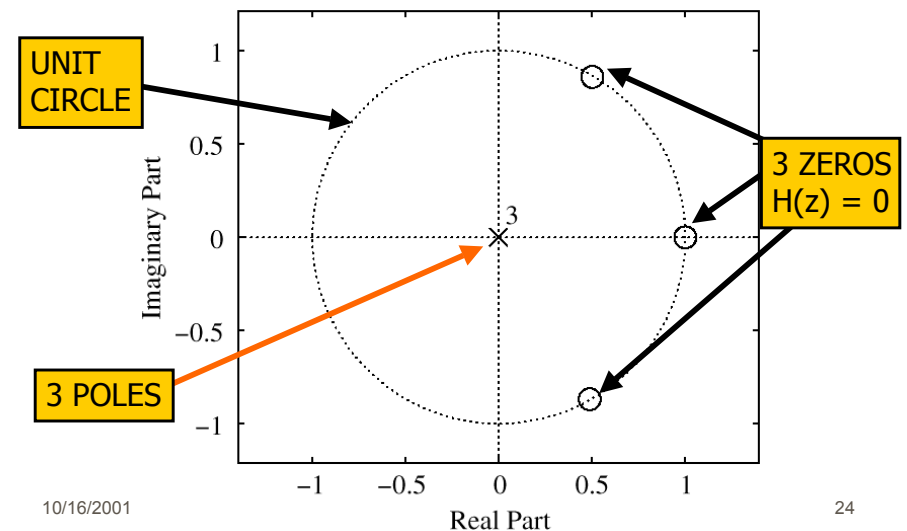
$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

$$H(e^{j\hat{\omega}}) = 1 - 2e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

$$x[n] \xrightarrow{H(z)} y[n] \quad H(e^{j\pi/3}) = ?$$

$$x[n] = e^{j(\pi/3)n} \quad y[n] = H(e^{j(\pi/3)}) \cdot e^{j(\pi/3)n}$$

# PLOT ZEROS in z-DOMAIN



## NULLING PROPERTY of H(z)

- Evaluate H(z) at the input "frequency"

$$H(e^{j\hat{\omega}}) = 1 - 2e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

$$y[n] = H(e^{j\pi/3}) \cdot e^{j(\pi/3)n}$$

$$y[n] = (1 - 2e^{-j\pi/3} + 2e^{-j2\pi/3} - e^{-j3\pi/3}) \cdot e^{j(\pi/3)n}$$

$$(1 - 2(\frac{1}{2} - j\frac{\sqrt{3}}{2}) + 2(-\frac{1}{2} - j\frac{\sqrt{3}}{2}) + 1)$$

$$y[n] = (1 - 1 + j\sqrt{3} - 1 - j\sqrt{3} + 1) \cdot e^{j(\pi/3)n} = 0$$

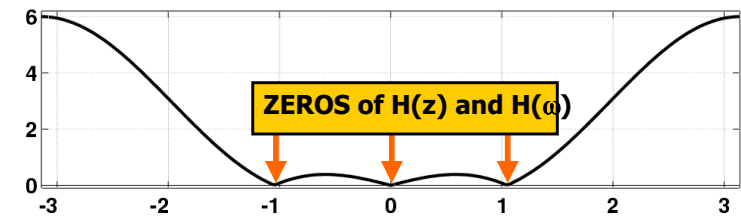
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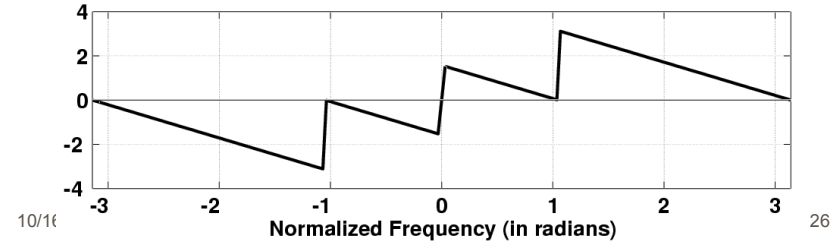
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## FIR Frequency Response

Magnitude of Frequency Response for h[n] = 1,-2,2,-1



Phase Angle of Frequency Response for h[n] = 1,-2,2,-1



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Normalized Frequency (in radians)

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