

EE-2025

Fall-2001

Lecture 4
Spectrum Representation
31-Aug-01

Web-CT Info

- Bulletin Board has all **OFFICIAL** msgs
- Lectures are being posted
 - PDF format (4 per page)
- **Revised** Version of Ch 3 available as PDF
 - Fourier Series material has been changed
- Upcoming Events:
 - Quiz #1 on 17-Sept (Monday)

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James THURBER

- It is better to know some of the questions than all of the answers

Lab Info

- Lab #1 Report
 - Turn in at beginning of your lab time
 - Write up section 3 only
- Lab #2 will be posted later today
 - For week of 4-Sept
- HARDER Pre-Lab Questions !!!
- Finish Instructor Verification in Lab
- Computer Problems? help@ece.gatech.edu

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Homework Info

- Written HW #1 due NEXT WEEK
 - In Recitation, at the beginning
 - Monday Rec?...turn in HW at Lab on Wed.
- HW #1 Solutions posted
 - Friday evening
- HW and MATLAB Help:
 - **Tues-11am, Wed-6pm VL-261**
 - The rest of the semester: Mon-6 and Tues-11

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Homework Formatting

- Cover page with
 - Name
 - Lab section, ie, L05, L21, etc.
 - Recitation Prof's name; Grader's name
- Write on ONE side only
 - Prefer Engineering paper, or plain
 - NO LINES
- STAPLE

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HISTORY

- Which company's first successful product was a sine-wave generator?
 - Variable frequency
 - Lab Instrument



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LECTURE OBJECTIVES

- Sinusoids with **DIFFERENT** Frequencies
 - SYNTHESIZE by Adding Sinusoids

$$x(t) = \sum_{k=1}^N A_k \cos(2\pi f_k t + \varphi_k)$$

- **SPECTRUM** Representation
 - Graphical Form shows **DIFFERENT** Freqs

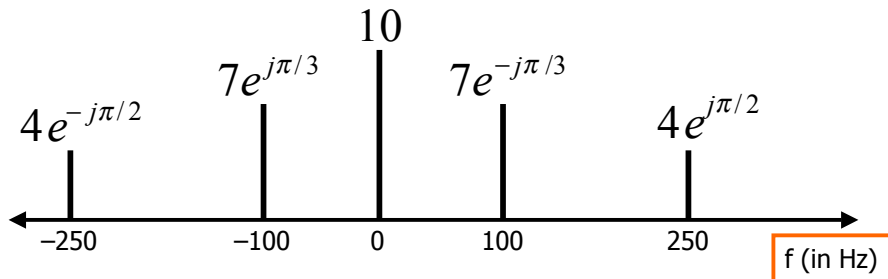
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FREQUENCY DIAGRAM

Plot Complex Amplitude vs. Freq



Another FREQ. Diagram

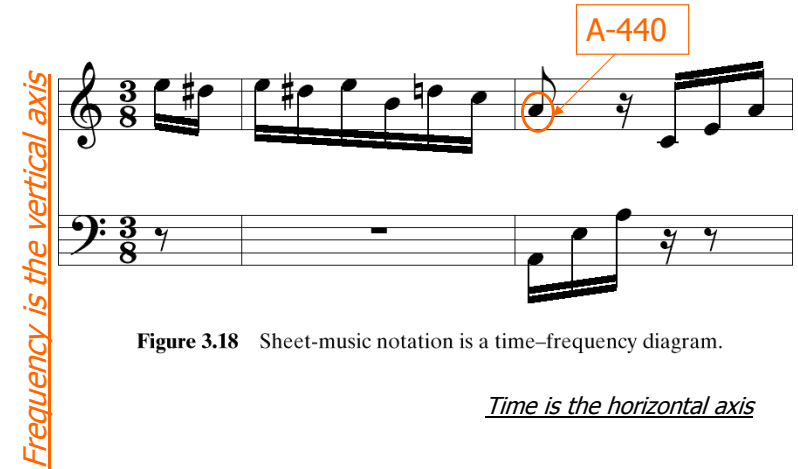




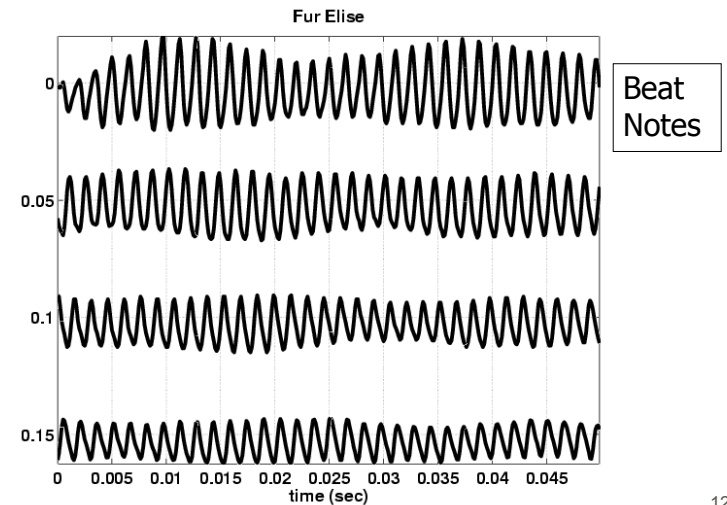
Figure 3.18 Sheet-music notation is a time–frequency diagram.

MOTIVATION

Synthesize Complicated Signals

- Musical Notes 
 - ┆ Piano uses 3 strings for many notes
 - ┆ Chords: play several notes simultaneously
- Human Speech 
 - ┆ Vowels have dominant frequencies
 - ┆ Application: computer generated speech
- Can **all** signals be generated this way?
 - ┆ Sum of sinusoids?

Fur Elise WAVEFORM

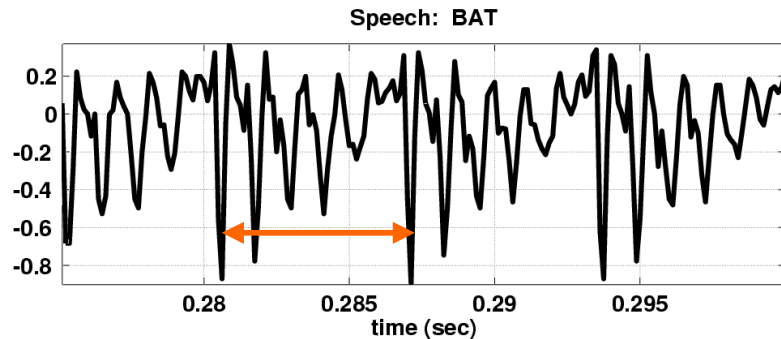


Speech Signal: BAT



- Nearly **Periodic** in Vowel Region

- Period is (Approximately) $T = 0.0065$ sec



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Euler's Formula Reversed

- Solve for **cosine** (or sine)

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

$$e^{-j\omega t} = \cos(-\omega t) + j \sin(-\omega t)$$

$$e^{-j\omega t} = \cos(\omega t) - j \sin(\omega t)$$

$$e^{j\omega t} + e^{-j\omega t} = 2 \cos(\omega t)$$

$$\cos(\omega t) = \frac{1}{2} (e^{j\omega t} + e^{-j\omega t})$$

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INVERSE Euler's Formula

- Solve for **cosine** (or sine)

$$\cos(\omega t) = \frac{1}{2} (e^{j\omega t} + e^{-j\omega t})$$

$$\sin(\omega t) = \frac{1}{2j} (e^{j\omega t} - e^{-j\omega t})$$

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SPECTRUM Interpretation

- Cosine = sum of 2 complex exponentials:

$$A \cos(7t) = \frac{A}{2} e^{j7t} + \frac{A}{2} e^{-j7t}$$

- One has a positive frequency
- The other has **negative** freq.
- Amplitude of each is half as big

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NEGATIVE FREQUENCY

- Is negative frequency real?
- Doppler Radar provides an example
 - Police radar measures speed by using the Doppler shift principle
 - Let's assume 400Hz <---> 60 mph
 - +400Hz means towards the radar
 - -400Hz means away (opposite direction)
 - Think of a train whistle

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SPECTRUM of SINE

- Sine = sum of 2 complex exponentials:

$$A \sin(7t) = \frac{A}{2j} e^{j7t} - \frac{A}{2j} e^{-j7t}$$

$$= \frac{1}{2} A e^{-j0.5\pi} e^{j7t} + \frac{1}{2} A e^{j0.5\pi} e^{-j7t}$$

$$\frac{-1}{j} = j = e^{j0.5\pi}$$

- Positive freq. has phase = -0.5π
- Negative freq. has phase = +0.5π

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GRAPHICAL SPECTRUM

EXAMPLE of SINE

$$A \sin(7t) = \frac{1}{2} A e^{-j0.5\pi} e^{j7t} + \frac{1}{2} A e^{j0.5\pi} e^{-j7t}$$



AMPLITUDE, PHASE & FREQUENCY are shown

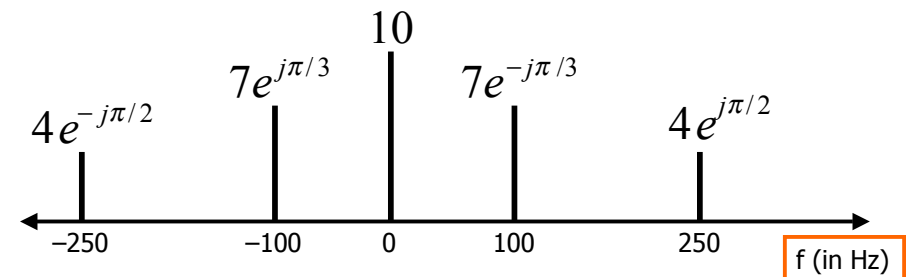
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SPECTRUM ---> SINUSOID

- Add the spectrum components:



What is the formula for the signal x(t)?

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Gather (A, ω, ϕ) information

Frequencies:

- -250 Hz
- -100 Hz
- 0 Hz
- 100 Hz
- 250 Hz

Amplitude & Phase

- 4 $-\pi/2$
- 7 $+\pi/3$
- 10 **0**
- 7 $-\pi/3$
- 4 $+\pi/2$

Note the **conjugate phase**

DC is another name for zero-freq component

DC component always has $\phi=0$ or π (for real $x(t)$)

Add Spectrum Components-1

Frequencies:

- -250 Hz
- -100 Hz
- 0 Hz
- 100 Hz
- 250 Hz

Amplitude & Phase

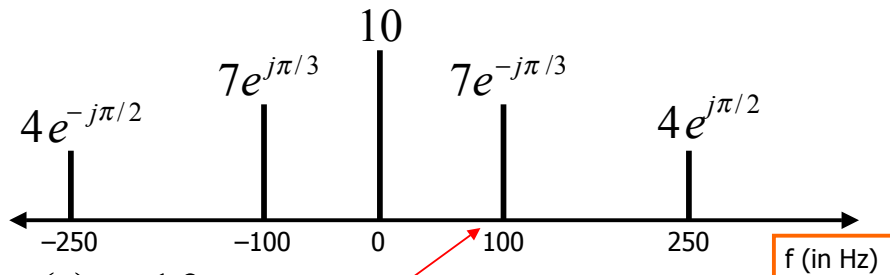
- 4 $-\pi/2$
- 7 $+\pi/3$
- 10 0
- 7 $-\pi/3$
- 4 $+\pi/2$

$$x(t) = 10 +$$

$$7e^{-j\pi/3} e^{j2\pi(100)t} + 7e^{j\pi/3} e^{-j2\pi(100)t}$$

$$+ 4e^{j\pi/2} e^{j2\pi(250)t} + 4e^{-j\pi/2} e^{-j2\pi(250)t}$$

Add Spectrum Components-2



$$x(t) = 10 +$$

$$7e^{-j\pi/3} e^{j2\pi(100)t} + 7e^{j\pi/3} e^{-j2\pi(100)t}$$

$$+ 4e^{j\pi/2} e^{j2\pi(250)t} + 4e^{-j\pi/2} e^{-j2\pi(250)t}$$

Simplify Components

$$x(t) = 10 +$$

$$7e^{-j\pi/3} e^{j2\pi(100)t} + 7e^{j\pi/3} e^{-j2\pi(100)t}$$

$$+ 4e^{j\pi/2} e^{j2\pi(250)t} + 4e^{-j\pi/2} e^{-j2\pi(250)t}$$

Use Euler's Formula to get **REAL** sinusoids:

$$A \cos(\omega t + \varphi) = \frac{A}{2} e^{j\varphi} e^{j\omega t} + \frac{A}{2} e^{-j\varphi} e^{-j\omega t}$$

FINAL ANSWER

$$x(t) = 10 + 14 \cos(2\pi(100)t - \pi/3) + 8 \cos(2\pi(250)t + \pi/2)$$

So, we get the general form:

$$x(t) = \sum_{k=1}^N A_k \cos(2\pi f_k t + \varphi_k)$$

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Summary: GENERAL FORM

$$x(t) = A_0 + \sum_{k=1}^N A_k \cos(2\pi f_k t + \varphi_k)$$

$$x(t) = X_0 + \sum_{k=1}^N \Re\{X_k e^{j2\pi f_k t}\}$$

$$X_k = A_k e^{j\varphi_k}$$

Frequency = f_k

$$\Re\{z\} = \frac{1}{2}z + \frac{1}{2}z^*$$

$$x(t) = X_0 + \sum_{k=1}^N \left\{ \frac{1}{2} X_k e^{j2\pi f_k t} + \frac{1}{2} X_k^* e^{-j2\pi f_k t} \right\}$$

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Example: Synthetic Vowel

Sum of 5 Frequency Components

f_k (Hz)	X_k	Mag	Phase (rad)
200	$(771 + j12202)$	12,226	1.508
400	$(-8865 + j28048)$	29,416	1.876
500	$(48001 - j8995)$	48,836	-0.185
1600	$(1657 - j13520)$	13,621	-1.449
1700	$4723 + j0$	4723	0

Table 3.1: Complex amplitudes for harmonic signal that approximates the vowel sound "ah".

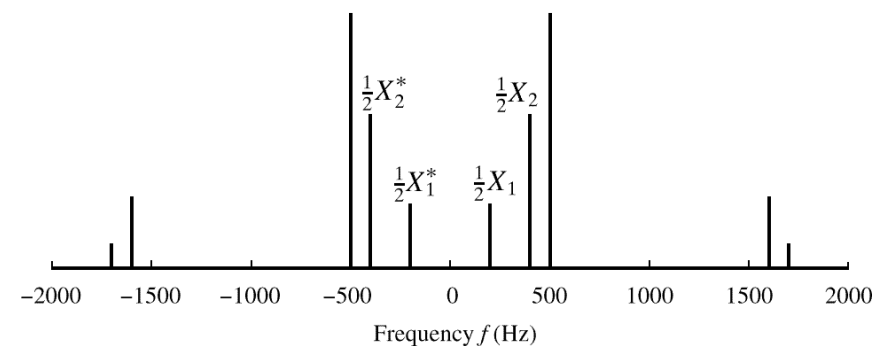
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SPECTRUM of VOWEL

- Note: Spectrum has $0.5X_k$ (except X_{DC})
- Conjugates in negative frequency

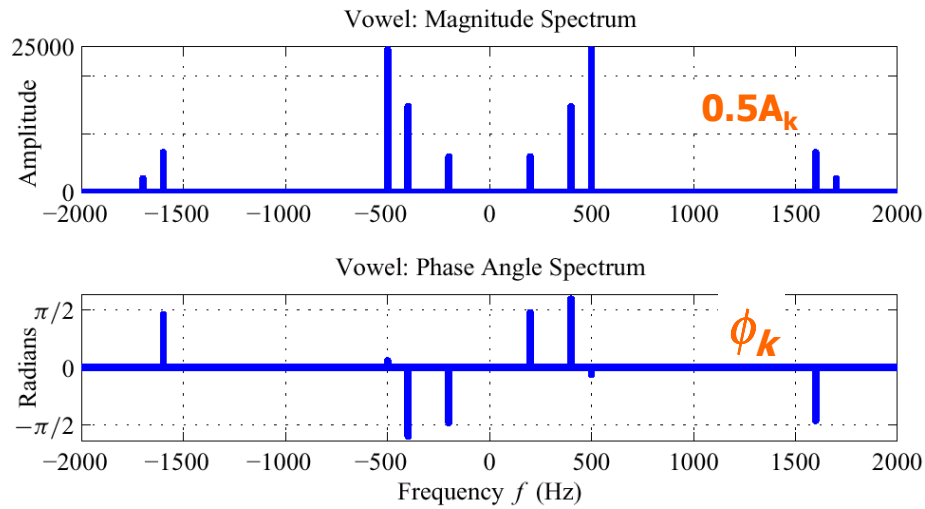


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SPECTRUM of VOWEL (Polar Format)



Vowel Waveform (sum of all 5 components)

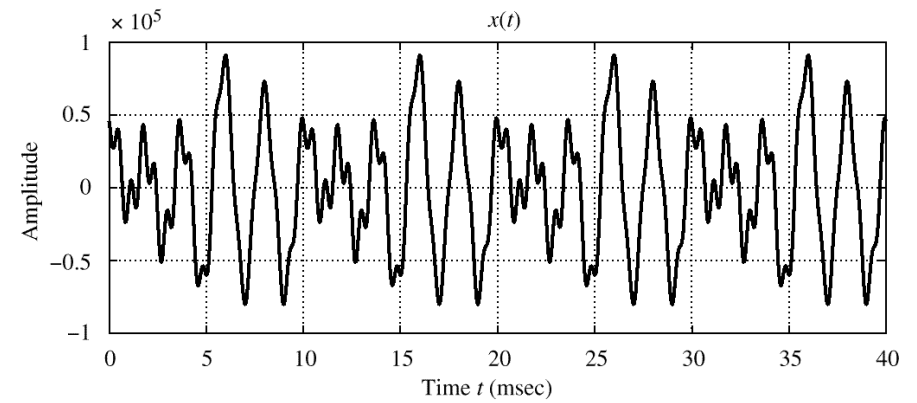


Figure 3.11 Sum of all of the terms in (3.3.4). Note that the period is 10 msec, which equals $1/f_0$.