EE-2025 Fall-01

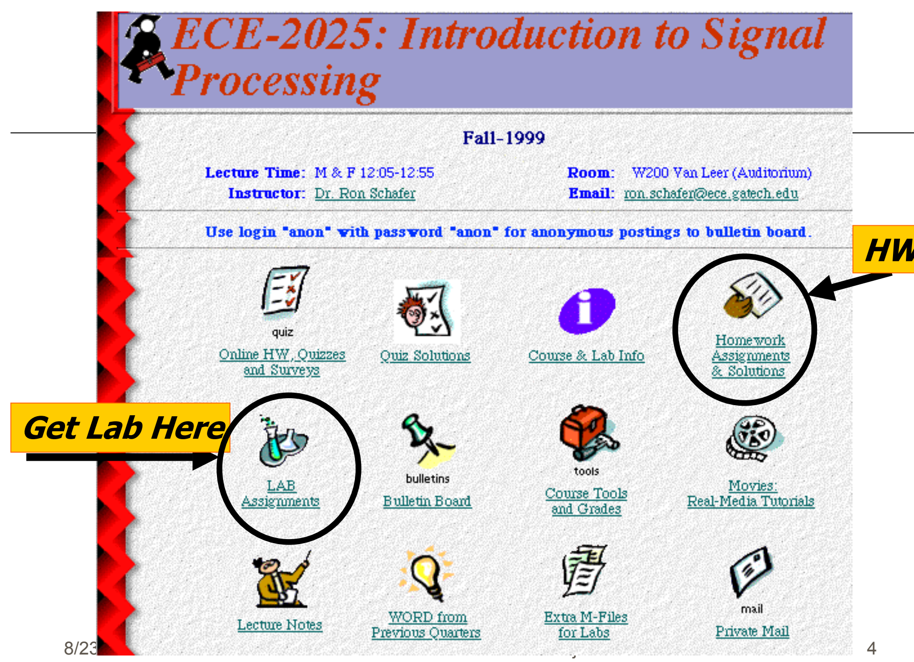
LECTURE #2
Phase & Time-Shift
Complex Exponentials
24-Aug-01

INFORMATION

- MATLAB: Mon,T,Wed in VL-456 (6pm)
- LABS start NEXT week **(MONDAY)**
  - Attend correct section (in VanLeer-252)
  - ECE Computer acct: gtxxxx, password: SSN
  - Verification must be signed during Lab
  - **Pre-Lab & Post-Lab ON-LINE Questions**
- RECITATIONS
  - Attend your assigned time

Web-CT Info

- Check the Bulletin Board for msgs
  - **MAKE YOUR OWN POSTINGS**
- Web-CT Password:
  - Last 4 digits of student number; change it soon
- PDF Files on WebCT
  - Lectures are being posted (4 per page)
  - Get PDF file of Lab#1 from WebCT
    - Hard copy of Instructor Verification Sheet
  - HW #1 posted as PDF
  - Adobe Acrobat version 4 required (it's free)



Homework Info

- On-Line HW #0 ends Monday nite
  - Last attempt is scored
- HWs will be posted on Friday/Sat
  - Covered in Rec during the following Week
  - Due the week after that (9+ days later)
- Prob Set #1 due **in RECITATION**
  - **At the beginning of class**
  - Solutions will be posted after last Recitation

JUMP

READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, pp. 17-32
- Appendix A: Complex Numbers
- Appendix B: MATLAB
- Next Lecture: finish Chap. 2, pp. 31-43

LECTURE OBJECTIVES

- Define Sinusoid Formula from a plot
- Relate TIME-SHIFT to PHASE
- Introduce an **ABSTRACTION**:
  - Complex Numbers **represent** Sinusoids
  - Complex Exponential Signal

$$z(t) = Xe^{j\omega t}$$

SINUSOIDAL SIGNAL

$$A\cos(\omega t + \varphi)$$

- **FREQUENCY** ω
  - **Radians/sec**
  - or, Hertz (cycles/sec)

$$\omega = (2\pi) f$$

- **PERIOD** (in sec)

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

- **AMPLITUDE** A
  - Magnitude
- **PHASE** φ

PLOTTING COSINE SIGNAL from the FORMULA

$$5\cos(0.3\pi t + 1.2\pi)$$

- Determine **period**:

$$T = 2\pi/\omega = 2\pi/0.3\pi = 20/3$$

- Determine a **peak** location by solving

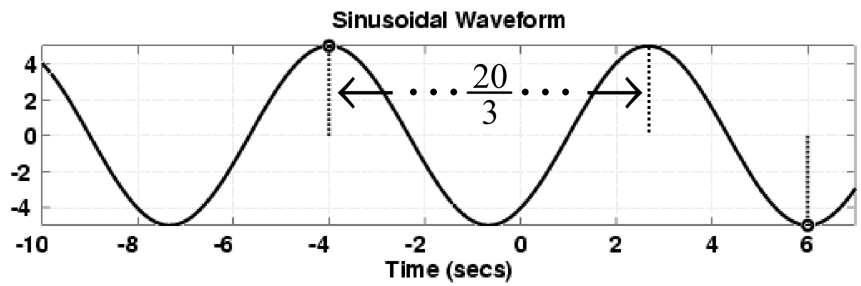
$$(\omega t + \varphi) = 0$$

- **Peak at t=-4**

ANSWER for the PLOT

$$5\cos(0.3\pi t + 1.2\pi)$$

- Use T=20/3 and the peak location at t=-4



TIME-SHIFT

- In a mathematical formula replace t with $t-t_m$

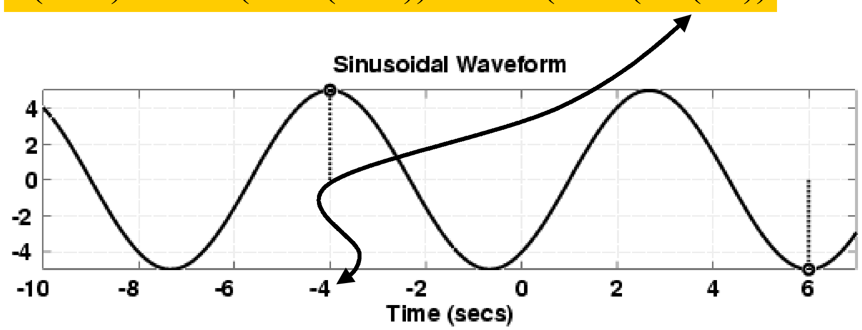
$$x(t-t_m) = A\cos(\omega(t-t_m))$$

- Then the t=0 point moves to $t=t_m$

- Peak value of $\cos(\omega(t-t_m))$ is now at $t=t_m$

TIME-SHIFTED SINUSOID

$$x(t+4) = 5\cos(0.3\pi(t+4)) = 5\cos(0.3\pi(t-(-4)))$$



PHASE <--> TIME-SHIFT

- Equate the formulas:

$$A\cos(\omega(t-t_m)) = A\cos(\omega t+\varphi)$$

- and we obtain: $$-\omega t_m = \varphi$$

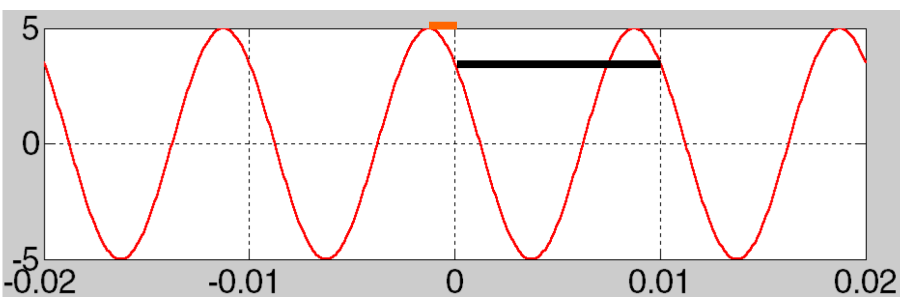
- or, $$t_m = -\frac{\varphi}{\omega}$$

SINUSOID from a PLOT

- Measure the period, T
  - Between peaks or zero crossings
  - Compute frequency: $\omega = 2\pi/T$
- Measure time of a peak: t_m
  - Compute phase: $\phi = -\omega\, t_m$
- Measure height of positive peak: A

3 steps

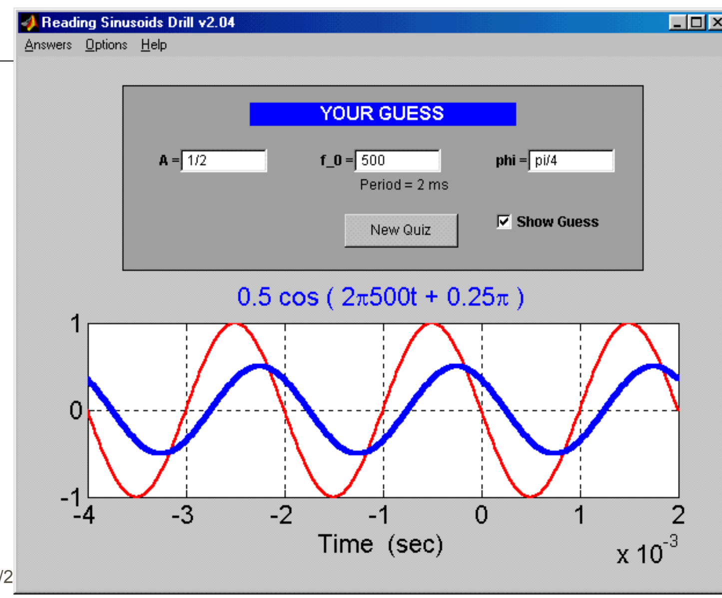
(A, ω, ϕ) from a PLOT



$$T = \frac{0.01\sec}{1\text{ period}} = \frac{1}{100} \rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{0.01} = 200\pi$$

$$t_m = -0.00125\sec \rightarrow \varphi = -\omega t_m = -(200\pi)(t_m) = 0.25\pi$$

SINE DRILL (MATLAB GUI)



PHASE is AMBIGUOUS

- The cosine signal is periodic
  - Period is 2π

$$A\cos(\omega t+\varphi+2\pi)=A\cos(\omega t+\varphi)$$

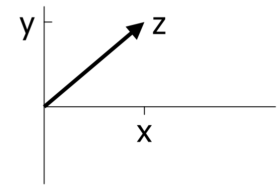
  - Thus adding any multiple of 2π leaves x(t) unchanged

$$\text{if } t_m = \frac{-\varphi}{\omega}, \text{ then}$$

$$t_{m_2} = \frac{-(\varphi+2\pi)}{\omega} = \frac{-\varphi}{\omega} - \frac{2\pi}{\omega} = t_m - T$$

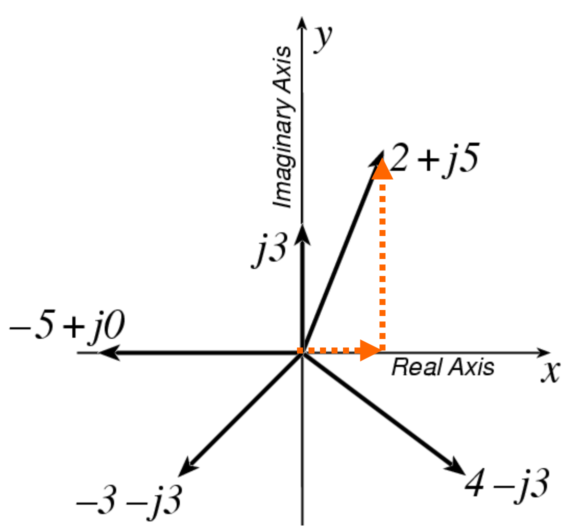
COMPLEX NUMBERS

- To solve: $z^2 = -1$
  - z = ***j***
  - Math and Physics use z = ***i***
- Complex number: z = x + ***j*** y

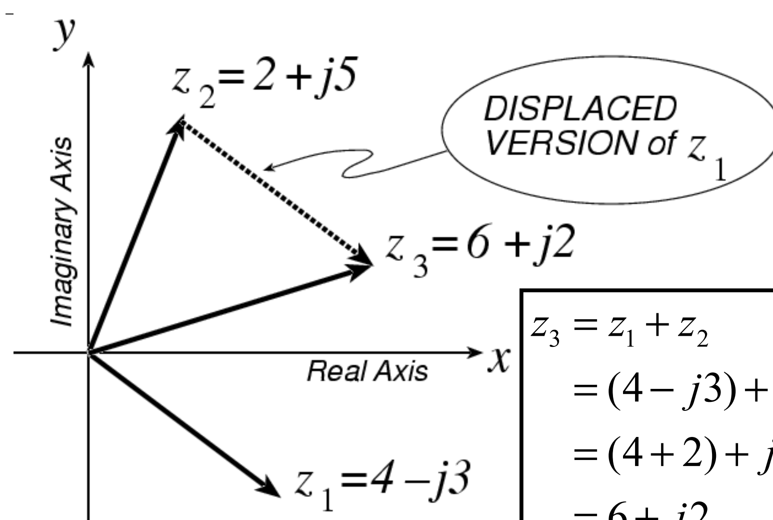


Cartesian coordinate system

PLOT COMPLEX NUMBERS



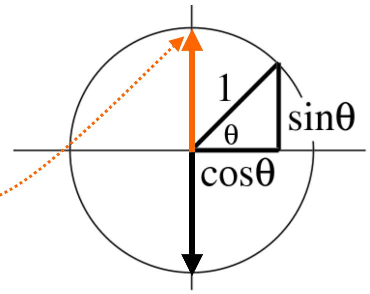
COMPLEX ADDITION = VECTOR Addition



$$\begin{aligned} z_3 &= z_1 + z_2 \\ &= (4 - j3) + (2 + j5) \\ &= (4+2) + j(-3+5) \\ &= 6 + j2 \end{aligned}$$

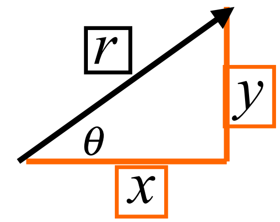
*** POLAR FORM ***

- Vector Form
  - **Length** =1
  - **Angle** = θ
- Common Values
  - **j** has angle of 0.5π
  - -1 has angle of π
  - $-$**j** has angle of 1.5π
  - also, angle of $-$**j** **could** be $-0.5\pi = 1.5\pi - 2\pi$
  - because the PHASE is **AMBIGUOUS**



POLAR <--> RECTANGULAR

- Relate (x,y) to (r,θ)



$$r^2 = x^2 + y^2$$

$$\theta = \mathrm{Tan}^{-1}\left(\tfrac{y}{x}\right)$$

$$x = r\cos\theta$$

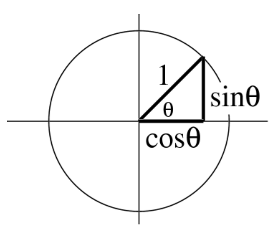
$$y = r\sin\theta$$

Most calculators do Polar-Rectangular

Need a notation for POLAR FORM

Euler's FORMULA

- **Complex Exponential**
  - Real part is cosine
  - Imaginary part is sine
  - Magnitude is one



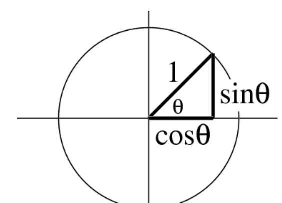
$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$re^{j\theta} = r\cos(\theta) + jr\sin(\theta)$$

COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

- Interpret this as a **Rotating Vector**
  - $\theta = \omega t$
  - Angle changes vs. time
  - ex: $\omega = 20\pi$ rad/s
  - Rotates 0.2π in 0.01 secs



$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

cos = REAL PART

Real Part of Euler's

$$\cos(\omega t) = \Re e\left\{e^{j\omega t}\right\}$$

General Sinusoid

$$x(t) = A\cos(\omega t + \varphi)$$

So,

$$A\cos(\omega t + \varphi) = \Re e\left\{Ae^{j(\omega t + \varphi)}\right\}$$

$$= \Re e\left\{Ae^{j\varphi}e^{j\omega t}\right\}$$

REAL PART EXAMPLE

$$A\cos(\omega t + \varphi) = \Re e\left\{Ae^{j\varphi}e^{j\omega t}\right\}$$

Evaluate: $x(t) = \Re e\left\{-3je^{j\omega t}\right\}$

Answer:

$$x(t) = \Re e\left\{(-3j)e^{j\omega t}\right\}$$

$$= \Re e\left\{3e^{-j0.5\pi}e^{j\omega t}\right\} = 3\cos(\omega t - 0.5\pi)$$

COMPLEX AMPLITUDE

General Sinusoid

$$x(t) = A\cos(\omega t + \varphi) = \Re e\left\{Ae^{j\varphi}e^{j\omega t}\right\}$$

Complex AMPLITUDE = X

$$z(t) = Xe^{j\omega t} \qquad X = Ae^{j\varphi}$$

Then, any Sinusoid = REAL PART of $Xe^{j\omega t}$

$$x(t) = \Re e\left\{Xe^{j\omega t}\right\} = \Re e\left\{Ae^{j\varphi}e^{j\omega t}\right\}$$