GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2001
Problem Set #12

Assigned: 21-November
Due Date: December 7, 2001

• The Final Exam will be on 10-December at 2:50 pm (Monday, period 3) for people in the 12:00 lecture and 13-December at 11:30 (Thursday, period 11) for people in the 11:00 lecture. **You must take the final exam during your assigned time.** The exam will cover the entire course, with slight emphasis on material since Quiz #3. You may bring one 8.5 × 11 inch sheet of handwritten notes (both sides) and a calculator.

• **Review Sessions:** There will be one **review session** before the Final Exam on Sunday evening December 9 at 6:30 in the EE Auditorium. A second will be held on Wednesday evening December 13 at a time and place to be announced.

•**Reading:** Finish reading Chapter 13 in the notes. Also in *DSP First*, read Chapter 8 on *IIR Filters*.

• **The final date for turning in any late labs Friday, December 7.**

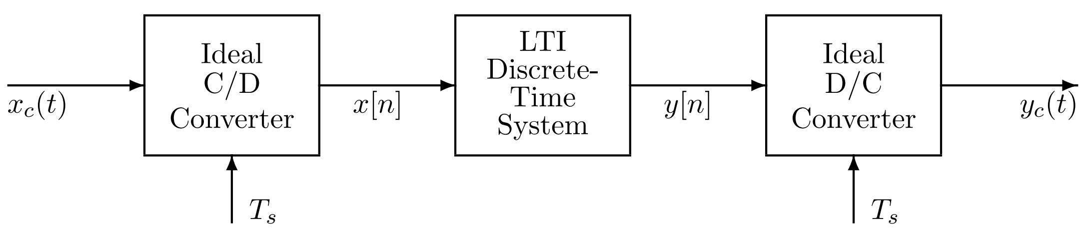
• Please check the "Bulletin Board" often. All official course announcements are posted there.

• All **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

This final problem set can be turned in at the last class on Dec. 7, 2001.

PROBLEM 12.1*:

All parts of this problem are concerned with the following system.



$$x[n] = x_c(nT) \qquad\qquad y_c(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin \frac{\pi}{T_s}(t - nT_s)}{\frac{\pi}{T_s}(t - nT_s)}$$

In all parts of this problem, assume that $X_c(j\omega) = 0$ for $|\omega| \geq 8000\pi$.

(a) Suppose that the discrete-time system is defined by $y[n] = x[n]$. What is the *minimum* value of $2\pi/T_s$ such that $y_c(t) = x_c(t)$?

(b) Suppose that the LTI D-T system has system function $H(z) = 3z^{-4}$ and assume that the sampling rate satisfies the condition of (a). Determine the overall effective frequency response $H_{\text{eff}}(j\omega)$ and from it determine a general relationship between $y_c(t)$ and $x_c(t)$.

(c) The system function for the discrete-time system is

$$H(z) = \frac{0.5}{1 - 0.75z^{-1}}.$$

For the value of T_s chosen in part (a), the input and output Fourier transforms are related by an equation of the form $Y_c(j\omega) = H_{eff}(j\omega)X_c(j\omega)$. Find an equation for the overall effective frequency response $H_{eff}(j\omega)$. Plot the magnitude and phase of $H_{eff}(j\omega)$. Use MATLAB to do this or sketch it by hand.

PROBLEM 12.2*:

Determine the z-transforms of the following. *Express your answer as the ratio of polynomials in* z^{-1}.

(a) $x_a[n] = \frac{1}{2}(-\frac{1}{2})^n u[n]$.

(b) $x_b[n] = 3(\frac{1}{3})^n u[n] - 2u[n-1]$.

(c) $x_c[n] = -\delta[n] + u[n-1]$.

PROBLEM 12.3*:

Determine the inverse z-transforms of the following:

(a) $H_a(z) = \dfrac{1 + z^{-1}}{1 - 0.5z^{-1}}$.

(b) $H_b(z) = \dfrac{0.5}{1 + 0.75e^{j0.3\pi}z^{-1}} + \dfrac{0.5}{1 - 0.75e^{-j0.3\pi}z^{-1}}$.

(c) $H_c(z) = \dfrac{0.6 + z^{-1}}{1 + 0.6z^{-1}}$.

PROBLEM 12.4:

For each of the difference equations below, determine the poles and zeros of the corresponding system function, $H(z)$. Plot the poles **(X)** and zeros **(O)** in the complex z-plane.

$$\mathcal{S}_1: \quad y[n] = 0.4y[n-1] + x[n] + x[n-1]$$

$$\mathcal{S}_2: \quad y[n] = -0.75y[n-1] + 1.5x[n] - x[n-1]$$

$$\mathcal{S}_3: \quad y[n] = -0.5y[n-2] + x[n] + 2.0x[n-1]$$

$$\mathcal{S}_4: \quad y[n] = x[n] + \tfrac{1}{4}x[n-1] - \tfrac{3}{4}x[n-2]$$

PROBLEM 12.5:

A causal LTI system has the following system function:

$$H(z) = \frac{1 - z^{-1}}{1 + 0.9z^{-1}}.$$

(a) Plot the poles and zeros of $H(z)$ in the z-plane.

(b) Use z-transforms to determine the impulse response $h[n]$ of the system; i.e., the output of the system when the input is $x[n] = \delta[n]$.

(c) Determine if the system is stable.

(d) Determine an expression for the frequency response $H(e^{j\hat{\omega}})$ of the system.

(e) Use the frequency response function to determine the output $y_1[n]$ of the system when the input is

$$x_1[n] = 2\cos(0.5\pi n) \qquad -\infty < n < \infty.$$

PROBLEM 12.6*:

We have developed several concepts that are useful in solving problems involving LTI systems. The main concepts are the *difference equation*, the *impulse response*, the *system function*, and the *frequency response* function. Most problem solving demands that you be able to go back and forth among these different mathematical representations of the LTI system because, as simple as it seems, the z-transform is *not* always the best tool for solving problems. Indeed, for a specific problem, one of these representations may be more convenient than the others, or we may need to use more than one of these representations in solving a given problem. The following is a simple problem that might be posed about an LTI system:

> *Given the input sequence $x[n]$ find the output sequence for all n when the system is an IIR filter:*
> $$y[n] = -.9y[n-1] + x[n] - x[n-1].$$

1. *Time-Domain:* Use the difference equation representation of the system to compute the output $y[n]$ for all required values of n. *For example, you could do this using* MATLAB.
2. *Z-Domain:* Multiply the z-transform of the input by the system function and determine $y[n]$ as the inverse z-transform of $Y(z)$.
3. *Frequency-Domain:* Break the input into a sum of complex exponential signals, use the frequency response function to determine the output due to each complex exponential signal separately, and finally, add the individual outputs together to get $y[n]$.

In each of these solution methods you would use one or more of the basic representations of the first-order IIR filter. Which method is easiest will have a lot to do with the nature of the input signal. For example, if you are given the difference equation and you want to use approach #2, you will have to determine the system function $H(z)$ from the difference equation coefficients.

Now in each of the following cases, the input will be given. In each case, determine which representation of the system and which of the above approaches will lead to the easiest solution of the problem, and detail the steps in using that approach to solve the problem. For example, if you choose approach #2 to solve the problem, your answer should be something like the following:

Step 1: Find $X(z)$, the z-transform of $x[n]$.

Step 2: Find $H(z)$, the system function of the first-order IIR filter.

Step 3: Multiply $X(z)H(z)$ to get $Y(z)$.

Step 4: Take the inverse z-transform of $Y(z)$ to get $y[n]$.

Now here are some possible inputs. In each case, state which of the approaches above (#1, #2, or #3) you would use. There may not be a clear cut answer. Give the approach that you *think* will yield the solution with least effort. Then carry out the method to get the output.

(a) $x[n] = 2\cos(0.8\pi n - \pi/3) + \cos(0.4\pi n - \pi)$ for $-\infty < n < \infty$.

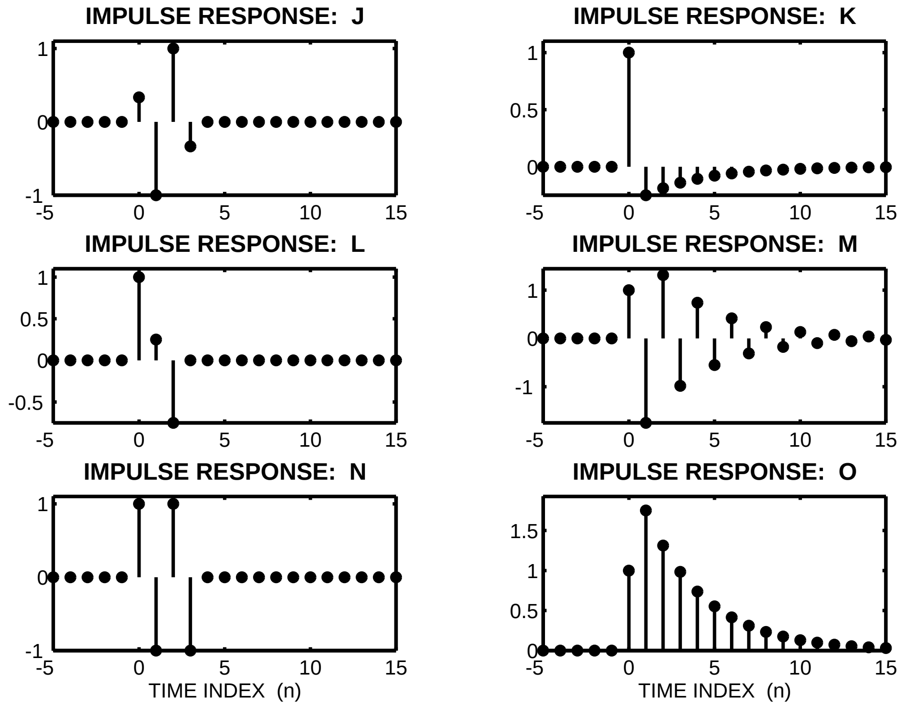
(b) $x[n] = \begin{cases} 1 & 0 \le n \le 5 \\ 0 & \text{otherwise.} \end{cases}$

(c) $x[n] = \delta[n] - \delta[n-2]$.

(d) $x[n]$ is a sampled speech signal. It is represented by a vector of 10000 numbers. In this case, you do not have to find the actual output.

PROBLEM 12.7:

This problem has been given before on exams. It is a good review.



For each of the impulse-response plots (J, K, L, M, N, O), determine which one of the following systems[1] (specified by either an $H(z)$ or a difference equation) matches the impulse response. In addition, derive a formula for the impulse response, $h[n]$, for $\mathcal{S}_1$ and $\mathcal{S}_4$.

$\mathcal{S}_1$: $y[n] = 0.4y[n-1] + x[n] + x[n-1]$

$\mathcal{S}_2$: $H(z) = \dfrac{1+z^{-1}}{1-0.75z^{-1}}$

$\mathcal{S}_3$: $y[n] = -0.75y[n-1] + x[n] - x[n-1]$

$\mathcal{S}_4$: $H(z) = \dfrac{1-z^{-1}}{1-0.75z^{-1}}$

$\mathcal{S}_5$: $y[n] = x[n] - x[n-1] + x[n-2]$

$\mathcal{S}_6$: $H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$

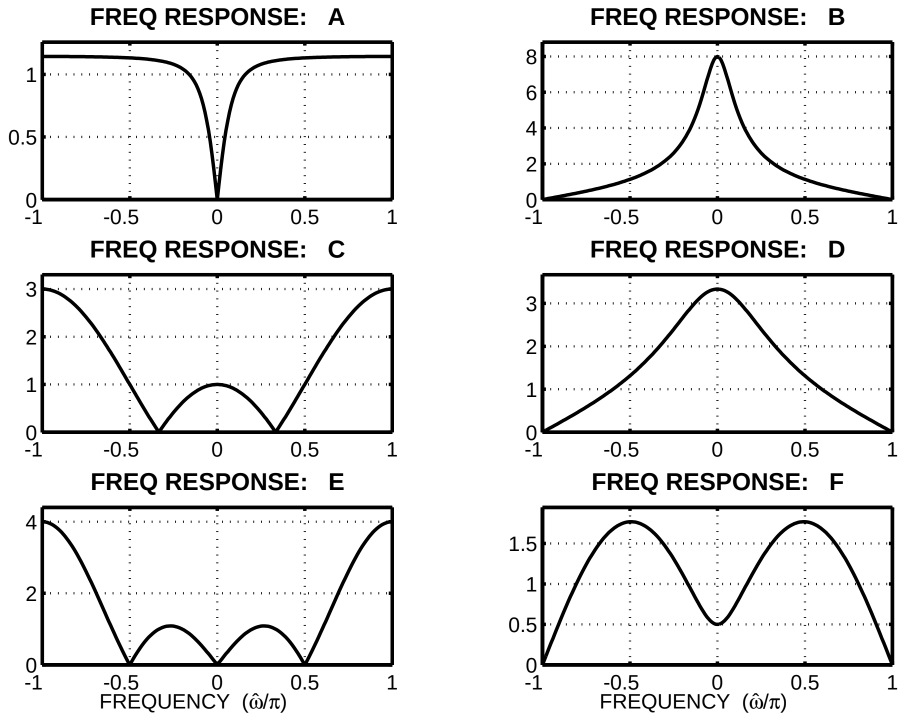
$\mathcal{S}_7$: $y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{4}x[n-2]$

$\mathcal{S}_8$: $H(z) = \frac{1}{3}(1-z^{-1})^3$

[1] These 8 systems are exactly the same as for the next problem.

PROBLEM 12.8*:

This problem has been given before on exams. It is a good review.



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an $H(z)$ or a difference equation) matches the frequency response (magnitude only) and write an expression for the magnitude of the frequency response. NOTE: the frequency axis is **normalized**; it is $\hat{\omega}/\pi$.

$\mathcal{S}_1: \quad y[n] = 0.4y[n-1] + x[n] + x[n-1]$

$\mathcal{S}_2: \quad H(z) = \dfrac{1 + z^{-1}}{1 - 0.75z^{-1}}$

$\mathcal{S}_3: \quad y[n] = -0.75y[n-1] + x[n] - x[n-1]$

$\mathcal{S}_4: \quad H(z) = \dfrac{1 - z^{-1}}{1 - 0.75z^{-1}}$

$\mathcal{S}_5: \quad y[n] = x[n] - x[n-1] + x[n-2]$

$\mathcal{S}_6: \quad H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$

$\mathcal{S}_7: \quad y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{4}x[n-2]$

$\mathcal{S}_8: \quad H(z) = \frac{1}{3}(1 - z^{-1})^3$