GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2001
Problem Set #11

Assigned: 9-November
Due Date: Week of 26-November-2001

• Quiz #3 will be held on 19-November (Monday). It will cover material represented in Problem Sets #8, #9 and #10.

• There will be a quiz review on at 6:30pm on Sunday November 18 in the ECE Auditorium.

• Reading: Finish reading Chapter 12 and read all of Chapter 13.

• **Please check the "Bulletin Board" often. All official course announcements are posted there.**

• All **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 11.1*:

In each of the following cases, use known Fourier transform pairs together with Fourier transform properties to complete the following Fourier transform pair relationships:

(a) $x(t) = \qquad\qquad \Longleftrightarrow \quad X(j\omega) = u(\omega + 10)u(10 - \omega)$

(b) $x(t) = \sin(5\pi t)\cos(100\pi t) \quad \Longleftrightarrow \quad X(j\omega) =$

(c) $x(t) = \dfrac{\sin 5\pi t}{\pi t}\sin(100\pi t) \quad \Longleftrightarrow \quad X(j\omega) =$

(d) $x(t) = \left(\dfrac{\sin(5t)}{\pi t}\right)^2 \quad \Longleftrightarrow \quad X(j\omega) =$

(e) $x(t) = \sin^2(t) \quad \Longleftrightarrow \quad X(j\omega) =$

PROBLEM 11.2:

Let $x(t)$ be a triangular pulse defined by

$$x(t) = \begin{cases} 1 - |t| & ; \ |t| < 1 \\ 0 & ; \ \text{else} \end{cases}$$

(a) By taking the derivative of $x(t)$, use the derivative property to find the Fourier transform of $x(t)$. Hint: Express the derivative as a sum of two pulses, one with an amplitude of one, and the other with an amplitude of minus one. From your table of Fourier transforms, and the delay property, you should be able to write down the transform without any integration.

(b) Find the Fourier transform of $x(t)$ by differentiating $x(t)$ twice and using the derivative property. Compare your results.

PROBLEM 11.3:

In the following, either the time-domain signal $x(t)$ or the Fourier transform is given. Use the tables of Fourier transforms and Fourier transform properties to determine the Fourier transform for each of the signals or the inverse Fourier transform for each of the given Fourier transforms. You may give your answer either as an equation or a carefully labelled plot, whichever is most convenient.

(a) $X(j\omega) = \dfrac{e^{-j\omega 2}}{3 + j\omega}$

(b) $X(j\omega) = \dfrac{j\omega}{3 + j\omega}$

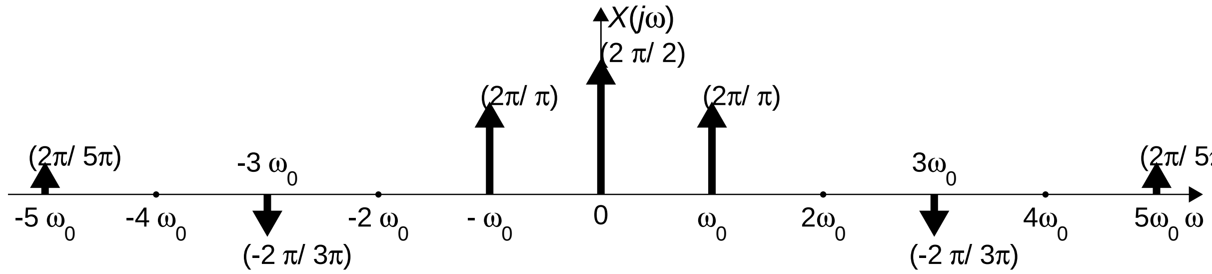
(c) $X(j\omega) = \dfrac{(j\omega)}{3 + j\omega} e^{-j\omega 2}$

(d) $x(t) = 10 \dfrac{d}{dt}\left(\dfrac{\sin(200\pi t)}{2\pi t}\right)$

(e) $x(t) = \displaystyle\sum_{n=-\infty}^{\infty} \delta(t - \frac{1}{5}n)$ (Periodic impulse train)

PROBLEM 11.4*:

The periodic input to a LTI system $H(j\omega)$ has Fourier transform $X(j\omega)$ as defined below:



where the dark arrows denote impulses.

There are six possible filters: each one is described by one of the equations or graphs below. In each case determine the output signal $y(t)$. Justify your answer by giving a derivation, or by explaining how you used the Fourier transform filtering property to get $y(t)$. Give a simple formula for $y(t)$.

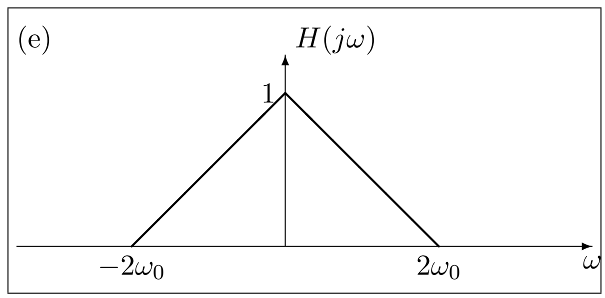
In most cases the output will be a sum of sinusoids, but it is permissible to write $y(t)$ in terms of $x(t)$, e.g., one of the answers can be written in the form $y(t) = Ax(t - t_d)$ and another one is $y(t) = x(t) - c$. Of course, you have to find the parameters, t_d, A and c.

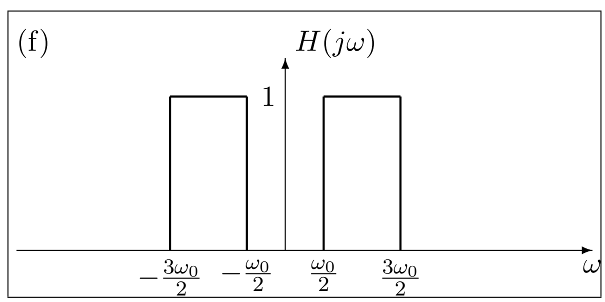
(a) $H(j\omega) = \begin{cases} 2 & |\omega| < \omega_0/2 \\ 0 & |\omega| > \omega_0/2 \end{cases}$

(b) $H(j\omega) = 0.5e^{-j4\omega/3}$

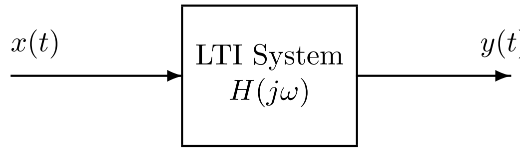
(c) $H(j\omega) = \begin{cases} e^{-j\omega/3} & |\omega| < 5\omega_0/2 \\ 0 & |\omega| > 5\omega_0/2 \end{cases}$

(d) $H(j\omega) = \begin{cases} 0 & |\omega| < \omega_0/2 \\ 1 & |\omega| > \omega_0/2 \end{cases}$





PROBLEM 11.5*:



The impulse response of the above system is

$$h(t) = \frac{10\sin(\omega_{\text{co}} t)}{\pi t},$$

and the input to this system is a periodic signal (with period $T_0 = 2$) given by the following equation:

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t-2n) = \sum_{k=-\infty}^{\infty} e^{j\pi k t}.$$

(a) Determine the Fourier transform $X(j\omega)$ of the input signal.[1] Plot $X(j\omega)$ over the range $-5\pi < \omega < 5\pi$.

(b) Determine the frequency response $H(j\omega)$ of the system and plot $|H(j\omega)|$ on the same graph as $X(j\omega)$ for the case $\omega_{\text{co}} = 2.5\pi$.

(c) Use your plot in (b) to determine $y(t)$, the output of the LTI system for the given input $x(t)$ when the cutoff frequency is $\omega_{\text{co}} = 2.5\pi$.

(d) How would you choose ω_{co} so that the output is a constant; i.e., $y(t) = C$ for all t. What is the constant C?

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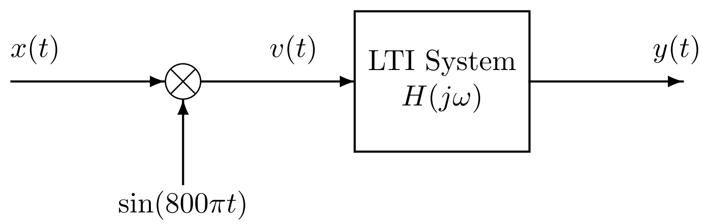
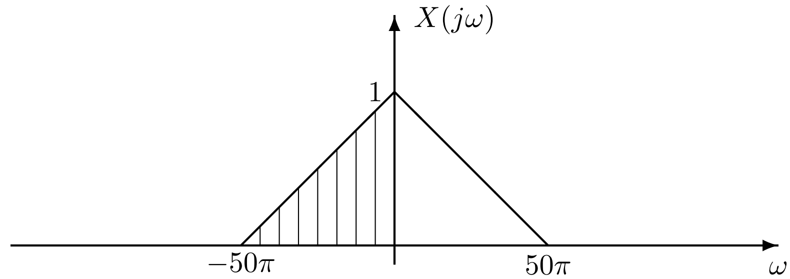
[1] Recall that if $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$, then the Fourier transform is $X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$, where $\omega_0 = 2\pi/T_0$.

PROBLEM 11.6*:

Consider the following amplitude modulation system:



Assume that the input signal $x(t)$ has a bandlimited Fourier transform as depicted below



and the linear system has the frequency response of an ideal bandpass filter:

$$H(j\omega) = \begin{cases} 1 & 750\pi < |\omega| < 800\pi \\ 0 & \text{otherwise.} \end{cases}$$

(a) Plot the Fourier transform $H(j\omega)$ of the ideal BPF specified above. *Be sure to plot for both negative and positive frequencies.*

(b) Plot the Fourier transform $V(j\omega)$ of the signal $v(t)$ at the output of the multiplier.

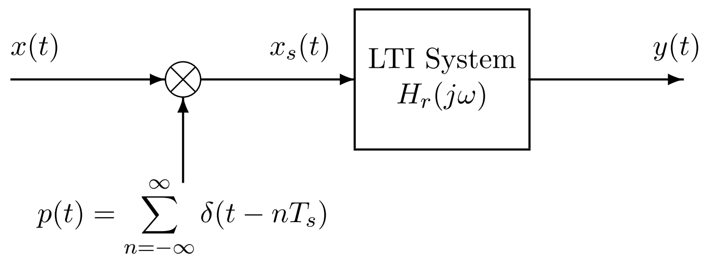
(c) Plot the Fourier transform $Y(j\omega)$ of the output signal $y(t)$ from the filter. *Do not try to find $y(t)$.*

(d) The output signal is called a "single side-band" signal. Can you see why?

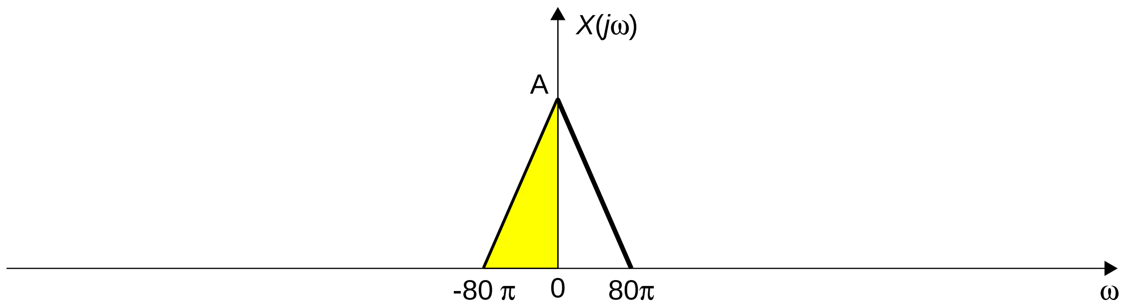
Note that the negative frequency portion of the Fourier transform $X(j\omega)$ is shaded. Mark the corresponding region or regions in your plots of $V(j\omega)$ and $Y(j\omega)$.

PROBLEM 11.7*:

The derivation of the Sampling Theorem involves the operations of impulse train sampling and reconstruction as shown in the following system:



The "typical" bandlimited Fourier transform of the input is depicted below:



(a) For the input with Fourier transform depicted above, use the Sampling Theorem to choose the sampling rate $\omega_s = 2\pi/T_s$ so that $x_r(t) = x(t)$. Plot $X_s(j\omega)$ for the value of $\omega_s = 2\pi/T_s$ that is equal to the Nyquist rate.

(b) If $\omega_s = 2\pi/T_s = 130\pi$ in the above system and $X(j\omega)$ is as depicted above, plot the Fourier transform $X_s(j\omega)$ and show that aliasing occurs. There will be an infinite number of shifted copies of $X(j\omega)$, so indicate what the pattern is versus ω.

(c) For the conditions of part (b), determine and sketch the Fourier transform of the output $X_r(j\omega)$ if the frequency response of the LTI system is

$$H_r(j\omega) = \begin{cases} T_s & |\omega| \le \pi/T_s \\ 0 & |\omega| > \pi/T_s \end{cases}$$