GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2001
Problem Set #10

Assigned: 2 November
Due Date: Week of 12-November-01

• **Quiz #3** will be on 19-November (Monday). One page of notes will be allowed.

• **Reading**: In *DSP First (notes)*, Chapter 11 and Chapter 12.

• **Please check the "Bulletin Board" often. All official course announcements are posted there.**

• **ALL** of the **STARRED** problems have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 10.1*:

The impulse response of a continuous-time linear time-invariant system is

$$h(t) = \delta(t) - 100e^{-100t}u(t).$$

(a) Find the frequency response $H(j\omega)$ of the system. Express your answer as a single rational function $N(j\omega)/D(j\omega)$ with powers of $(j\omega)$ in the numerator and denominator.

(b) Plot the magnitude squared, $|H(j\omega)|^2 = H(j\omega)H^*(j\omega)$, versus ω. Also plot the phase $\angle H(j\omega)$ as a function of ω.

(c) At what frequency ω does the magnitude squared of the frequency response have its largest value? At what frequency is the magnitude squared of the frequency response equal to one half of its maximum value? (This is referred to as the 3-dB point of the filter since the frequency response magnitude measured in decibels, $10\log_{10}|H(j\omega)|^2$, is 3.01dB smaller at this frequency compared to its maximum value when measured in decibels. Specifically, $10\log_{10}(\frac{1}{2}) = -3.01$ dB.)

(d) Suppose that the input to this system is

$$x(t) = -2 + 5\cos(1000t) + \delta(t-3).$$

Use superposition to find the output $y(t)$. *Hint: To find the response of each term, use the easier method, i.e., impulse response or frequency response.*

PROBLEM 10.2*:

In each of the following cases, use known Fourier transform pairs together with some Fourier transform properties to complete the following Fourier transform pair relationships:

(a) $x(t) = \qquad \Longleftrightarrow \quad X(j\omega) = 2\delta(\omega - 2)e^{\omega}$

(b) $x(t) = \delta(t-4) + \delta(t+4) \quad \Longleftrightarrow \quad X(j\omega) =$

(c) $x(t) = 20\dfrac{\sin(200t)}{t} \quad \Longleftrightarrow \quad X(j\omega) =$

(d) $x(t) = \qquad \Longleftrightarrow \quad X(j\omega) = u(\omega+3)u(3-\omega)$

PROBLEM 10.3*:

The delay property of Fourier transform states that if $X(j\omega)$ is the Fourier transform of $x(t)$, the the Fourier transform of $x(t-t_d)$ is $e^{-j\omega t_d}X(j\omega)$, i.e.,

$$x(t-t_d) \Longleftrightarrow e^{-j\omega t_d}X(j\omega).$$

Use this property to find the Fourier transforms of the following signals:

(a) $x(t) = \delta(t-2)\cos(2t)$

(b) $x(t) = \delta(t-2) * \cos(2t)$

(c) $x(t) = -\delta(t+1) + 2\delta(t) - \delta(t-1)$

(d) $x(t) = e^{-3t}u(t) - e^{-3t}u(t-3) = e^{-3t}u(t) - e^{-9}e^{-3(t-3)}u(t-3)$

(e) $x(t) = [u(t) - u(t-5)]\cos(10\pi t)$
In this part, use the *frequency shifting property:* $x(t)e^{j\omega_c t} \Longleftrightarrow X(j(\omega-\omega_c))$.

PROBLEM 10.4:

A continuous-time LTI system is defined by the following input/output relation:

$$y(t) = x(t+1) + 2x(t) + x(t-2). \tag{1}$$

(a) Find the impulse response $h(t)$ of the system; i.e., determine the output when the input is an impulse.

(b) Substitute your answer for $h(t)$ into the integral formula

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$$

to find the frequency response.

(c) Apply the system definition given in Eq. (1) directly to the input $x(t) = e^{j\omega t}$ for $-\infty < t < \infty$ and show that $y(t) = H(j\omega)e^{j\omega t}$, where $H(j\omega)$ is as determined in part (b).

PROBLEM 10.5*:

For each of the following cases, use the table of known Fourier transform pairs to complete the following Fourier transform pair relationships.

(a) Find $x(t)$ if $X(j\omega) = \dfrac{1}{1 + j0.2\omega} e^{-0.2j\omega}$.

(b) Find $x(t)$ if $X(j\omega) = j10\sin(\omega)$.

(c) Find $x(t)$ if $X(j\omega) = \dfrac{1 + j\omega}{2 + j\omega}$.

(d) Find $x(t)$ if $X(j\omega) = \delta(\omega - 50\pi) + \delta(\omega + 50\pi)$.

PROBLEM 10.6:

Here is a Fourier transform that will be useful in an upcoming lab project.

(a) Define the finite duration signal $x(t)$ to be one half cycle of a sine wave:

$$x(t) = \begin{cases} \sin(\pi t) & \text{for } 0 \leq t \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Make a plot of $x(t)$ over the range $-3 \leq t \leq 3$.

(b) Determine $X(j\omega)$, the Fourier transform of $x(t)$. One approach is to use Euler's formula to break the integral down into integrating two complex exponentials.

(c) Plot the magnitude and phase of $X(j\omega)$ from the previous part. You will probably have to use MATLAB to make these plots from the formula that you derive.

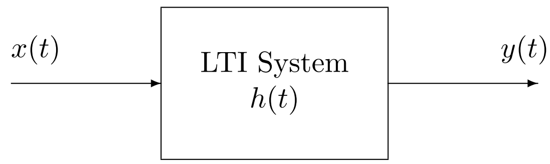
(d) Define a new signal

$$q(t) = \begin{cases} \cos(100\pi t) & \text{for } |t| \leq 0.005 \\ 0 & \text{elsewhere} \end{cases}$$

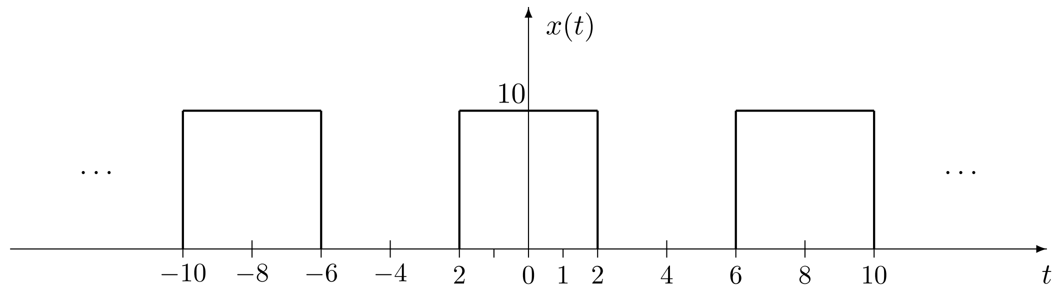
Use the scaling and shifting properties of the Fourier transform to write the formula for $Q(j\omega)$. *Hint: if you write $q(t)$ in terms of $x(t)$ as $q(t) = x(\alpha t - \beta)$, then you can apply the scaling and shifting properties. Which order should you use? Scale first and then shift, or vice versa?*

(e) Prove that the $Q(j\omega)$ from the previous part is a purely real function; no imaginary part.

PROBLEM 10.7*:
Consider the LTI system below:



The input to this system is the periodic pulse wave $x(t)$ depicted below:



(a) Determine ω_0 and the coefficients a_k in the Fourier series representation of $x(t)$.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.$$

Feel free to use known results from the text, lecture, or earlier problem sets.

(b) Plot the spectrum of the signal $x(t)$; i.e., make a plot showing the a_k's plotted at the frequencies $k\omega_0$ for $-4\omega_0 \le \omega \le 4\omega_0$.

(c) If the frequency response of the system is the ideal *highpass* filter

$$H(j\omega) = \begin{cases} 0 & |\omega| < \pi/8 \\ 1 & |\omega| \ge \pi/8 \end{cases}$$

plot the output of the system, $y(t)$, when the input is $x(t)$ as plotted above. *Hint: First determine which frequency is removed by the filter, and then determine what effect this will have on the waveform.*

(d) If the frequency response of the system is an ideal lowpass filter

$$H(j\omega) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

where ω_c is the *cutoff frequency*, for what values of ω_c will the output of the system have the form

$$y(t) = A + B\cos(\omega_0 t + \phi)$$

where A and B are nonzero?