GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2001
Problem Set #9

Assigned: 26-October
Due Date: Week of 5-November-01

• Any grading questions on Quiz #2 must be resolved no later than 2-November; after that date the scores will not be changed.

• Quiz #3 will be on 19-November (Monday). Coverage will be Homeworks #8, #9 and #10; as well as Chapter 7 in *DSP First* plus the Continuous-Time Signals & Systems notes (Chapters 10–12).

• Reading: Read Chapter 10 in Notes.

• Please check the "Bulletin Board" often. All official course announcements are posted there.

• ALL of the **STARRED** problems have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 9.1*:

Try your hand at expressing each of the following in a simpler form:

(a) $\delta(t+5) * [\delta(t-10) + 3e^{-t}\cos(5\pi t)u(t) + \sin(50\pi t)] =$

(b) $u(-t+4)u(t)[\delta(t+1) + \delta(t-1) + \delta(t-5)] =$
In addition, plot the function $u(t)u(-t+4)$.

(c) $\dfrac{d}{dt}\left[e^{-(t+1)}\sin(5\pi t)u(t+1)\right] =$

(d) $\displaystyle\int_{-\infty}^{t} e^{-(\tau-1)}[\delta(\tau) + \delta(\tau-1)]\,d\tau =$

Note: use properties of the impulse signal $\delta(t)$ and the unit-step signal $u(t)$ to perform the simplifications. For example, recall

$$\delta(t) = \frac{d}{dt}u(t) \qquad \text{where} \qquad u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Be careful to distinguish between multiplication and convolution. Convolution is denoted by a "star", as in $x(t) * \delta(t-2) = x(t-2)$ and multiplication is usually indicated as in $x(t)\delta(t-2) = x(2)\delta(t-2)$.

PROBLEM 9.2:

This is Problem 9.2 of Problem Set #9 of ECE2025 from the Spring of 2001. You should be sure that you understand this problem. The best way to work it is to draw pictures with "typical" input and impulse response signals.

The impulse response of an LTI continuous-time system is such that $h(t) = 0$ for $t \leq T_1$ and for $t \geq T_2$. By drawing appropriate figures as recommended for evaluating convolution integrals, show that if $x(t) = 0$ for $t \leq T_3$ and for $t \geq T_4$ then $y(t) = x(t) * h(t) = 0$ for $t \leq T_5$ and for $t \geq T_6$. In the process of proving this result you should obtain expressions for T_5 and T_6 in terms of T_1, T_2, T_3, and T_4.

PROBLEM 9.3*:

A linear time-invariant system has impulse response:

$$h(t) = \begin{cases} e^{-t} & -1 \leq t < 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $h(\tau)$ and $h(t-\tau)$ as a functions of τ for $t = -5$, 0, and 5.

(b) Is the system stable? Justify your answer.

(c) Is the system causal? Justify your answer.

(d) Find the output $y(t)$ when the input is $x(t) = \delta(t-1)$.

(e) Use the convolution integral to determine the output $y(t)$ when the input is

$$x(t) = \begin{cases} e^{-t/4} & 0 \leq t < 6 \\ 0 & \text{otherwise} \end{cases}$$

(f) (*Optional:*)Use the GUI `cconvdemo` to verify your answer in part (e)

PROBLEM 9.4*:

A continuous-time system is defined by the input/output relation

$$y(t) = \int_{t-3}^{t} x(\tau)d\tau$$

(a) Determine the impulse response, $h(t)$, of this system.

(b) Is this a stable system? Explain with a proof (if true) or counter-example (if false).

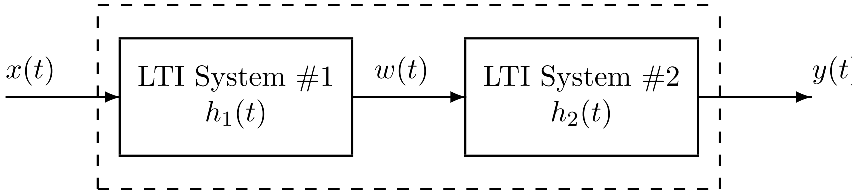
(c) Is it a causal system? Explain with a proof (if true) or counter-example (if false).

(d) Use the convolution integral to determine the output of the system when the input is the pulse

$$x(t) = u(t-1) - u(t+1)$$

(e) (*Optional:*) Use the GUI `cconvdemo` to verify your answer to part (d).

PROBLEM 9.5*:



In the cascade of two LTI systems shown in the figure above, the first system has an impulse response

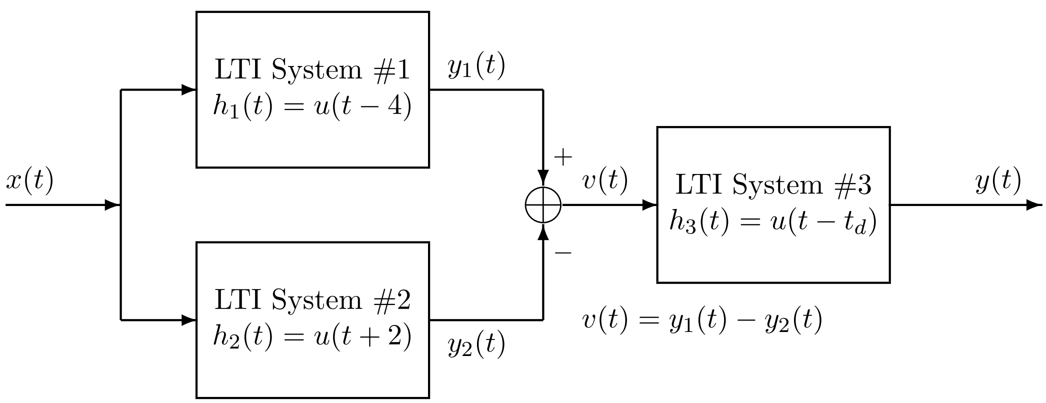
$$h_1(t) = \begin{cases} e^{-2t} & 0 \le t < 1 \\ 0 & \text{otherwise,} \end{cases}$$

and the second system is described by the input/output relation

$$y(t) = \frac{dw(t)}{dt} + 3w(t)$$

Find the impulse response of the overall system; i.e., find the output $y(t) = h(t)$ when the input is $x(t) = \delta(t)$. Give your answer both as an equation and as a carefully labeled sketch.

PROBLEM 9.6*:



(a) What is the impulse response of the overall LTI system (i.e., from $x(t)$ to $y(t)$)? Give your answer both as an equation and as a carefully labeled sketch.

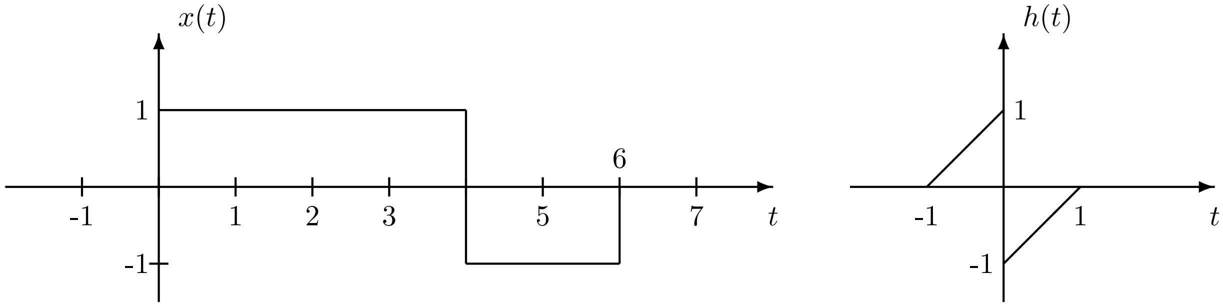
(b) How should the time delay t_d be chosen so that the overall system is causal?

(c) Which systems (#1, #2, #3) are stable? Is the overall system a stable system? Explain to receive credit.

PROBLEM 9.7:

This is a problem from Problem Set #9 of Fall 2000. Try working it first before checking the answer.

If the input $x(t)$ and the impulse response $h(t)$ of an LTI system are the following:



(a) Determine $y(0)$, the value of the output at $t = 0$.

(b) Find all the values of t for which the outpout $y(t) = 0$. *Note: You do not need to find $y(t)$ at any other values of t.*