GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2001
Problem Set #8

Assigned: 12-Oct-01
Due Date: Week of 29-Oct-01

Quiz #2 on 22-October (Monday). Covers Problem Sets #3 – #7.

• Reading: In *DSP First*, Chapter 7 on *The z-Transform.*

• Please check the "Bulletin Board" often. All official course announcements are posted there.

• **ALL** of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM or in the archives.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 8.1*:

This problem is almost identical to Problem 8.1 of Spring 2001. Try working it without looking at the answer. If you have trouble consult the solution, but don't print it out. Then try this one again. It is very important that you understand this problem.

We have developed several concepts that are useful in solving problems involving LTI systems. The main concepts are the *difference equation*, the *impulse response*, the *system function*, and the *frequency response* function. You need to be able to go back and forth among these different mathematical representations of the LTI system because, as simple as it seems, the z-transform is *not* always the best tool for solving these problems. Indeed for a specific problem, one of these representations may be more convenient than the others, or we may need to use more than one of these representations in solving a given problem. The following is a simple problem that might be posed about an LTI system:

> *Given the input sequence $x[n]$ find the output sequence $y[n]$ of a 5-point running average filter for all values of n.*

The following is a partial list of possible approaches to solving this problem:

1. Use the difference equation representation of the system to compute the output $y[n]$ for all required values of n.

2. Multiply the z-transform of the input by the system function and determine $y[n]$ as the inverse z-transform of $Y(z)$.

3. Break the input into a sum of complex exponential signals, use the frequency response function to determine the output due to each complex exponential signal separately, and finally, add the individual outputs together to get $y[n]$.

In each of these solution methods you would use one or more of the basic representations of the 5-point running average filter. Which method is easiest will have a lot to do with the nature of the input signal. This may require that you convert a given representation of the system into one of the other forms. For example, if you are given the difference equation and you want to use approach #2, you will have to determine the system function $H(z)$ from the difference equation coefficients.

Now in each of the following cases, the input will be given. In each case, determine which representation of the system and which of the above approaches will lead to the easiest solution of the problem, and detail the steps in using that approach to solve the problem. For example, if you choose approach #2 to solve the problem, your answer should be something like the following:

Step 1 Find $X(z)$, the z-transform of $x[n]$.

Step 2 Find $H(z)$, the system function of the 5-point running averager.

Step 3 Multiply $X(z)H(z)$ to get $Y(z)$.

Step 4 Take the inverse z-transform of $Y(z)$ to get $y[n]$.

Now here are some possible inputs. In each case, state which of the above (#1, #2, or #3) approaches you would use. There may not be a clear cut answer. Give the approach that you *think* will yield the solution with least effort. Outline your approach to solving the problem of finding the output of the 5-point moving averager. **You do not have to actually find the output—just tell how you would solve it in a step-by-step procedure described as illustrated above.**

(a) $x[n] = 10\delta[n-50]$.

(b) $x[n]$ is a sampled audio signal. It is represented by a vector of 20000 numbers.

(c) $x[n] = \begin{cases} 1 & 0 \le n \le 10 \\ 0 & \text{otherwise.} \end{cases}$

(d) $x[n] = 3\cos(0.1\pi n - \pi/3) + 2\cos(0.4\pi n - \pi) \quad \text{for} \quad -\infty < n < \infty.$

(e) $x[n] = 10\delta[n-50] + 4\cos(0.1\pi n + \pi/2) + 3\cos(0.4\pi n - \pi) \quad \text{for} \quad -\infty < n < \infty.$

PROBLEM 8.2:

Consider the following MATLAB program:

```
nn = 0:16000;
xx = 3 + 2*cos(0.75*pi*nn-pi/4) + 11*cos(1.5*pi*nn-pi/3);
yy = conv([1,0,0,0,-1]/4,xx);
soundsc(yy,8000)
```

(a) What is the system function $H(z)$ of the system that corresponds to the `conv( )` statement?

(b) What is the frequency response $H(e^{j\hat{\omega}})$ of the system?

(c) Neglecting the end effects in the convolution, determine $y(t)$ that describes the signal produced by the `soundsc( )` statement.

PROBLEM 8.3*:

We now have four ways of describing an LTI system: the difference equation; the impulse response, $h[n]$; the frequency response, $H(e^{j\hat{\omega}})$; and the system function, $H(z)$. In the following, you are given one of these representations and you must find the other three.

(a) $H(z) = 1 + 2z^{-2} - z^{-4} + 2z^{-5}$.

(b) $y[n] = x[n] + 2x[n-2] - 3x[n-3]$.

(c) $h[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3] + \delta[n-4]$.

(d) $H(e^{j\hat{\omega}}) = [1 - \cos(3\hat{\omega})]e^{-j\hat{\omega}4}$.

(e) For each of the four systems in parts (a) through (d), determine the poles and zeros and make a plot of the pole-zero locations in the z-plane. Show the unit circle for reference. *Note:* you can check your work with the MATLAB function `zplane()` (or `zzplane()` in the DSP-First toolbox).

PROBLEM 8.4*:

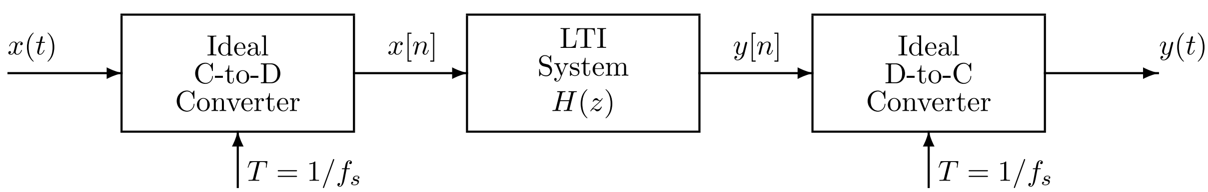
The input to the C-to-D converter in the figure below is

$$x(t) = 5 + 3\cos(4000\pi t - \pi/4) + 3\cos(9000\pi t - \pi/3)$$

The system function of the LTI system is

$$H(z) = (1 + z^{-2})$$

If $f_s = 8000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter.



PROBLEM 8.5*:

A linear time-invariant filter is described by the difference equation

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4] = \sum_{k=0}^{4} x[n-k]$$

(a) Determine the system function $H(z)$ for this system.

(b) Plot all the zeros and poles of $H(z)$ in the complex z-plane.

(c) Find an expression for the frequency response $H(e^{j\hat{\omega}})$ of the system. Simplify your answer so that it can be expressed in the form:

$$H(e^{j\hat{\omega}}) = \frac{\sin(5\hat{\omega}/2)}{\sin(\hat{\omega}/2)} e^{-j2\hat{\omega}}.$$

(d) Suppose that the input signal is

$$x[n] = 1 + 2\cos(n\hat{\omega}_0) \text{ for } -\infty < n < \infty$$

Find a non-zero frequency $0 < \hat{\omega}_0 < \pi$ for which the output $y[n]$ will be a constant for all n, i.e.,

$$y[n] = c \qquad \text{for } -\infty < n < \infty$$

and find the value for c. (In other words, the sinusoid is removed by the filter.)

PROBLEM 8.6*:

The system function of a linear time-invariant filter is given by the formula

$$H(z) = (1 - z^{-1})(1 - e^{j\pi/3}z^{-1})(1 - e^{-j\pi/3}z^{-1})$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$. Simplify your answer so that all the filter coefficients are real numbers.

(b) What is the output of this LTI system if the input is $x[n] = \delta[n]$?

(c) Use multiplication of z-transform polynomials to find the output when the input is

$$x[n] = \delta[n-1] - 2\delta[n-3] + \delta[n-4].$$

(d) Plot the poles and zeros of this LTI system in the complex z-plane. Note that the factored form should make this relatively easy.

(e) If the input to the system is of the form

$$x[n] = \cos(\hat{\omega}_o n) \qquad -\infty < n < \infty,$$

for which values of $\hat{\omega}_o$ will the output be zero for all n? Find all possible frequencies in the range from 0 to π.
Comment: We can use the z-transform to solve this problem if we use the pole-zero plot, and then relate the frequency response to the zeros that appear on the unit-circle in the z-plane.