

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2001
Problem Set #7

Assigned: 5-Oct-01

Due Date: Week of 15-Oct-01

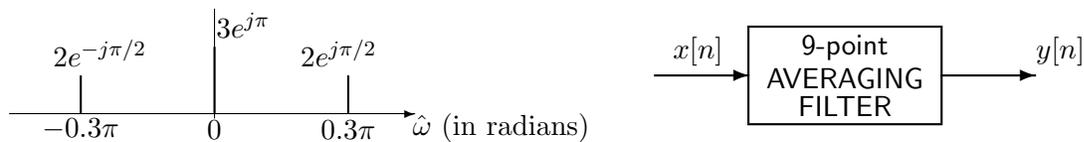
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- Quiz #2 on 22-October (Monday). Covers Problem Sets #3 – #7.
 - Reading: In *DSP First*, Chapter 6 on *Frequency Response*.
 - Please check the “Bulletin Board” often. All official course announcements are posted there.
 - **ALL** of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM or in the archives.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

If you have Recitation on 15-Oct (Monday) or 16-Oct (Tuesday), please turn in your homework at the Recitation that you attend on Wednesday or Thursday. Mark your name and section clearly on a title page. The last Recitation meets from 4:30 to 6 on Thursday.

PROBLEM 7.1:

A discrete-time signal $x[n]$ has the two-sided spectrum representation shown below.



- Write an equation for $x[n]$. Make sure to express $x[n]$ as a real-valued signal.
- Determine the formula for the output signal $y[n]$.

See Problem 6.1 of Spring 1999 for a solution to this problem.

PROBLEM 7.2:

A discrete-time system is defined by the input/output relation

$$y[n] = x[n] + 3x[n - 1] + x[n - 2]. \quad (1)$$

- (a) Determine whether or not the system defined by Equation (1) is (i) linear; (ii) time-invariant; (iii) causal. Explain your answers.
- (b) Obtain an expression for the frequency response of this system.
- (c) Make a sketch of the frequency response (magnitude and phase) as a function of frequency.
Hint: Use symmetry to simplify your expression before determining the magnitude and phase.
- (d) For the system of Equation (1), determine the output $y_1[n]$ when the input is

$$x_1[n] = 2 \cos(0.75\pi n) = e^{j0.75\pi n} + e^{-j0.75\pi n}.$$

Express your answer in terms of cosine functions.

- (e) For the system of Equation (1), determine the output $y_2[n]$ when the input is

$$x_2[n] = 4 + 4 \cos(0.75\pi(n - 1)).$$

Hint: use the linearity and time-invariance properties.

PROBLEM 7.3*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

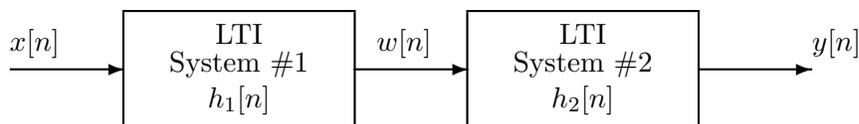


Figure 1: Cascade connection of two LTI systems.

- (a) Suppose that LTI System #1 is described by the difference equation

$$w[n] = x[n] - \alpha x[n - 1].$$

Determine the impulse response $h_1[n]$ of the first system.

- (b) The LTI System #2 is described by the impulse response

$$h_2[n] = \alpha^n (u[n] - u[n - L]) = \sum_{k=0}^{L-1} \alpha^k \delta[n - k] = \begin{cases} \alpha^n & n = 0, 1, \dots, L - 1 \\ 0 & \text{otherwise.} \end{cases}$$

For the special case of $L = 8$, use convolution to show that the impulse response sequence of the overall cascade system is

$$h[n] = h_1[n] * h_2[n] = \delta[n] - \alpha^8 \delta[n - 8].$$

- (c) Generalize your result in part (b) for the general case where L is any integer value.
- (d) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Fig. 1.
- (e) Assuming that $0 < \alpha < 1$, how would you choose L so that $y[n] = x[n]$ in Figure 1; i.e., how would you choose L so that the second system “undoes” the effect of the first system?

You will use the results of this problem in Lab #6.

PROBLEM 7.4*:

Consider again the cascade system in Figure 1 with

$$h_1[n] = \delta[n] - \delta[n - 1] \quad \text{and} \quad h_2[n] = u[n] - u[n - 8].$$

- (a) Determine $H_1(\hat{\omega})$, the frequency response of the first system.
- (b) Determine $H_2(\hat{\omega})$, the frequency response of the second system.
- (c) By using numerical convolution, show that $h[n] = h_1[n] * h_2[n] = \delta[n] - \delta[n - 8]$.
- (d) From $h[n]$ determine $H(\hat{\omega})$ the frequency response of the overall system (from $x[n]$ to $y[n]$).
- (e) Show that your result in part (d) is the product of the results in parts (a) and (b); i.e., $H_1(\hat{\omega})H_2(\hat{\omega}) = H(\hat{\omega})$.

PROBLEM 7.5*:

Suppose that three systems are hooked together in “cascade.” In other words, the output of \mathcal{S}_1 is the input to \mathcal{S}_2 , and the output of \mathcal{S}_2 is the input to \mathcal{S}_3 . The three systems are specified as follows:

$$\begin{aligned}\mathcal{S}_1 : \quad \mathcal{H}_1(\hat{\omega}) &= e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} \\ \mathcal{S}_2 : \quad y_2[n] &= x_2[n] - x_2[n - 2] \\ \mathcal{S}_3 : \quad y_3[n] &= 5x_3[n - 4] + 5x_3[n - 5]\end{aligned}$$

NOTE: the output of \mathcal{S}_i is $y_i[n]$ and the input is $x_i[n]$.

The objective in this problem is to determine the equivalent system that is a single operation from the input $x[n]$ (into \mathcal{S}_1) to the output $y[n]$ which is the output of \mathcal{S}_3 . Thus $x[n]$ is $x_1[n]$ and $y[n]$ is $y_3[n]$.

- Determine the difference equation for \mathcal{S}_1 , i.e., express $y_1[n]$ in terms of $x_1[n]$, $x_1[n - 1]$, $x_1[n - 2]$, etc.
- Determine the frequency response of the other two systems: $\mathcal{H}_i(\hat{\omega})$ for $i = 2, 3$.
- Determine the frequency response of the overall cascaded system.
- Write *one difference equation* that defines the overall system in terms of $x[n]$ and $y[n]$ only.

PROBLEM 7.6*:

The intention of the following MATLAB program is to filter a sinusoid via the `conv` function. However, the cosine signal has a starting point at $n = 0$; so we assume that it is zero for $n < 0$.

```
omegahat = pi/2;
nn = [ 0:4000 ];
xn = 3*cos(omegahat*nn + pi/2);
bb = ones(1,6)/6;
yn = conv( bb, xn );
```

- Determine a formula for $\mathcal{H}(\hat{\omega})$ for this FIR filter.
- Make a plot of the magnitude of $\mathcal{H}(\hat{\omega})$ and label *all* the frequencies where $|\mathcal{H}(\hat{\omega})|$ is zero. Use `freqz(bb, 1, ww)` in MATLAB, where `ww` is a vector of frequencies that defines a dense grid for $\hat{\omega}$.
- Use convolution to determine a formula (or table) for $y[n]$, the signal contained in the vector `yn`. Give the individual values for $n = 0, 1, 2, 3, 4$, and then provide a general formula for $y[n]$ that is correct for $5 \leq n \leq 4000$. This formula should give numerical values for the amplitude, phase and frequency of $y[n]$. *Hint*: the formula is a sinusoid for $n \geq 5$.
- It is possible to choose `omegahat` so that almost all of the output is zero. Give at least one value of `omegahat` such that the output is guaranteed to be zero, for $n \geq 5$ and $n \leq 4000$.

PROBLEM 7.7*:

The frequency response of a linear time-invariant filter is given by the formula

$$H(\hat{\omega}) = (1 - e^{-j\hat{\omega}})(1 + e^{-j\pi/2}e^{-j\hat{\omega}})(1 + e^{j\pi/2}e^{-j\hat{\omega}}). \quad (2)$$

- (a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$. *Hint:* Multiply out the factors to obtain a sum of powers of $e^{-j\hat{\omega}}$.
- (b) What is the impulse response of this system?
- (c) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of $-\pi \leq \hat{\omega} \leq \pi$ will $y[n] = 0$ for all n ?
- (d) Use superposition to determine the output of this system when the input is

$$x[n] = 1 + 2\delta[n - 3] + 7 \cos(0.5\pi n) \quad \text{for } -\infty < n < \infty$$

Hint: Divide the input into three parts and find the outputs separately each by the easiest method and then add the results.