GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2001
Problem Set #6

Assigned: 28-Sept-01
Due Date: Week of 8-Oct-01

- Quiz #2 on 22-October (Monday).
- Reading: In *DSP First*, Chapter 5 on *FIR Filters*
- Please check the "Bulletin Board" often. All official course announcements are posted there.
- **ALL** of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 6.1*:

This problem is concerned with finding the output of an FIR filter for a given input signal. A linear time-invariant system is described by the difference equation

$$y[n] = \sum_{k=0}^{4} |2-k|\, x[n-k]$$

(a) Determine the filter coefficients $\{b_k\}$ of this FIR filter.

(b) Find the impulse response, $h[n]$, for this FIR filter. The impulse response is a discrete-time signal, so make a (stem) plot of $h[n]$ versus n.

(c) Use the above difference equation to compute the output $y[n]$ when the input is

$$x[n] = \begin{cases} 0 & n < 0 \\ 1 & 0 \le n \le 5 \\ -1 & 6 \le n \le 10 \\ 0 & n \ge 11. \end{cases}$$

Make a plot of both $x[n]$ and $y[n]$ vs. n. (*Hint:* you might find it useful to check your results with MATLAB's `conv()` function.)

PROBLEM 6.2*:

The *unit step* sequence, denoted by $u[n]$, is defined as

$$u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0. \end{cases}$$

(a) Make a plot of $u[n]$ for $-5 \leq n \leq 10$. Describe the plot of $u[n]$ outside this range.

(b) We can use the unit step sequence as a convenient representation for sequences that are given by formulas over a range of values. For example, make a plot of the sequence

$$x[n] = \left(\frac{2}{3}\right)^n (u[n] - u[n-6])$$

for $-5 \leq n \leq 10$. *Hint:* First determine the values of the sequence $(u[n] - u[n-6])$.

(c) Suppose that $x[n]$ in part (b) is the input to a 4-point running average system. Compute and plot $y[n]$, the output of the system for $-5 \leq n \leq 10$.

PROBLEM 6.3:

Consider a system defined by $\qquad y[n] = \sum_{k=10}^{20} b_k x[n-k]$

Notice that the filter coefficients b_0, b_1, b_2, ..., b_9 are all zero.

Suppose that the input $x[n]$ is non-zero only for $5 \leq n \leq 20$. Show that $y[n]$ is non-zero at most over a finite interval of the form $N_3 \leq n \leq N_4$. Determine N_3 and N_4. *Hint*: consult Figs. 5.5 and 5.6 in the book for the sliding window interpretation of the FIR filter.

PROBLEM 6.4*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

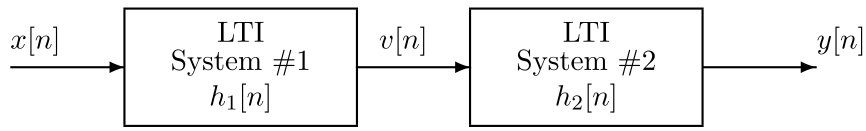

Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 has impulse response,

$$h_1[n] = \begin{cases} 0 & n < 0 \\ 0.125 & 0 \le n \le 7 \\ 0 & n > 7 \end{cases}$$

and System #2 is described by the difference equation

$$y[n] = -2v[n] + 2v[n-1]$$

(a) Determine the difference equation of System #1; i.e., the equation that relates $v[n]$ to $x[n]$.

(b) When the input signal $x[n]$ is an impulse, $\delta[n]$, determine the signal $v[n]$ and make a plot.

(c) Determine $h_2[n]$, the impulse response of System #2.

(d) Determine the impulse response of the overall cascade system, i.e., find $y[n]$ when $x[n] = \delta[n]$.

(e) From the impulse response of the overall cascade system as obtained in part (d), obtain a single difference equation that relates $y[n]$ directly to $x[n]$ in Fig. 1.

PROBLEM 6.5:

For each of the following systems, determine if they are (1) linear; (2) time-invariant; (3) causal.

(a) $y[n] = x[n-2] + 2x[n] + x[n+2]$

(b) $y[n] = nx[n]$

(c) $y[n] = (x[-n])^2$

PROBLEM 6.6*:

A discrete-time system is defined by the input/output relation

$$y[n] = (x[n+1])^3.$$

(a) Determine whether or not this system is (i) linear; (ii) time-invariant; (iii) causal.

(b) For this system determine the output $y_1[n]$ when the input is

$$x_1[n] = 2\sin(0.75\pi n) = -je^{j0.75\pi n} + je^{-j0.75\pi n}.$$

Express your answer in terms of cosine functions. Do not leave any powers of cosine functions in your answers. Notice that this system produces an output that contains frequencies that are not present in the input signal. Explain how this system might cause "aliasing" of sinusoidal components of the input.

PROBLEM 6.7*:

A linear time-invariant discrete-time system is described by the difference equation

$$y[n] = x[n] + x[n-1] + 2x[n-2] - 3x[n-3].$$

(a) Draw a block diagram that represents this system in terms of unit-delay elements, coefficient multipliers, and adders as in Figure 5.13 in the text.

(b) Determine the impulse response $h[n]$ for this system.

(c) Use convolution to determine the output due to the input

$$x[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] = \begin{cases} 1 & n = 0, 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

Plot the output sequence $y[n]$ for $-3 \le n \le 12$.