GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2001
Problem Set #3

Assigned: 7-September-01
Due Date: Week of 17-September-01

• **Quiz #1 will be held in lecture on Monday 17-September-01.** It will cover material from Chapters 2 and 3, as represented in Problem Sets #1 and #2.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$, **both sides)**

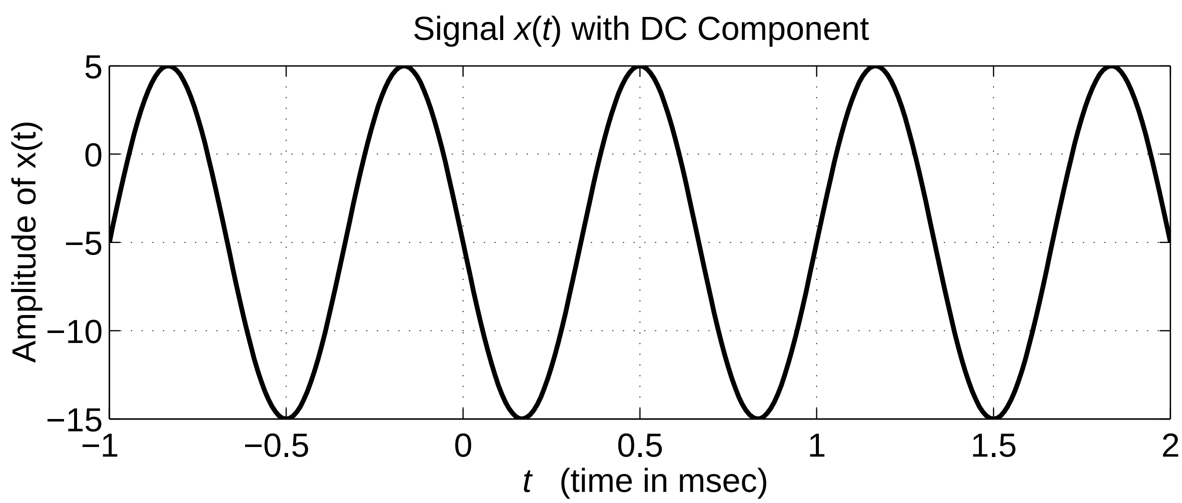
• Reading: In *DSP First*, all of Chapter 3 on *Spectrum Representation,* especially pp. 3001–3036 (revised version) or pp. 48–73 (MSY classic) .

• **Please check the "Bulletin Board" often. All official course announcements are posted there.**

• **ALL** of the **STARRED** problems should be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 3.1*:



The above signal $x(t)$ consists of a DC component plus a cosine signal. The terminology *DC component* means a component that is constant versus time.

(a) What is the frequency of the DC component? What is the frequency of the cosine component?

(b) Write an equation for the signal $x(t)$. You should be able to determine numerical values for all the amplitudes, frequencies, and phases in your equation by inspection of the above graph.

(c) Expand the equation obtained in the previous part into a sum of positive and negative frequency complex exponential signals.

(d) Plot the two-sided spectrum of the signal $x(t)$. Show the complex amplitudes for each positive and negative frequency contained in $x(t)$.

PROBLEM 3.2:

Consider the signal

$$x(t) = 20[\sin(1000\pi t)]^2.$$

(a) Using the inverse Euler relation for the sine function, express $x(t)$ as a sum of complex exponential signals with positive and negative frequencies.

(b) Use your result in part (a) to express $x(t)$ in the form $x(t) = A_0 + A_1 \cos(\omega_0 t)$.

(c) Determine the period T_0 of $x(t)$ and sketch its waveform over the interval $-T_0 \le t \le 2T_0$. Carefully label the graph.

(d) Plot the spectrum of $x(t)$.

PROBLEM 3.3*:

A real periodic signal $x(t)$ is created by the following sum of complex exponentials:

$$x(t) = \sum_{k=-3}^{3} \left(\frac{j^k}{2\pi}\right) e^{j2\pi 50kt}.$$

(a) Write an equation for $x(t)$ as a sum of cosines.

(b) Plot the spectrum of $x(t)$.

(c) Find the fundamental period of $x(t)$.

PROBLEM 3.4*:

(Similar to *DSP First*, Chapter 3, Problem 8, page 80.)

We have seen that musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano you are aware of the fact that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Since middle C is 9 tones below A440, its frequency is approximately $(440)2^{-9/12} \approx 262$ Hz. The names of the tones (notes) of the octave starting with A-440 and ending with A-880 are:

note name	A	$B^\flat$	B	C	$C\#$	D	$E^\flat$	E	F	$F\#$	G	$G\#$	A
note number	49	50	51	52	53	54	55	56	57	58	59	60	61
frequency													

(a) Make a table of the frequencies of the tones of the octave beginning with A-440 and ending at A-880. Recall that A-440 is the A above middle C (note #49) which is tuned to 440 Hz.

(b) The notes (from part (a)) on a piano keyboard are numbered 49 through 61. If n denotes the note number, and f denotes the frequency of the corresponding tone in hertz, give a formula for the frequency of the tone as a function of the note number.

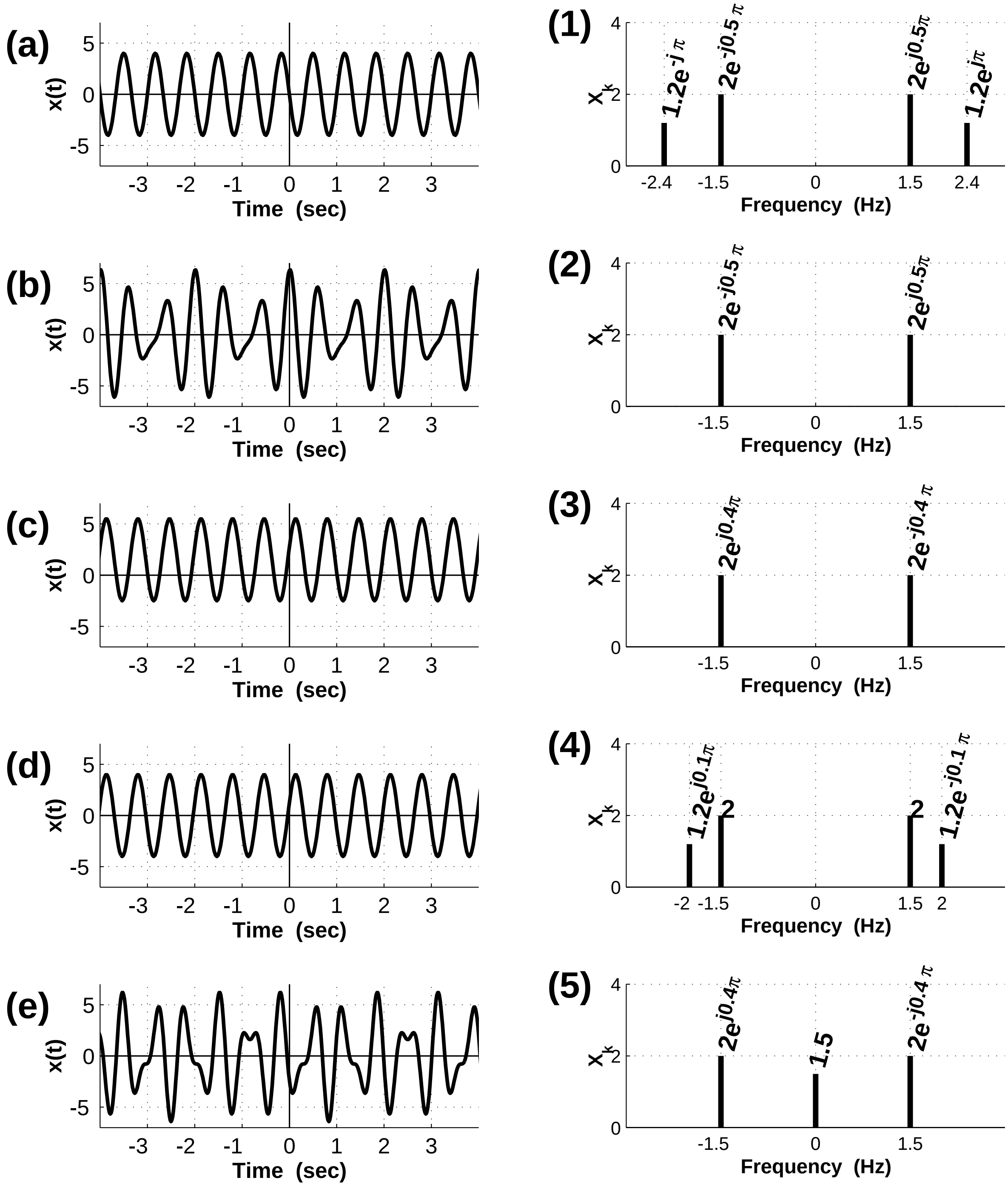
(c) A *chord* is a combination of musical notes sounded simultaneously. A *triad* is a three note chord. The A Minor triad is composed of the tones of A, C, E sounded simultaneously. From the set of corresponding frequencies determined in part (a), make a sketch of the essential features of the spectrum of the A-Minor chord assuming that each note is realized by a pure sinusoidal tone and that each note is equally loud. (You do not have to specify the complex amplitudes precisely.)

PROBLEM 3.5*:

The following plots show waveforms on the left and spectra on the right.

(a) Hand in a table matching the waveform letter with its corresponding spectrum number.

(b) For each waveform on the left, write its formula as a sum of sinusoids.



PROBLEM 3.6:

In this problem you will consider the general case of the "beating" phenomenon. When you multiply two sinusoids:

$$x(t) = \cos(2\pi(40)t - \pi/3)\cos(2\pi(600)t + \pi/4)$$

the signal can still be expressed as a "spectrum." In order to do this, you need an *additive* combination of sinusoids.

(a) Use the inverse Euler formula to obtain a set of complex exponential signals that sum together to make $x(t)$.

(b) Plot the spectrum of $x(t)$.

(c) Find a complex signal $z(t)$ such that $x(t) = \Re e\{z(t)\}$.

(d) Use the spectrum to write an alternate formula for $x(t)$ as:

$$x(t) = A\cos[2\pi(f_c - \Delta)t + \phi_1] + B\cos[2\pi(f_c + \Delta)t + \phi_2]$$

Find the numerical values for all the parameters: A, B, f_c, Δ, ϕ_1, and ϕ_2.

(e) This signal is periodic; determine its fundamental period.

PROBLEM 3.7*:

A periodic signal $x(t)$ with a period $T_0 = 10$ is described *over one period,* $0 \leq t \leq 10$, by the equation

$$x(t) = \begin{cases} 0 & 0 \leq t \leq 5 \\ 2 & 5 < t \leq 10. \end{cases}$$

This signal can be represented by the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t},$$

which is valid for all time $-\infty < t < \infty$.

(a) Sketch the periodic function $x(t)$ for $-10 < t < 20$.

(b) Determine the D.C. coefficient of the Fourier Series, a_0.

(c) Use the Fourier *analysis* integral [1] (for $k \neq 0$)

$$a_k = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-jk\omega_0 t} dt$$

to find the first ($k = 1$) Fourier series coefficient, a_1. Note: $\omega_0 = 2\pi/T_0$.

(d) If we add a constant value of one to $x(t)$, we obtain the signal $y(t) = 1 + x(t)$ with $y(t)$ given over one period by

$$y(t) = \begin{cases} 1 & 0 \leq t \leq 5 \\ 3 & 5 < t \leq 10. \end{cases}$$

This signal can also be represented by a Fourier series,

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}.$$

Explain how b_0 and b_1 are related to a_0 and a_1. (Note: You should not have to evaluate any new integrals explicitly to answer this question.)

[1] The Fourier integral can be done over any period of the signal; in this case, the most convenient choice is from 0 to 10.