GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2001
Problem Set #2

Assigned: 31-Aug-01
Due Date: Week of 10-Sep-01

Reading: In *DSP First*, all of Chapter 2 on *Sinusoids*; and start reading in Chapter 3: *Spectrum Representation,* especially pp. 48–61. Look for a new version of Ch. 3 to be posted on webct. It will contain updates for the Fourier series material.

$\Longrightarrow$ Please check the "Bulletin Board" often. All official course announcements are posted there.

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 2.1*:

Each of the following signals may be simplified and expressed as a single sinusoid of the form: $A\cos(\omega t + \phi)$. For each signal, draw a vector diagram of the complex amplitudes (phasors) and use vector addition to estimate the amplitude A and phase ϕ of the sinusoid. Then use the phasor addition theorem to find the exact values for A and ϕ.

(a) $x_a(t) = 2\cos(40\pi t - 3\pi/4) + 2\cos(40\pi t - \pi/4) + 2\cos(40\pi t + \pi/2)$

(b) $x_b(t) = \sqrt{3}\cos(50\pi t + 11\pi) + 2\cos(50\pi t - 11.5\pi) + \cos(50\pi t + 12\pi)$

(c) $x_c(t) = 10\cos(60\pi t + \pi/6) + 10\cos(60\pi t + 5\pi/6) + 10\cos(60\pi t - 5\pi/6) + 10\cos(60\pi t - \pi/6)$

PROBLEM 2.2*:

Define $x(t)$ as

$$x(t) = 2\cos(20\pi t + \pi/6) + 2\sqrt{3}\sin(20\pi(t - 1/120))$$

(a) Find a complex-valued signal $z(t)$ such that $x(t) = \Re e\{z(t)\}$. Simplify $z(t)$ as much as possible, so that you can identify its complex amplitude. *Hint:* Be careful to note that the second term in $x(t)$ is a sine rather than a cosine.

(b) Make a plot of $\Re e\{\sqrt{2}(1 + j)e^{j0.2\pi t}\}$ over the range $-10 \le t \le 10$ secs. How many periods are included in the plot?

PROBLEM 2.3:

Define $x(t)$ as

$$x(t) = 2\sqrt{3}\cos(10\pi t - 5\pi/6) + A\cos(10\pi t + \phi), \tag{1}$$

where A is a *positive* number. In addition, assume that $x(t)$ has a phase of π, so that it may be written as

$$x(t) = B\cos(10\pi t + \pi), \tag{2}$$

where B is a *positive* number.

(a) What relationship must exist between A and ϕ in order for $x(t)$ to have the phase indicated in Eq. 2?

(b) If $B = 4$, what are the values for A and ϕ?

(c) Now assume that B is unspecified. Find the values for A, B, and ϕ so that the value of A is *minimized*. Draw a plot of the complex amplitudes to prove using a geometrical argument that you have found the minimum for A. *Hint:* Recall the geometrical "theorem" that tells you how to find the shortest distance between a line and a point that is not on the line.

PROBLEM 2.4*:

Complex exponentials obey the expected rules of algebra when doing operations such as integrals, derivatives, and time-shifts. Consider the complex signal $z(t) = Ze^{j\pi t}$ where $Z = e^{-j\pi/4}$.

(a) Show that the first derivative of $z(t)$ with respect to time can be represented as a new complex exponential $Qe^{j\pi t}$, i.e., $\frac{d}{dt}z(t) = Qe^{j\pi t}$. Determine the value for the complex amplitude Q.

(b) Plot both Z and Q in the complex plane. How much greater (or smaller) is the angle of Q than the angle of Z?

(c) Compare $\Re e\{\frac{d}{dt}z(t)\}$ to $\frac{d}{dt}\Re e\{z\}$ for the given signal. Would this relationship still be true if you changed the frequency of the exponential? Would it still be true if you changed the phase?

(d) Evaluate the definite integral of $z(t)$ over the range $-0.5 \le t \le 0.5$:

$$\int_{-0.5}^{0.5} z(t)dt = ?$$

Note that integrating a complex quantity follows the expected rules of algebra: you could integrate the real and imaginary parts separately, but you can also *use the integration formula for an exponential* directly on $z(t)$.

(e) Show that the time-shifted version of $z(t)$ can be represented as a new complex exponential $Qe^{j\pi t}$, i.e., $z(t - t_d) = Qe^{j\pi t}$. Determine the value for the complex amplitude Q.

PROBLEM 2.5*:

Consider the signal

$$x(t) = 2 + 2\cos(5000\pi t + 3\pi/7) + 3\cos(7000\pi t - \pi/5).$$

This signal has three sinusoidal components (including the "DC" component, which has frequency 0).

(a) Express the signal $x(t)$ as a sum of complex exponential components using the relationship

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}.$$

(b) What frequencies (in Hz) are present in this signal?

(c) For each frequency identified in part (b), give the complex amplitude of the corresponding complex exponential component. Make a table of your analysis with frequency in one column and complex amplitude in a second column.

PROBLEM 2.6*:

In AM radio, the transmitted signal is voice (or music) mixed with a *carrier signal.* The carrier is a sinusoid at the assigned broadcast frequency of the AM station. For example, WCNN in Atlanta has a *carrier frequency* of 680 kHz. If we use the notation $v(t)$ to denote the voice/music signal, then the actual transmitted signal for WCNN might be:

$$x(t) = (v(t) + A)\cos(2\pi(680 \times 10^3)t)$$

where A is a constant. (A is introduced to make the AM receiver design easier, in which case A must be chosen to be larger than the maximum value of $v(t)$.)

(a) Voice-band signals tend to contain frequencies less than 4000 Hz (4 kHz). Suppose that $v(t)$ is a 1 kHz sinusoid, $v(t) = 2\sin(2\pi(1000)t)$. Draw the spectrum for $v(t)$.

(b) Now draw the spectrum for $x(t)$, assuming a carrier at 680 kHz. Use $v(t)$ from part (a) and assume that $A = 2.5$. *Hint:* Substitute for $v(t)$ and expand $x(t)$ into a sum of cosine terms of three different frequencies.

(c) How would the spectrum of the AM radio signal change if the carrier frequency is changed to 750 kHz (WSB) and $v(t)$ and A are the same as defined in parts (a) and (b)?