Problem 9.1

A) $\delta(t-3) * [\delta(t) + 2e^{-t}\cos(5\pi t)u(t)]$

$= \delta(t-3) * \delta(t) + \delta(t-3) * 2e^{-t}\cos(5\pi t)\,u(t)$

$= \delta(t-3) + 2e^{-(t-3)}\cos(5\pi(t-3))\,u(t-3)$

$= \delta(t-3) + 2e^{-(t-3)}\cos(5\pi t - \pi)\,u(t-3)$

for the phase:
$-15\pi = -\pi$

B) $[u(-t+3) - u(t)][\delta(t-1) + \delta(t-4)]$

$= u(-t+3)\,\delta(t-1) + u(-t+3)\,\delta(t-4) - u(t)\,\delta(t-1) - u(t)\,\delta(t-4)$

$= u(2)\,\delta(t-1) + u(-1)\,\delta(t-4) - u(1)\,\delta(t-1) - u(4)\,\delta(t-4)$

$= \delta(t-1) - \delta(t-1) - \delta(t-4)$

$= -\delta(t-4)$

C) $\frac{d}{dt}[\cos(5\pi t)\,u(t-1)] = \cos 5\pi t\left[\frac{d\,u(t-1)}{dt}\right] + \left[\frac{d}{dt}(\cos 5\pi t)\right]u(t-1)$

$= (\cos(5\pi t))\,\delta(t-1) - 5\pi(\sin(5\pi t))\,u(t-1)$

$= \cos(5\pi)\,\delta(t-1) - 5\pi(\sin(5\pi t))\,u(t-1)$

$= -\delta(t-1) - 5\pi(\sin(5\pi t))\,u(t-1)$

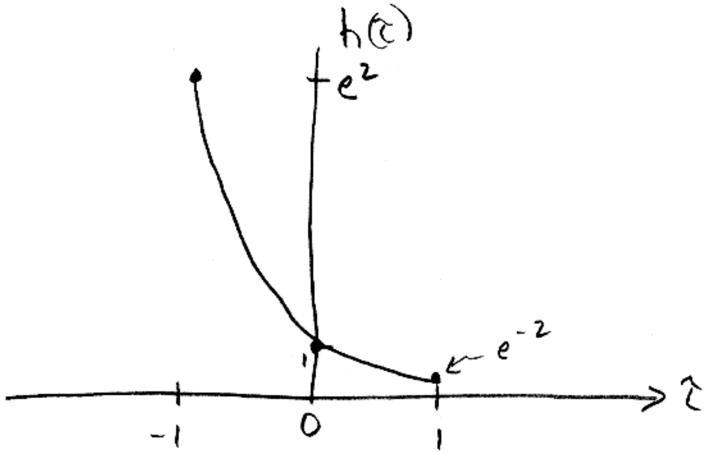
D) $\int_{-\infty}^{t} e^{-(\tau-1)}\,\delta(\tau-1)\,d\tau$ using $u(t) = \int_{-\infty}^{t}\delta(\tau)\,d\tau$

$= \int_{-\infty}^{t} e^{-(1-1)}\,\delta(\tau-1)\,d\tau = \int_{-\infty}^{t}\delta(\tau-1)\,d\tau$

$= u(t-1)$

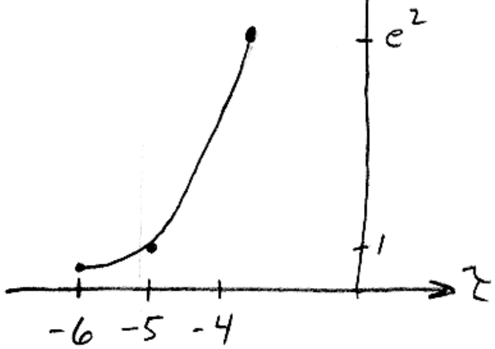
Problem 9.2

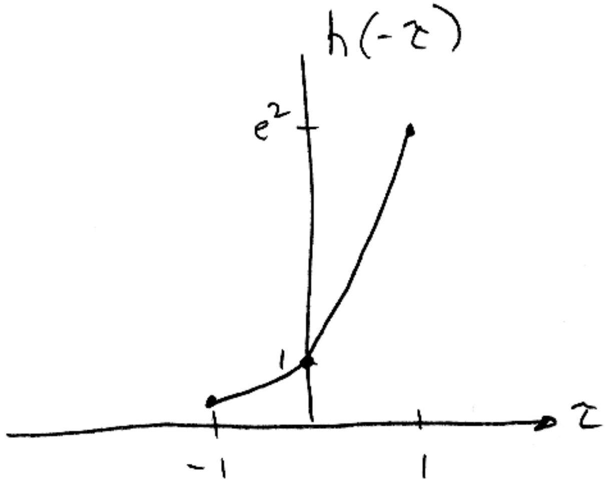
A) Plot $h(\tau)$ and $h(t-\tau)$ for $t = -5, 0, 5$

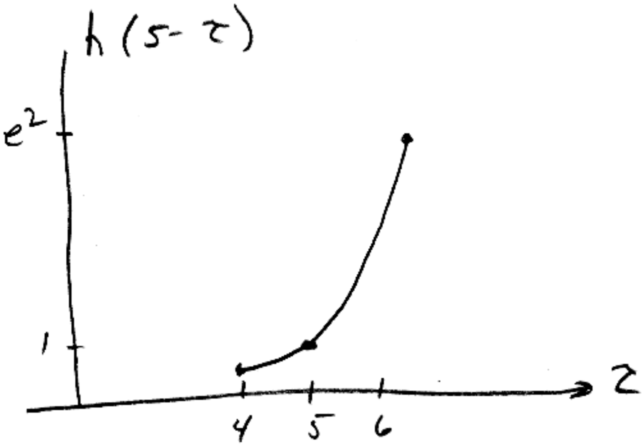


$-5-\tau \geq -1$ & $-5-\tau \leq 1$

$\tau \leq -4$ & $\tau \geq -6$







B) The system is not causal. $h(t) \neq 0$ for $t < 0$.

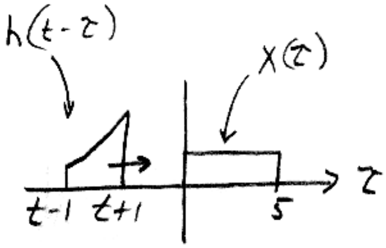
c) $y(t) = x(t) * h(t) = \delta(t+5) * e^{-2t} = e^{-2(t+5)}$ for $-6 \leq t \leq -4$

$$\Rightarrow y(t) = \begin{cases} e^{-10} e^{-2t} & -6 \leq t \leq -4 \\ 0 & \text{otherwise} \end{cases}$$

Problem 9.2 (continued)

D) $x(t) = \begin{cases} 1 & 0 \le t \le 5 \\ 0 & \text{otherwise} \end{cases}$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = x(t) * h(t) =$$



Interval 1 For $t+1<0$
$t<-1$, functions don't overlap

$$y(t) = 0 \qquad t<-1$$

Interval 2

$t+1 \ge 0$ & $t-1<0 \Rightarrow -1 \le t \le 1$

$$y(t) = \int_0^{t+1} 1\, e^{-2(t-\tau)}\,d\tau = e^{-2t}\int_0^{t+1} e^{2\tau}\,d\tau$$

$$= e^{-2t}\left(\frac{e^{2(t+1)}}{2} - \frac{1}{2}\right)$$

$$y(t) = \frac{e^{2}}{2} - \frac{e^{-2t}}{2}$$

Interval 3

$t+1 \le 5$ & $t-1 \ge 0 \Rightarrow 1 \le t \le 4$

$$y(t) = \int_{t-1}^{t+1} 1 e^{-2(t-\tau)}\,d\tau = e^{-2t}\int_{t-1}^{t+1} e^{2\tau}\,d\tau = e^{-2t}\left(\frac{e^{2(t+1)}}{2} - \frac{e^{2(t-1)}}{2}\right)$$

$$y(t) = \frac{e^{2}}{2} - \frac{e^{-2}}{2}$$

Interval 4 $t+1 \ge 5$ + $t-1<5 \Rightarrow 4 \le t \le 6$

$$y(t) = \int_{t-1}^{5} 1 e^{-2(t-\tau)}\,d\tau = \frac{e^{-2(t-5)}}{2} - \frac{e^{-2}}{2}$$

Interval 5 $t-1 \ge 5 \Rightarrow t \ge 6$

$y(t) = 0$, functions don't overlap

Problem 9.3

A) If $y(t) = \int_{-\infty}^{t+4} x(\tau)\,d\tau$, then $h(t) = \int_{-\infty}^{t+4} \delta(\tau)\,d\tau$

$\Rightarrow h(t) = u(t+4)$

B) A system is stable if:

1. a bounded input produces bounded outputs
2. $\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$

Using criteria 2, $\int_{-\infty}^{\infty} u(t+4)\,dt = \int_{-4}^{\infty} dt = \infty$

Therefore, the system is <u>unstable</u>.

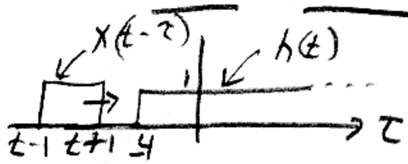
C) A system is causal if:

1. $y(t_0)$ depends only on $x(\tau)$ for $\tau \le t_0$
2. $h(t) = 0$ for $t < 0$

Using criteria 2, $h(t) = u(t+4) \ne 0$ for all $t < 0$

Therefore, the system is <u>not</u> <u>causal</u>.

D) $y(t) = x(t) * h(t) =$



Interval 1

$t+1 \le -4 \Rightarrow t \le -5$

$y(t) = 0$

Interval 2

$t+1 \ge -4$ & $t-1 \le -4 \Rightarrow -5 \le t \le -3$

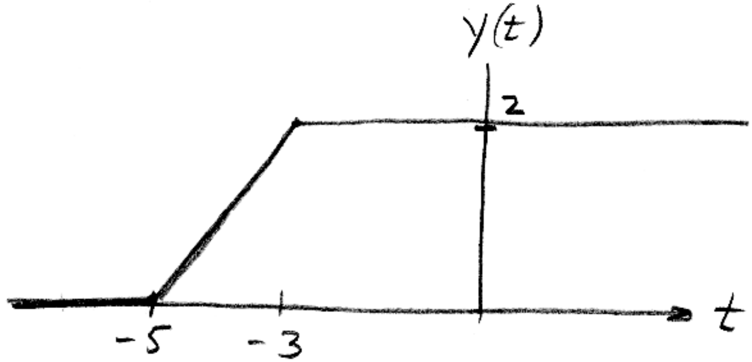
$y(t) = \int_{-4}^{t+1} d\tau = t+5$

Problem 9.3 (continued)

Interval 3 $\quad t-1 \ge -4 \Rightarrow t \ge -3$

$$y(t) = \int_{t-1}^{t+1} d\tau = 2$$

Plot of $y(t)$



E) If $x(t) = u(t)$ produced $y(t) = (t+4)u(t+4)$, then by time invariance $x_1(t) = u(t+1)$ produces $y_1(t) = (t+5)u(t+5)$ and $x_2(t) = u(t-1)$ produces $y_2(t) = (t+3)u(t+3)$. Using linearity (superposition), if $x(t) = x_1(t) - x_2(t)$, then $y(t) = y_1(t) - y_2(t)$.

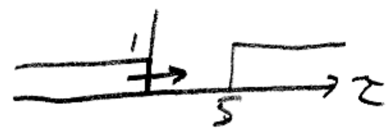
Therefore $y(t) = (t+5)u(t+5) - (t+3)u(t+3)$

Problem 9.4

$$x(t) \rightarrow \boxed{h_1(t)} \xrightarrow{w(t)} \boxed{h_2(t)} \rightarrow y(t)$$

A) If $w(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$, then $h_1(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau = u(t)$

Therefore $h_{\substack{\text{overall}\\ \text{system}}}(t) = h_1(t) * h_2(t) = u(t) * u(t-5)$



Interval 1 $\quad t < 5$

$h(t) = y(t) = 0$

Interval 2 $\quad t \geq 5$

$h(t) = y(t) = \int_{5}^{t} d\tau = t - 5$

B) If $w(t) = \frac{d\,x(t)}{dt} + 3x(t)$ and $h_{\substack{\text{overall}\\ \text{system}}}(t) = \delta(t)$

$$h_{\substack{\text{overall}\\ \text{system}}}(t) = \left(\frac{d\,\delta(t)}{dt} + 3\,\delta(t)\right) * h_2(t)$$

$$= \left(\frac{d\,\delta(t)}{dt} + 3\,\delta(t)\right) * e^{\alpha t} u(t) = \frac{d}{dt}\left(e^{\alpha t} u(t)\right) + 3e^{\alpha t} u(t)$$

$$= \alpha e^{\alpha t} u(t) + e^{\alpha t} \delta(t) + 3e^{\alpha t} u(t)$$

$$= (3 + \alpha) e^{\alpha t} u(t) + \delta(t)$$

Therefore for $h_{\substack{\text{overall}\\ \text{system}}}(t) = \delta(t)$, then $\alpha = -3$

$$h_2(t) = e^{-3t} u(t)$$

Problem 9.5

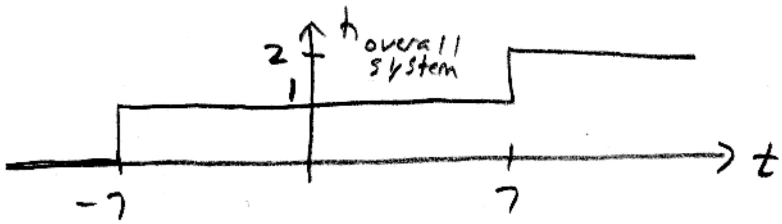
A) $h_1(t) = \delta(t+7)$, $h_2(t) = \delta(t-7)$, $h_3(t) = u(t)$

$$h_{\substack{\text{overall}\\ \text{system}}}(t) = \left(h_1(t) + h_2(t)\right) * h_3(t)$$

$$= \left(\delta(t+7) + \delta(t-7)\right) * u(t)$$

$$= \delta(t+7) * u(t) + \delta(t-7) * u(t)$$

$$= u(t+7) + u(t-7)$$



B) Causal if $h(t) = 0$ for $t < 0$. From graph above, $h(t) \neq 0$ for $t < 0$. Therefore, the system is <u>not causal</u>.

C) Stable if $\int_{-\infty}^{\infty} |h(t)| dt < \infty$

For this system $\int_{-\infty}^{\infty} |h(t)| dt = \int_{-7}^{7} dt + \int_{7}^{\infty} 2\, dt = \infty$

Therefore the system is <u>not stable</u>.