Problem 3.1

(A) The DC component has a frequency of zero, $f_{DC} = 0$. The cosine component has a frequency of $f = \frac{1}{T_0}$, where $T_0 \equiv$ period

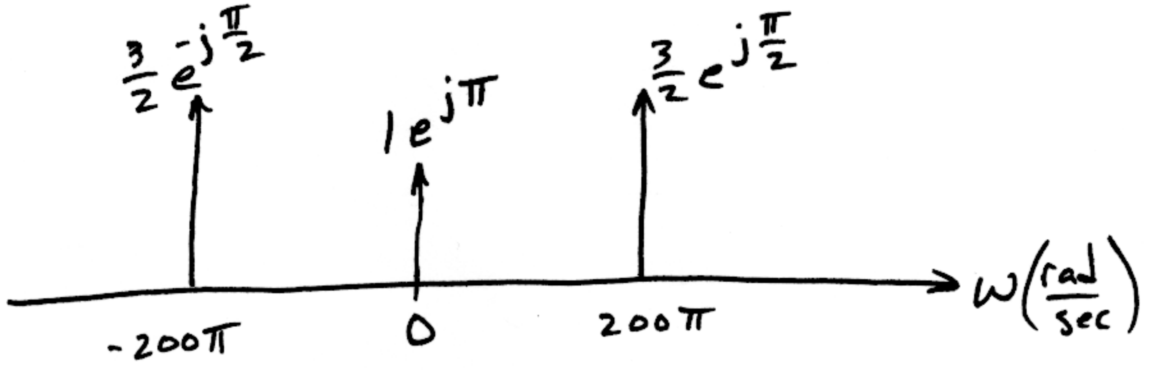
$$\Rightarrow f_{cosine} = \frac{1}{T_0} = \frac{1}{0.01} = 100 \text{ Hz}$$

(B) $$x(t) = -1 + 3\cos\left(200\pi t + \frac{\pi}{2}\right)$$

(C) $$x(t) = -1 + 3\left(\frac{e^{j\frac{\pi}{2}} e^{j200\pi t} + e^{-j\frac{\pi}{2}} e^{-j200\pi t}}{2}\right)$$

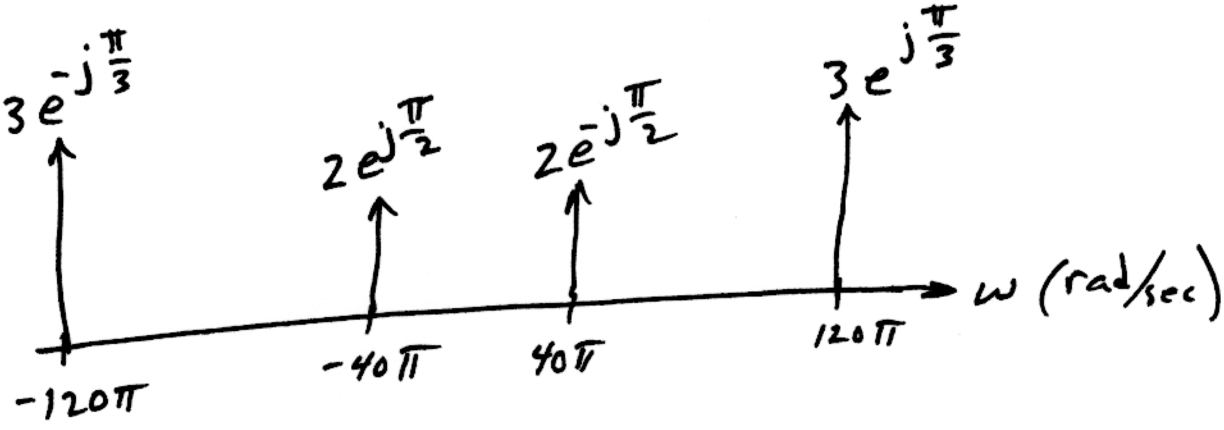
$$= -1 + \frac{3}{2} e^{j\frac{\pi}{2}} e^{j200\pi t} + \frac{3}{2} e^{-j\frac{\pi}{2}} e^{-j200\pi t}$$

(D)



Problem 3.2

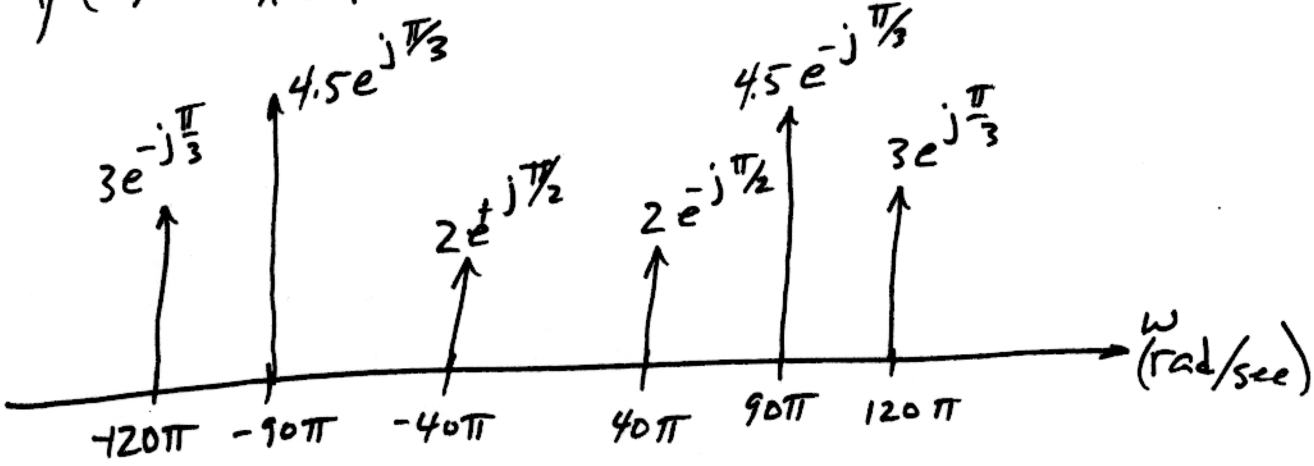
(A)



(B) $x(t)$ is periodic. The fundamental frequency is $\omega = 40\pi \Rightarrow f = 20$ Hz $\Rightarrow T_0 = 0.05$ sec

The fundamental ($k=1$) and 3^{rd} harmonic ($k=3$) at 120π are present.

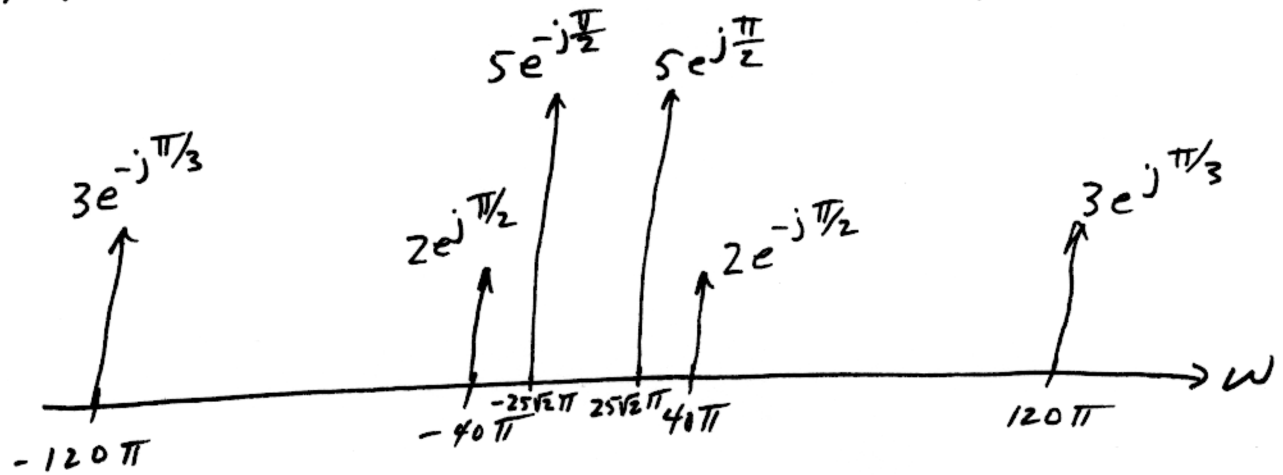
(C) $y(t) = x(t) + 9\cos(90\pi t - \pi/3)$



$y(t)$ is periodic. The period is $T_0 = \frac{1}{f_0} = \frac{1}{5}$ sec for $\omega_0 = 10\pi$. Note the 4^{th}, 9^{th} and 12^{th} harmonics are present.

Problem 3.2. (cont.)

(D) $w(t) = x(t) + 10\cos(25\sqrt{2}\pi t + \frac{\pi}{2})$



$w(t)$ is not periodic, because $\frac{40}{25\sqrt{2}}$ is not a rational number.

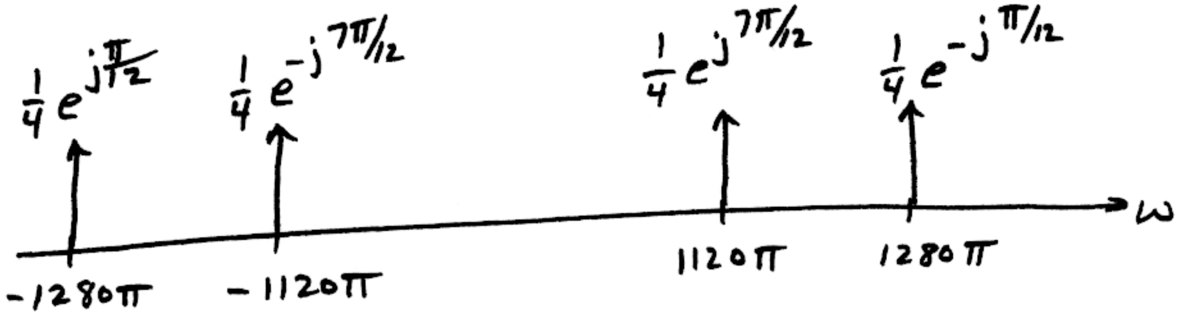
Problem 3.3

(A) $x(t) = \cos(2\pi(40)t - \pi/3)\cos(2\pi(600)t + \pi/4)$

$$x(t) = \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)\left[\left(e^{j80\pi t}e^{-j\pi/3} + e^{-j80\pi t}e^{j\pi/3}\right)\left(e^{j1200\pi t}e^{j\pi/4} + e^{-j1200\pi t}e^{-j\frac{\pi}{4}}\right)\right]$$

$$= \frac{1}{4}\left[e^{j1280\pi t}e^{-j\pi/12} + e^{j1120\pi t}e^{j7\pi/12} + e^{-j1120\pi t}e^{-j7\pi/12} + e^{-j1280\pi t}e^{j\pi/12}\right]$$

(B)



(C) Using $Ae^{j(\omega t+\phi)} = A\cos(\omega t+\phi) + Aj\sin(\omega t+\phi)$

then if $z(t) = Ae^{j(\omega t+\phi)} \Rightarrow \mathrm{Re}\{z(t)\} = A\cos(\omega t+\phi)$

Combining this result with the solutions to Parts (A) + (B) yields:

$$z(t) = \frac{1}{2}e^{j(1120\pi t + \frac{7\pi}{12})} + \frac{1}{2}e^{j(1280\pi t - \frac{\pi}{12})}$$

(D) $x(t) = \cos(80\pi t - \frac{\pi}{3})\cos(1200\pi t + \frac{\pi}{4})$

Using $\cos(\omega_1 t + \phi_1)\cos(\omega_2 t + \phi_2) = \frac{1}{2}\cos((\omega_1+\omega_2)t + \phi_1 + \phi_2) + \frac{1}{2}\cos((\omega_1-\omega_2)t + \phi_1 - \phi_2)$

or using the spectrum from Part (B)

$$x(t) = \frac{1}{2}\cos(1280\pi t - \frac{\pi}{12}) + \frac{1}{2}\cos(1120\pi t + \frac{7\pi}{12})$$

(E) Fundamental Frequency, $\omega_0 = 160\pi \Rightarrow$ Fund. Period, $T_0 = \frac{1}{80}$ sec.

Problem 3.4

Signal		Spectrum	Time Signal
a	$\longrightarrow$	4	$x(t) = 3\cos(2.4\pi t - \frac{\pi}{4}) + 3\cos(4\pi t + \pi)$
b	$\longrightarrow$	1	$x(t) = 3\cos(1.8\pi t - \frac{\pi}{4}) + 3\cos(3\pi t + \pi)$
c	$\longrightarrow$	2	$x(t) = 2 + 3\cos(2.4\pi t + \pi/2)$
d	$\longrightarrow$	5	$x(t) = 3\cos(3\pi t + \pi)$
e	$\longrightarrow$	3	$x(t) = 2 + 3\cos(2.4\pi t - \pi/4)$

Problem 3.5

(A) The ratio of frequencies of succesive notes must be $2^{1/12}$ because every 12 notes corresponds to a doubling of the frequency.

$$\left(2^{1/12}\right)^{12} = 2$$

$(440)\,2^{k/12}$ where $k \equiv$ # of keys above or below 440 Hz (A)

(B)

Note Name	A	B^b	B	C	$C^\#$	D	E^b	E	F	$F^\#$	G	$G^\#$	A
Note #	49	50	51	52	53	54	55	56	57	58	59	60	61
Frequency (Hz)	440	466	494	523	554	587	622	659	699	739	784	831	880
K	0	1	2	3	4	5	6	7	8	9	10	11	12

(C) $$f = (440)\,2^{k/12} = (440)\,2^{(n-49/12)}$$

since $k = n-49$

(D)