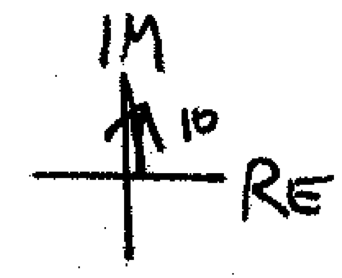
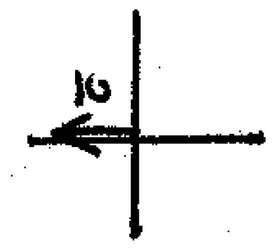
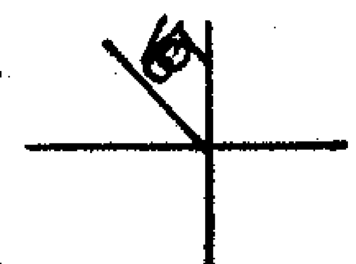
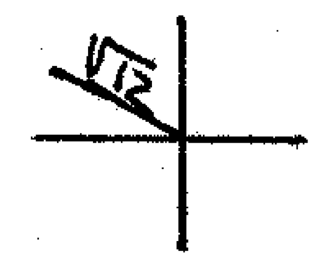
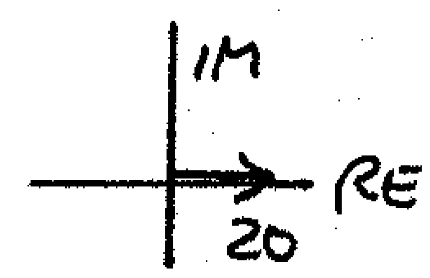
EE2025: HW1 : SPRING 1999+1

1.1

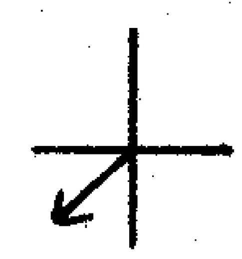
	RECTANGULAR	POLAR	
a)	$j10$	$10e^{j\frac{\pi}{2}}$	
b)	-10	$-10e^{j0}$ $+10e^{j\pi}$	
c)	$-10-j10$	$\sqrt{(-10)^2+(-10)^2}\cdot e^{j\tan^{-1}\left(\frac{Im=-10}{RE=-10}\right)}$ $10\sqrt{2}\ e^{-j\frac{3}{4}\pi}$	
d)	$-2+j2$	$\sqrt{8}\ e^{j3/4\pi}$	
e)	$-3+j\sqrt{3}$	$\sqrt{12}e^{j(150° = \frac{5}{6}\pi \text{ rad})}$	
f)	20	$20e^{j0} = 20$	

BE AWARE OF POSSIBLE CONFUSION BETWEEN 1ST + 3RD QUADRANT SOLUTIONS

1.2

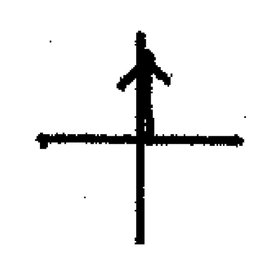
a) $3\sqrt{2}\left(\cos\left(-\frac{3}{4}\pi\right) + j\sin\left(-\frac{3}{4}\pi\right)\right) = 3\sqrt{2}e^{-j\frac{3}{4}\pi}$

$-3 \quad - j3$



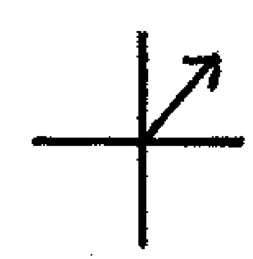
b) $5\left(\cancel{\cos\left(\frac{\pi}{2}\right)}^{0} + j\sin\left(\frac{\pi}{2}\right)\right) = 5e^{j\frac{\pi}{2}}$

$0 + j5$

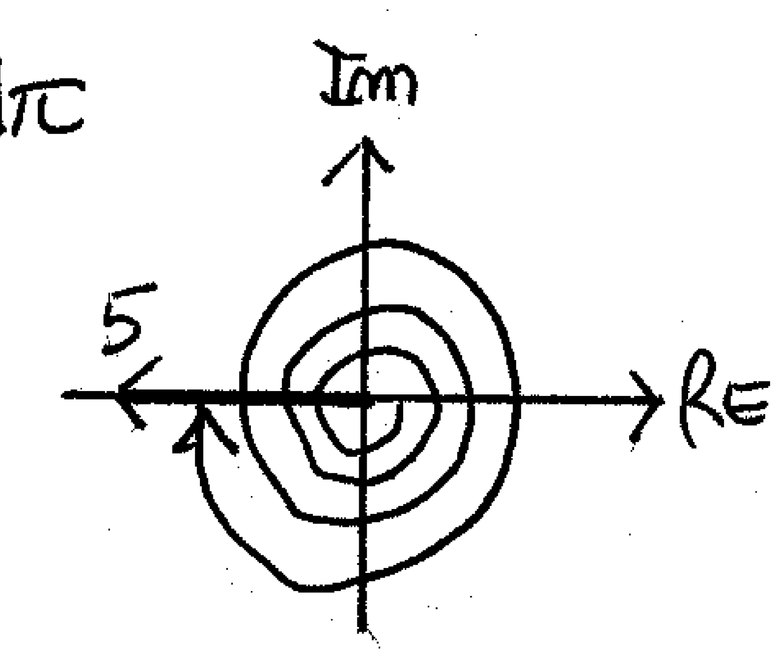


c) $4\left(\cos\left(\frac{\pi}{3}\right) + j\sin\left(\frac{\pi}{3}\right)\right) = 4e^{j\frac{\pi}{3}} = 4\angle\frac{\pi}{3}$

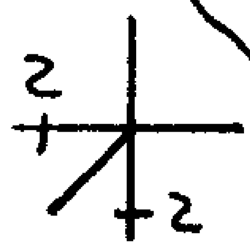
$2 + j2\sqrt{3}$

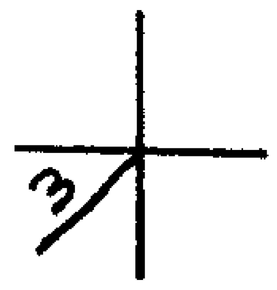


d) $-5 = 5\angle+\pi = 5\angle-\pi = 5\angle-61\pi$



(1.3) $z_1 = -2 - j2 = 2\sqrt{2}\, e^{-j\frac{3}{4}\pi}$ $\quad$ $z_2 = 3e^{-j\frac{3}{4}\pi} = \frac{3}{\sqrt{2}}(-1-j)$





a) $z_1^* = -2 + j2 = 2\sqrt{2}\, e^{+j\frac{3}{4}\pi}$

b) $jz_2 = e^{j\frac{\pi}{2}} \cdot 3e^{-j\frac{3\pi}{4}} = 3e^{-j\frac{\pi}{4}} = (-j+1)\frac{3}{\sqrt{2}}$

c) $\frac{z_2}{z_1} = \frac{3e^{-j\frac{3}{4}\pi}}{2\sqrt{2}\, e^{-j\frac{3}{4}\pi}} = \frac{3}{2\sqrt{2}} e^{j\left(-\frac{3}{4}\pi - \left(-\frac{3\pi}{4}\right)\right)} = \frac{3}{2\sqrt{2}} e^{j0} = \frac{3}{2\sqrt{2}}$

d) $z_2^2 = \left(3e^{-j\frac{3}{4}\pi}\right)^2 = 3e^{-j\frac{3}{4}\pi}\, 3e^{-j\frac{3}{4}\pi} = 9e^{-j\frac{6}{4}\pi} = 9e^{j\frac{\pi}{2}} = j9$

e) $z_1^{-1} = \frac{1}{2\sqrt{2}\, e^{-j\frac{3}{4}\pi}} = \frac{1}{2\sqrt{2}} e^{+j\frac{3}{4}\pi} = -\frac{1}{4} + j\frac{1}{4}$

f) $z_1 \cdot z_2 = 2\sqrt{2}\, e^{-j\frac{3}{4}\pi} \cdot 3e^{-j\frac{3}{4}\pi} = 6\sqrt{2}\, e^{-j\frac{6}{4}\pi} = 6\sqrt{2}\, e^{+j\frac{\pi}{2}} = j6\sqrt{2}$

g) $z_1 + z_2^* = (-2 - j2) + \left(-\frac{3}{\sqrt{2}} + j\frac{3}{\sqrt{2}}\right) = -\left(2 + \frac{3}{\sqrt{2}}\right) + \left(\frac{3}{\sqrt{2}} - 2\right)j \cong 4.12\, e^{j0.99\pi}$

$\simeq -4.12 + j0.121$

CONJUGATE PROPERTIES

h) $|z_2|^2 = z_2 z_2^*$

$|3|^2 = 3e^{-j\frac{3}{4}\pi} \cdot 3e^{+j\frac{3}{4}\pi}$

$3^2 = 3 \cdot 3 \cdot e^{j\pi\left(+\frac{3}{4} - \frac{3}{4}\right)}$

$3^2 = 3^2 \cdot \left(e^{j0} = 1\right)$

$9 = 9$ ✓

i) $z_2 + z_2^*$

$[Re(z_2) + j\cancel{Im(z_2)}] + [Re(z_2) - \cancel{Im(z_2)}]$

$2\, Re(z_2)$

$2 \cdot \frac{-3}{\sqrt{2}}$

$-\frac{6}{\sqrt{2}}$

1.4

a) $z = Ae^{-j\frac{\pi}{3}}$

$z^* = Ae^{+j\frac{\pi}{3}}$

$Im(z^*) = A \sin\left(\frac{\pi}{3}\right) = A\frac{\sqrt{3}}{2}$

b) $z = Ae^{-j\frac{\pi}{3}}$ BORROW FROM PROBLEM 1.3 $\Rightarrow$ $z + z^* = 2\,Re(z)$

$z + z^* = 2A\cos\left(\frac{\pi}{3}\right) = A$

c) $z = 10e^{j\phi}$

$jz = 10e^{j(\phi + \frac{\pi}{2})}$

$Re(jz) = 10\cos\left(\phi + \frac{\pi}{2}\right)$

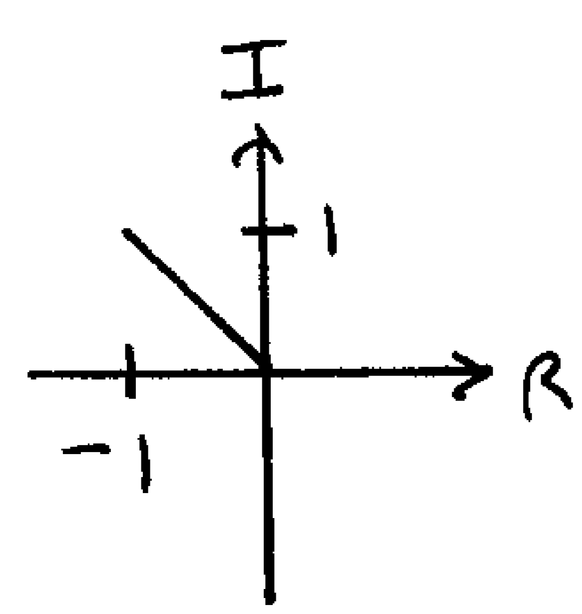
$= -10\sin(\phi)$

NOTE: $-\sin(\theta) = \cos\left(\phi + \frac{\pi}{2}\right)$

d) $z = -\alpha + j\alpha$

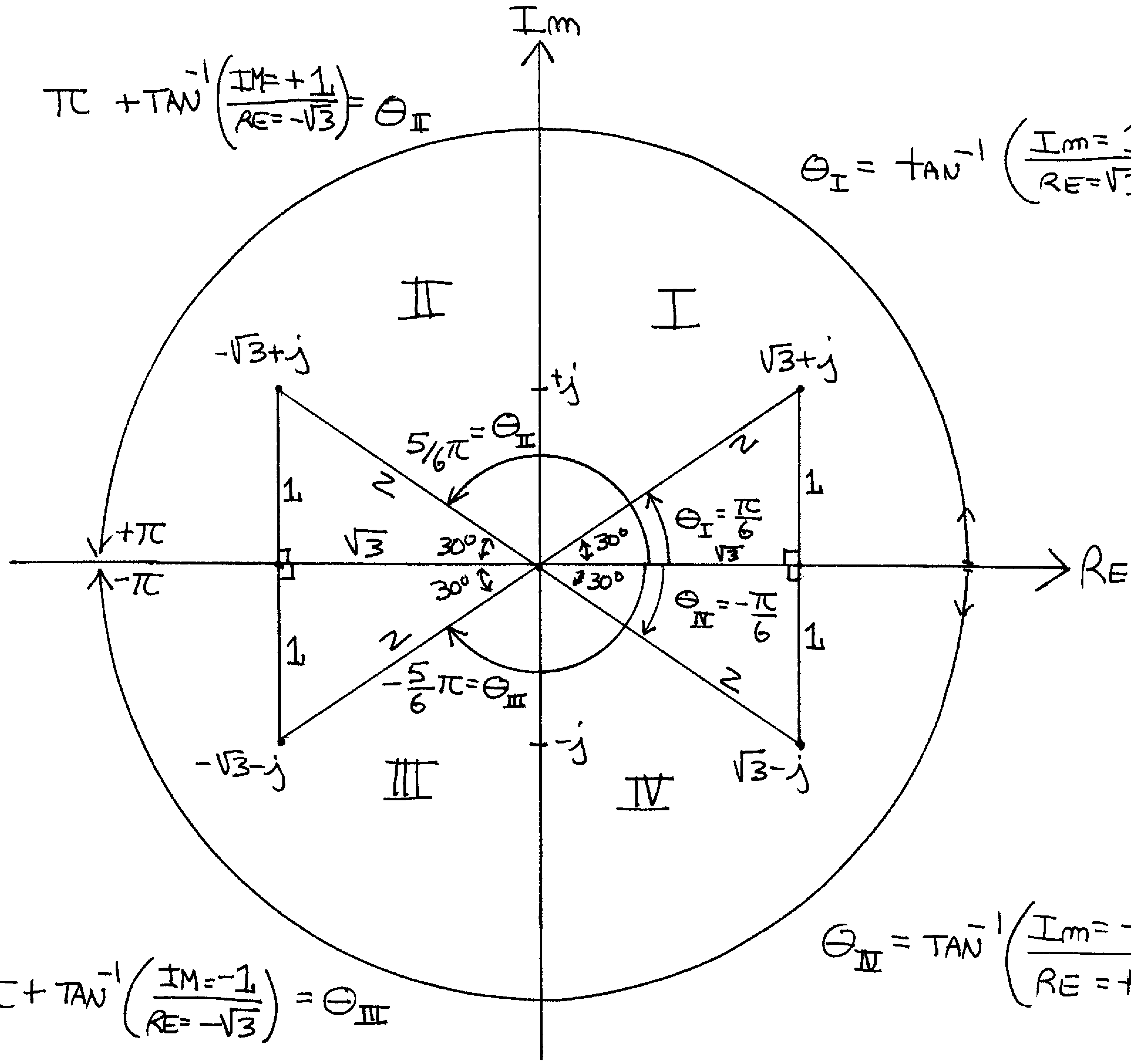
$= \alpha(-1 + j)$

$= \sqrt{2}\alpha\, e^{j\frac{3}{4}\pi}$



$$\pi + \tan^{-1}\left(\frac{IM=+1}{RE=-\sqrt{3}}\right) = \Theta_{II}$$

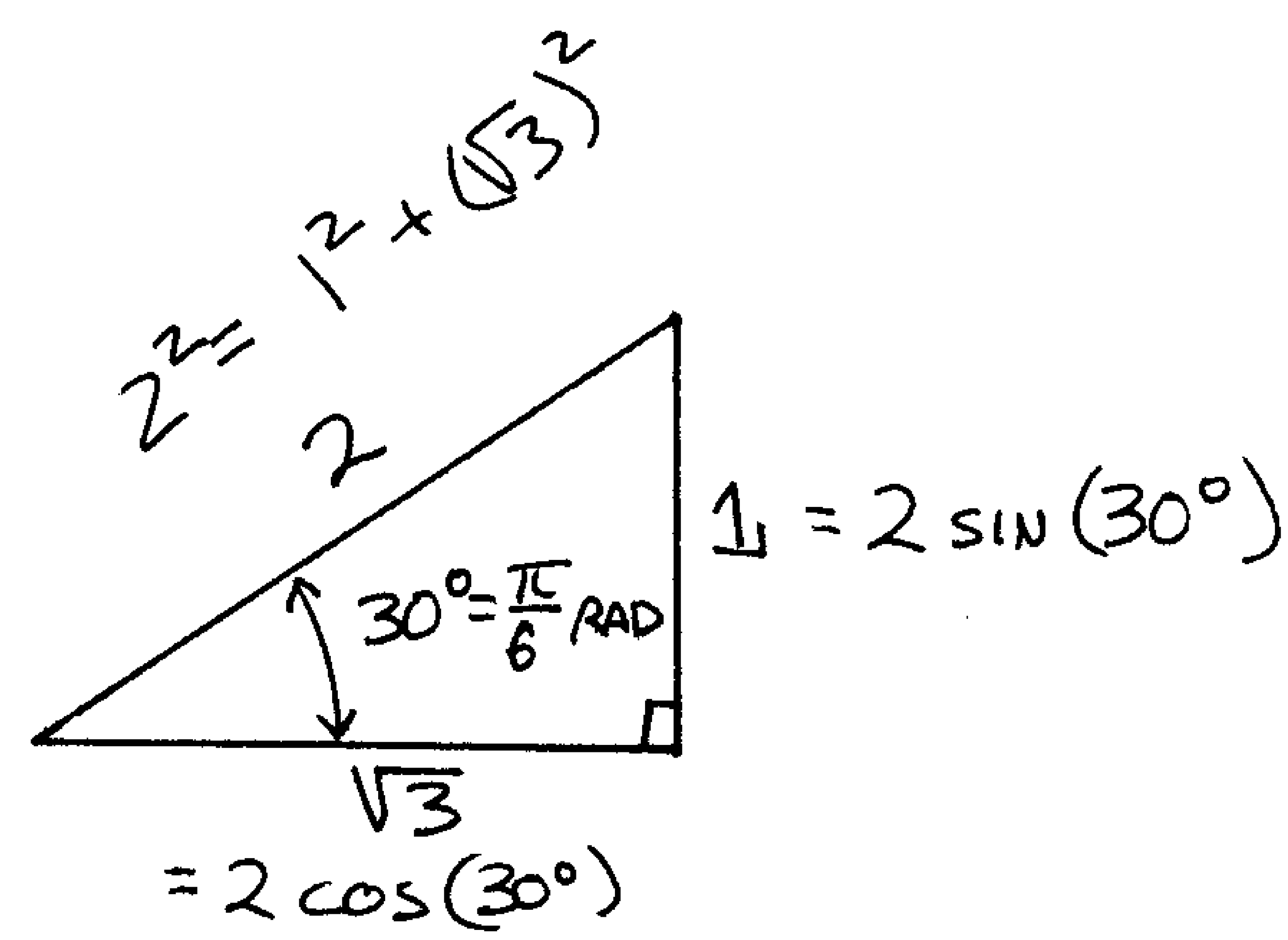
$$\Theta_{I} = \tan^{-1}\left(\frac{Im=1}{RE=\sqrt{3}}\right)$$



$$\Theta_{IV} = \tan^{-1}\left(\frac{Im=-1}{RE=+\sqrt{3}}\right)$$

$$-\pi + \tan^{-1}\left(\frac{IM=-1}{RE=-\sqrt{3}}\right) = \Theta_{III}$$

TANGENTS
+
QUADRANTS



```
» % Homework 1.5a
» z1=2*exp(-j*5/3*pi);
» z2=1*exp(+j*5/6*pi);
» za=z1+z2;
» abs(za)
ans =
    2.2361  = A

» angle(za)/pi
ans =
    0.4809 π[rad] =Θ
```

z_a IN POLAR FORM

```
» % Homework 1.5b
» zb=exp(j*pi/4)-exp(-j*pi/4)-sqrt(2)*exp(-j*pi)
» abs(zb)
ans =
     2  = A

» angle(zb)/pi
ans =
Θ = 0.2500 π[rad]
```

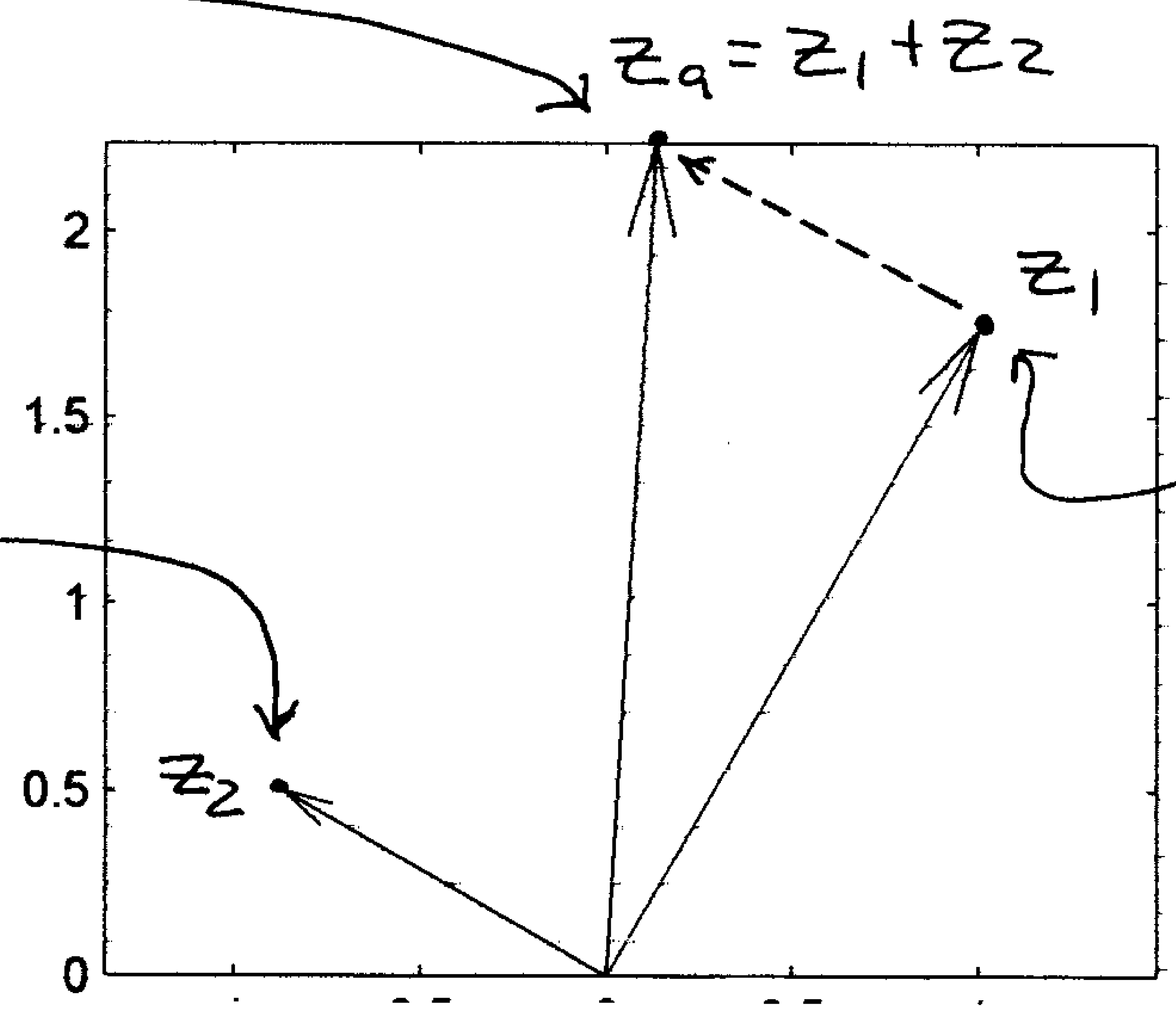
z_b IN POLAR FORM

```
» zprint([za, zb])
```

Z = X	+ jY	Magnitude	Phase	Ph/pi	Ph(deg)	
0.134	2.232	2.236	1.511	0.481	86.57	z_a
1.414	1.414	2	0.785	0.250	45.00	z_b

```
» zvect([z1, z2, z1+z2])
```

MATLAB PLOT FOR 1.5a



DO YOU GET THE SAME NUMBER BOTH WITH MATLAB and YOUR CALCULATOR ?

Use t_3 or t_1 for t_m?

From the definition, t_3 (Chapt 2.3) is the "right" answer. But t_3 is more than π radians away from $t=0$. Therefore, let $t_m = -t_1$. Because our wave is cyclical, we can use whichever peak works best.

PROBLEM 1.6*:

The waveform in the following figure can be expressed as

$$x(t) = A\cos[\omega_0(t - t_m)] = A\cos(\omega_0 t + \phi) = A\cos(2\pi f_0 t + \phi)$$

From the waveform, determine A, ω_0, t_m, and ϕ. Choose the value of ϕ such that $-\pi < \phi \leq \pi$.

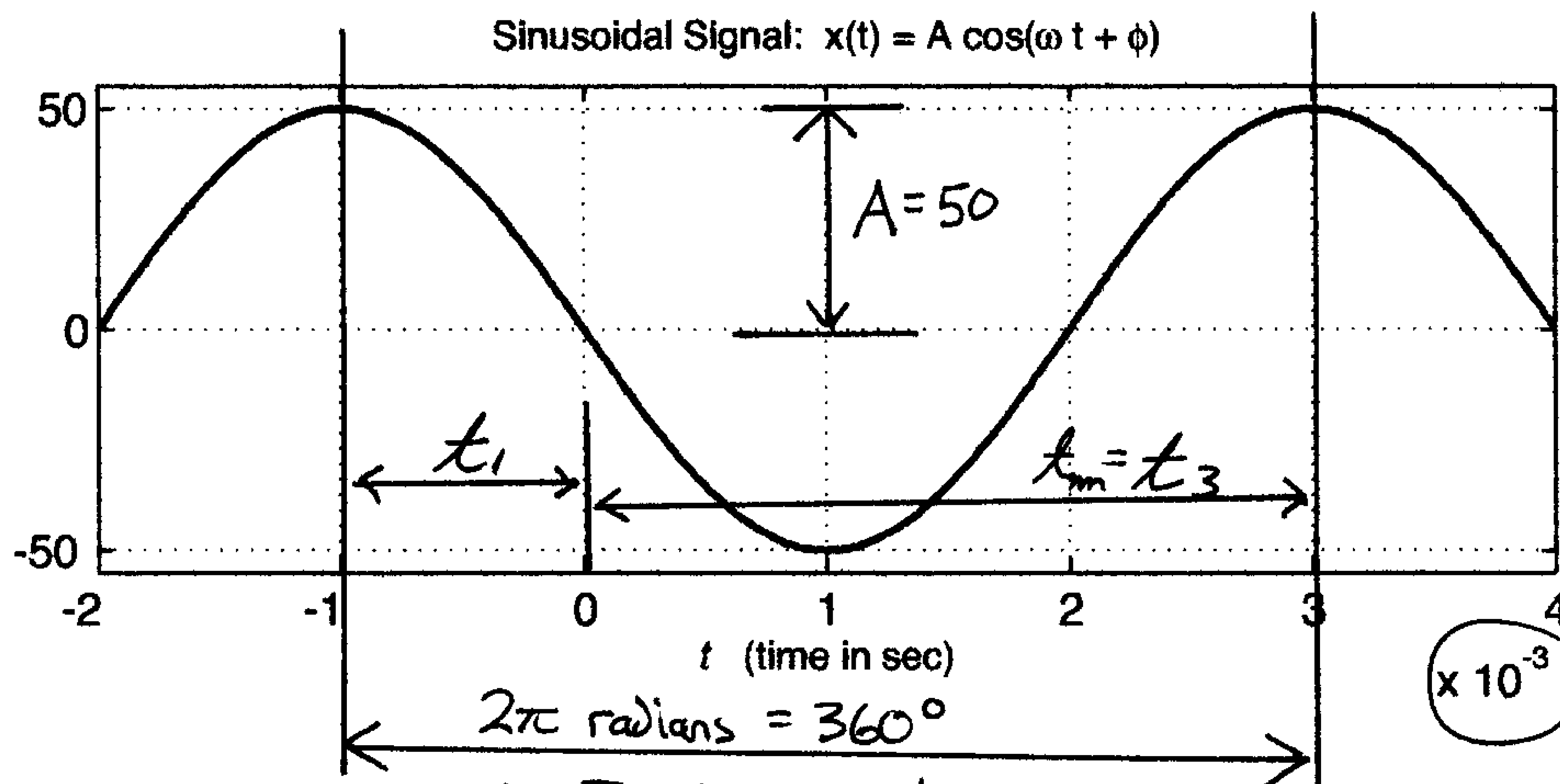
$t_1 = -1$ mSec

$\ominus = -2\pi\left(\frac{t_1=-1}{T_0=4}\right)$

eqn 2.3.6

$\Rightarrow \ominus = \frac{\pi}{2}$ [rad]

$= \ominus(t=0)$



mSec

$3-(-1) = T_0 = 4$ msec $= \frac{1}{f_0} \Rightarrow f_0 = 250$ Hz

$\omega_0 = 2\pi f_0 = 500\pi$ rad/sec

PROBLEM 1.7*:

Suppose that MATLAB is used to plot a sinusoidal signal. The following MATLAB code generates the signal and makes the plot. Draw a sketch of the plot that will be done by MATLAB. Determine the amplitude (A), phase (ϕ), and period of the sinusoid and label the period on your plot.

```
Fo = 12;
Z = -3 - 2i;
dt = 1/(50*Fo);
tt = -0.05 : dt : 0.15;
xx = real( Z*exp( 2j*pi*Fo*tt ) );
%
plot( tt, xx ),   grid
title( 'SECTION of a SINUSOID' ),   xlabel('TIME   (sec)')
```

Complex amplitude (includes phase at $t=0$)

Samples occur: $\frac{1}{50 \cdot 12}$ [sec] intervals or at $50 \cdot 12$ Hz (600 Hz)

x axis data = tt

y axis data = f(x = sample index)

tt

not the same!

Plot + Label Graph

Assume tt is in seconds

$\ominus$ in radians

FREQ

$2\pi f_0 t = 2 pi \cdot F_o \cdot tt$

$\Rightarrow f_0 = 12$ Hz

$\Rightarrow T_0 \cong 83$ mSec

Confirm on graph

$\text{Real}\{ZZ\} = |ZZ|\cos(\angle ZZ)$

Complex Amplitude $= -3 - j2 \cong 3.61\angle -0.813\pi$

A

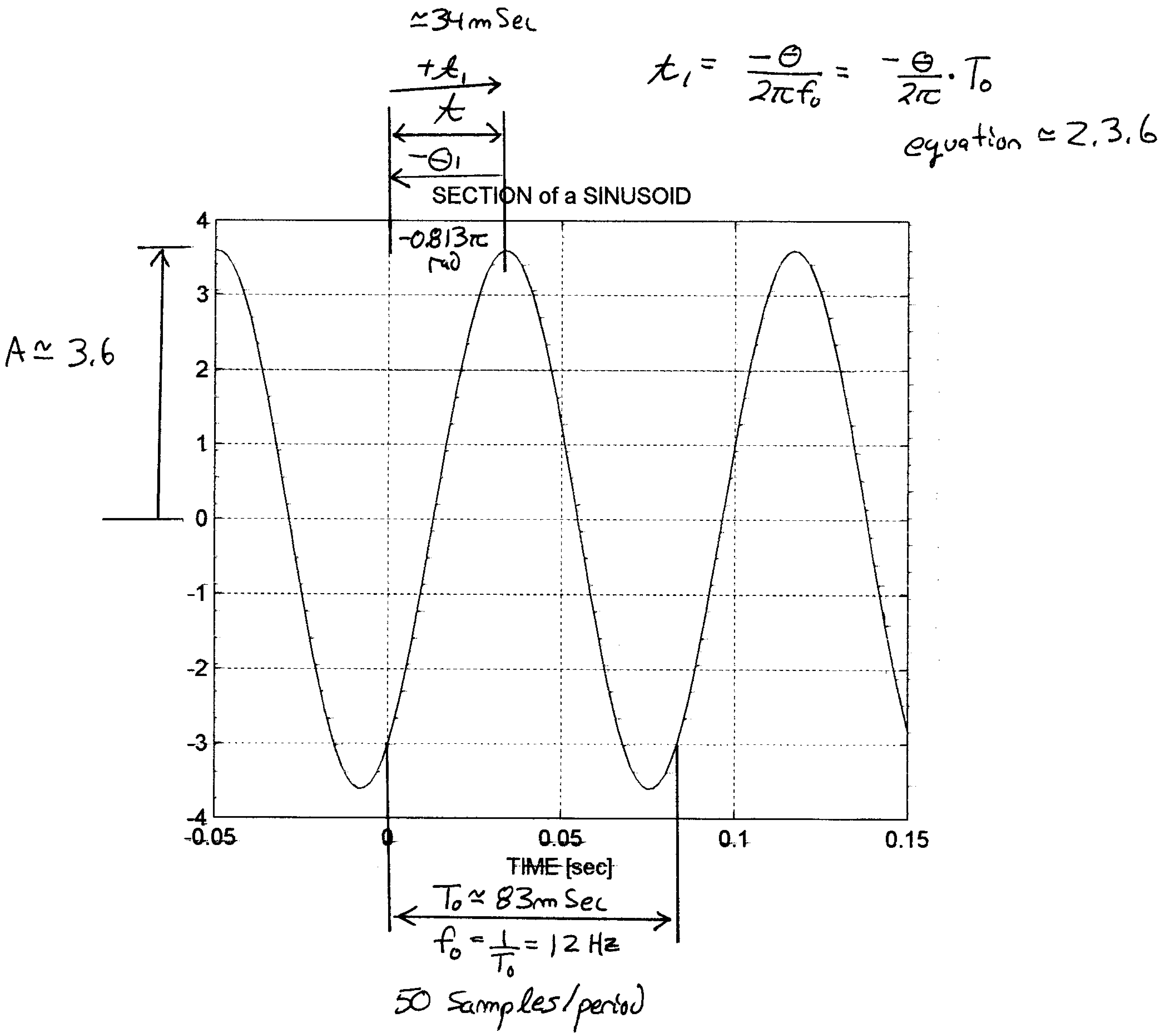
$\ominus(t=0)$

Amplitude

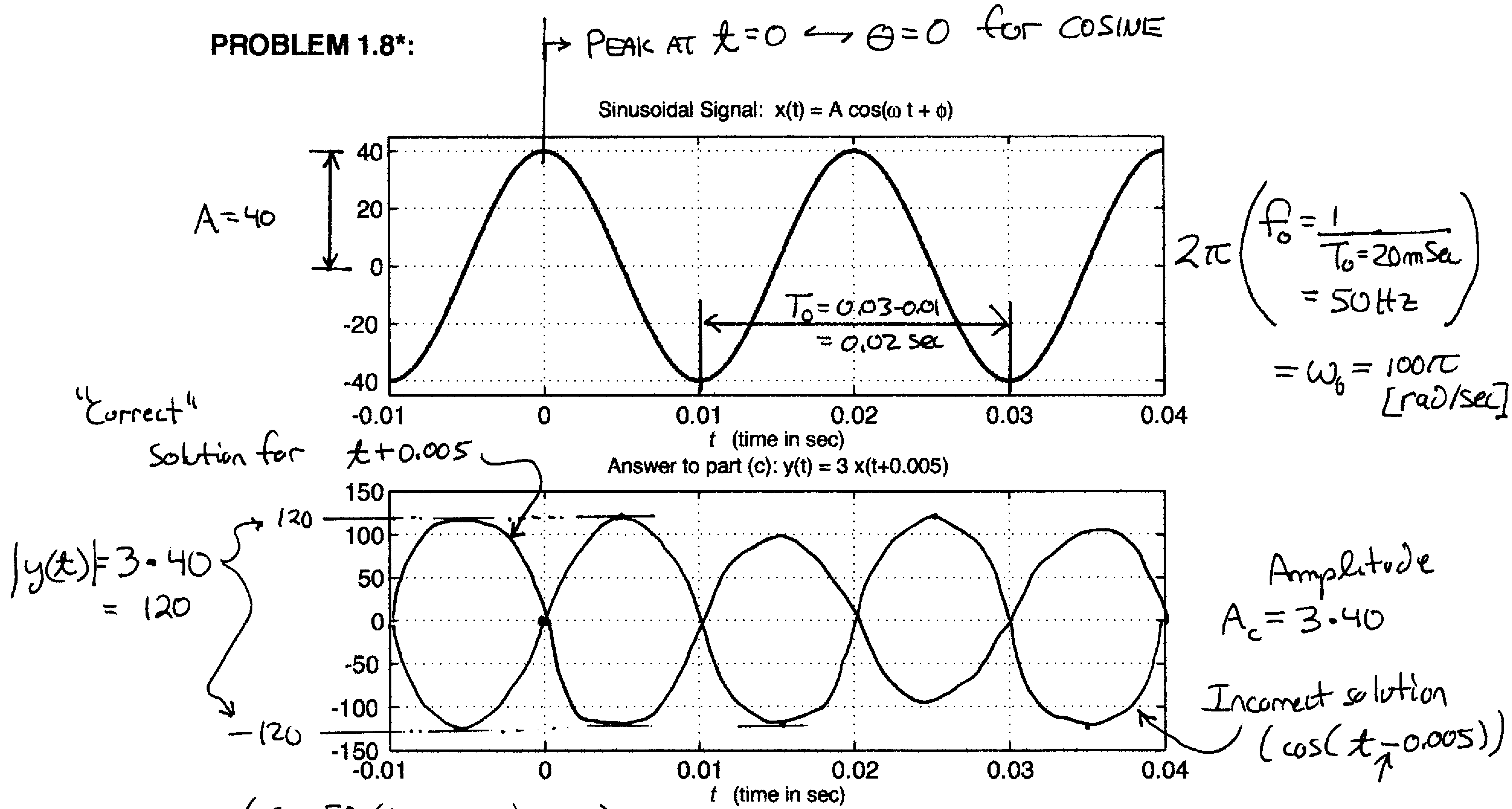
Confirm A and $\ominus$ by viewing plot

ECE2025: Spring 2000: HW Solutions: 1.7

```
» Fo=12;
» Z= -3 - 2j;
» dt = 1/(50*Fo);
» tt = -0.05 : dt : 0.15;
» xx = real( Z*exp( 2j*pi*Fo*tt ) );
»
» plot( tt, xx), grid
» title( 'SECTION of a SINUSOID' ), xlabel('TIME [sec]')
»
```



PROBLEM 1.8*:



$y(t)=3\cdot 40\cos(2\pi 50(t+0.005)-0)$

(a) The above figure shows a plot of a sinusoidal wave $x(t)$. From the plot, determine the values of A, ω_0, and $-\pi < \phi \le \pi$ in the representation

$$x(t) = A\cos(\omega_0 t + \phi)$$

Where appropriate, be sure to indicate the units of the sinusoidal signal parameters.

(b) The signal $x(t)$ in part (a) can be written as the real part of a complex exponential. Determine Z for the complex signal $z(t) = Ze^{j\omega_0 t}$ such that $x(t) = \Re e\{z(t)\}$. BECAUSE $\theta=0$, $Z = 40e^{j0} = 40$

(c) Sketch the signal $y(t) = 3x(t+0.005)$, where $x(t)$ is the signal from part (a). Use the axes provided above or make your own axes covering the same time interval.

NOTE: 0.005 SEC $= \frac{1}{4}(T=20\text{mSec}) \Rightarrow -\frac{1}{4}2\pi = -\frac{\pi}{2} = \theta$

So WHEN $t=0$, $y(t)$ will be at $\cos(\frac{\pi}{2})$ eqn 2.3.6

PROBLEM 1.9*:

Simplify the following and give the answer as a single sinusoid: $x(t) = A\cos(\omega t + \phi)$. Draw the vector diagram of the complex amplitudes (phasors) to show how you obtained the answer.

(a) $x_a(t) = 2\cos(222\pi t) - \sin(222\pi t)$

(b) $x_b(t) = 7\cos(377t + 3\pi/4) + 7\cos(377t + \pi/4)$

NOTE: $\sin(\theta) = \cos(\theta - \frac{\pi}{2})$

Can't remember? Plug in a few points to check...

a) $x_a = 2\cos(222\pi t) - \cos(222\pi t - \frac{\pi}{2})$

$X_a = 2e^{j0} - 1e^{-j\frac{\pi}{2}}$

b) $X_b = 7e^{j\frac{3}{4}\pi} + 7e^{j\frac{\pi}{4}}$ USE MATLAB TO PERFORM ADDITION

NOTE: WE COULD NOT ADD COMPLEX AMPLITUDES IF SINUSOIDS WERE AT DIFFERENT FREQUENCIES.

```
» xa1=2;
» xa2= -1*exp(-j*pi/2);
» xa = xa1 + xa2
xa =
   2.0000 + 1.0000i

» xb1=7*exp(j*3/4*pi);
» xb2=7*exp(j*1/4*pi);
» xb = xb1 + xb2;
» xb
xb =
   0.0000 + 9.8995i

» zprint([xa, xb])
 Z =          X     +       jY      Magnitude   Phase     Ph/pi    Ph(deg)
              2              1        2.236     0.464     0.148     26.57
   8.882e-016            9.899        9.899     1.571     0.500     90.00

» subplot(1,2,1), zvect([xa1, xa2, xa])
» subplot(1,2,2), zvect([xb1, xb2, xb])
```

COMPLEX AMPLITUDES FOR x_a, x_b

$x_a(t)=2.236\cos(377t+0.148\pi)$

$x_b(t)=9.899\cos(377t+0.5\pi)$