GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 1999
Problem Set #9

Assigned: 22 October 99
Due Date: 29 October 99 (FRIDAY)

Quiz #2 will be given on Monday, October 25-th in your regular class time.

Review Sessions are planned for Saturday 11-1pm and Sunday at 7 pm in the ECE Auditorium.

Reading: In *DSP First*, Chapters 7 and 8 on *Z-Transforms* and *IIR Filters*.

$\Longrightarrow$ The two (2) **STARRED** problems will have to be turned in for grading. The other problems are from EE2200, Spring 1999, Problem Set #7. They are excellent exercises on the current material. You will see more of these type of problems on Quiz #3.

Next week a solution will be posted. Some similar problems solutions can be found on the CD-ROM and in old homeworks, especially the "unstarred" problems.

PROBLEM 9.1*:

A linear time-invariant system has system function

$$H(z) = \frac{(1 - z^{-1})}{(1 + 0.5z^{-1})} = \frac{1}{(1 + 0.5z^{-1})} - \frac{z^{-1}}{(1 + 0.5z^{-1})}.$$

(a) Determine the difference equation that is satisfied by the output $y[n]$ and the input $x[n]$ of this system.

(b) Plot the poles and zeros of $H(z)$ in the complex z-plane.

(c) Determine the frequency response $H(e^{j\hat{\omega}})$. Use `freqz( )` in MATLAB to evaluate the frequency response of this system for $-\pi \le \hat{\omega} \le \pi$. Plot $|H(e^{j\hat{\omega}})|$ and $\angle H(e^{j\hat{\omega}})$ in a two part plot and hand it in with your solution to this problem.

(d) Determine $h[n]$, the impulse response of this system. (We did this in lecture 15 on Friday, October 22). Plot the values of this sequence for $-3 \le n \le 10$.

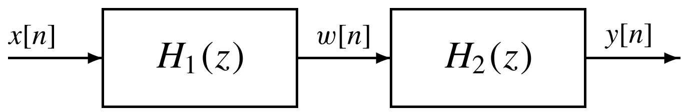
(e) The input to this system is

$$x[n] = 3 + 7\delta[n] + 10\cos(\pi n/2).$$

Use superposition to determine the output of the system $y[n]$ corresponding to the above input $x[n]$. Give an equation for $y[n]$ that is valid for all n. (*Note: This is an easy problem if you approach it correctly!*)

PROBLEM 9.2*:

An *inverse filter* is an LTI system which, when cascaded with another LTI system, "undoes" the effects of the other LTI system. You saw an approximate inverse filter in one of the Laboratory experiments. The reason that you were only able to do an approximation then was that we had not yet studied IIR systems.



(a) Suppose that $H_1(z)$ describes a given LTI system. Determine $H_2(z)$ so that the output is $y[n] = x[n]$; i.e., so that the second system compensates exactly for the effects of the first system.

(b) If $H_1(z)$ represents an FIR system, what can you say about the second system; i.e., is it FIR or IIR? Explain.

(c) Suppose that the first system is defined by the difference equation

$$w[n] = \sum_{k=0}^{9} \alpha^k x[n-k] \qquad \text{where } 0 < \alpha < 1.$$

Show that $H_1(z)$ can be expressed the following ratio of polynomials in the variable z^{-1}:

$$H_1(z) = \frac{1 - \alpha^{10} z^{-10}}{1 - \alpha z^{-1}}.$$

Plot the poles and zeros of $H_1(z)$ in the complex z-plane. *Hint: You will need to find the values of z that satisfy the equation* $1 - \alpha^{10} z^{-10} = 0$. *This is done just as in the case of the moving average filter. You should find that the zeros are not on the unit circle, but on a circle of a different radius.*

(d) Now, for the system $H_1(z)$ of part (c), determine the inverse system function $H_2(z)$ and plot its poles and zeros in the complex z-plane. What happens to the poles and zeros of $H_1(z)$ and $H_2(z)$ when we form the product $H_1(z)H_2(z)$?

(e) From $H_2(z)$ obtained in part (d), determine the difference equation that relates the output $y[n]$ to $w[n]$, the input to the second system.

(f) The system of part (e) could be implemented in Matlab by the statement

```
yy=filter(b,a,ww);
```

What should `b` and `a` be in order to implement the inverse system for the example of part (c)?

PROBLEM 9.3:

For each of the difference equations below, determine the poles and zeros of the corresponding system function, $H(z)$. [1]

$$S_1: \quad y[n] = 0.4y[n-1] + x[n] + x[n-1]$$

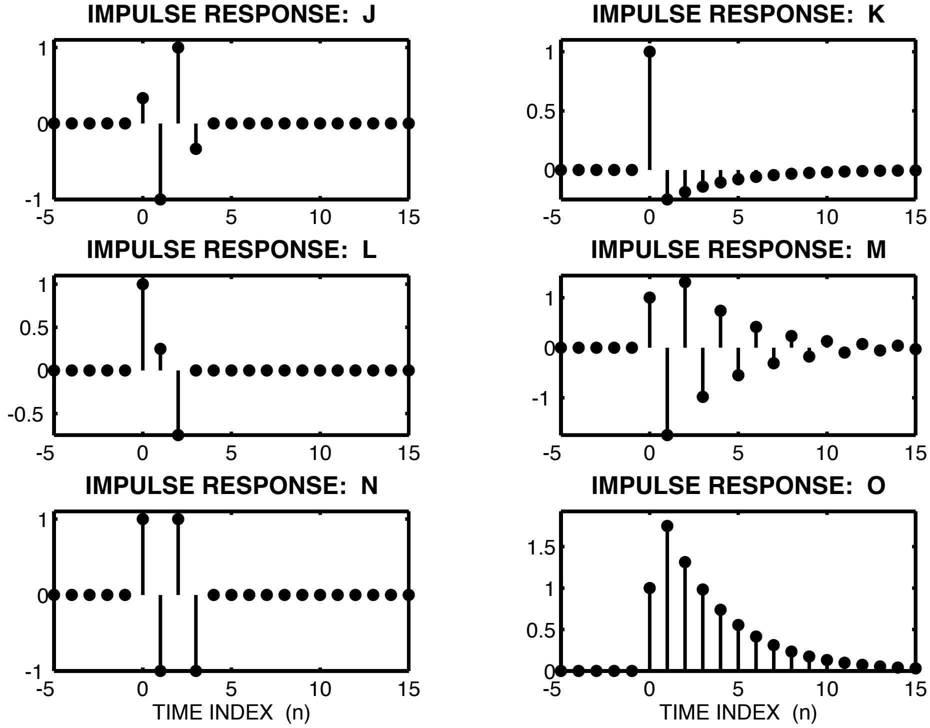
$$S_3: \quad y[n] = -0.75y[n-1] + x[n] - x[n-1]$$

$$S_6: \quad y[n] = x[n] - x[n-1] + x[n-2] - x[n-3]$$

$$S_7: \quad y[n] = x[n] + \tfrac{1}{4}x[n-1] - \tfrac{3}{4}x[n-2]$$

[1] These systems are a subset of those in the following two problems.

PROBLEM 9.4:



For each of the impulse-response plots (J, K, L, M, N, O), determine which one of the following systems[2] (specified by either an $H(z)$ or a difference equation) matches the impulse response. In addition, derive a formula for the impulse response, $h[n]$, for S_1 and S_4.

S_1 : $\quad y[n] = 0.4y[n-1] + x[n] + x[n-1]$

S_2 : $\quad H(z) = \dfrac{1+z^{-1}}{1-0.75z^{-1}}$

S_3 : $\quad y[n] = -0.75y[n-1] + x[n] - x[n-1]$

S_4 : $\quad H(z) = \dfrac{1-z^{-1}}{1-0.75z^{-1}}$

S_5 : $\quad y[n] = x[n] - x[n-1] + x[n-2]$

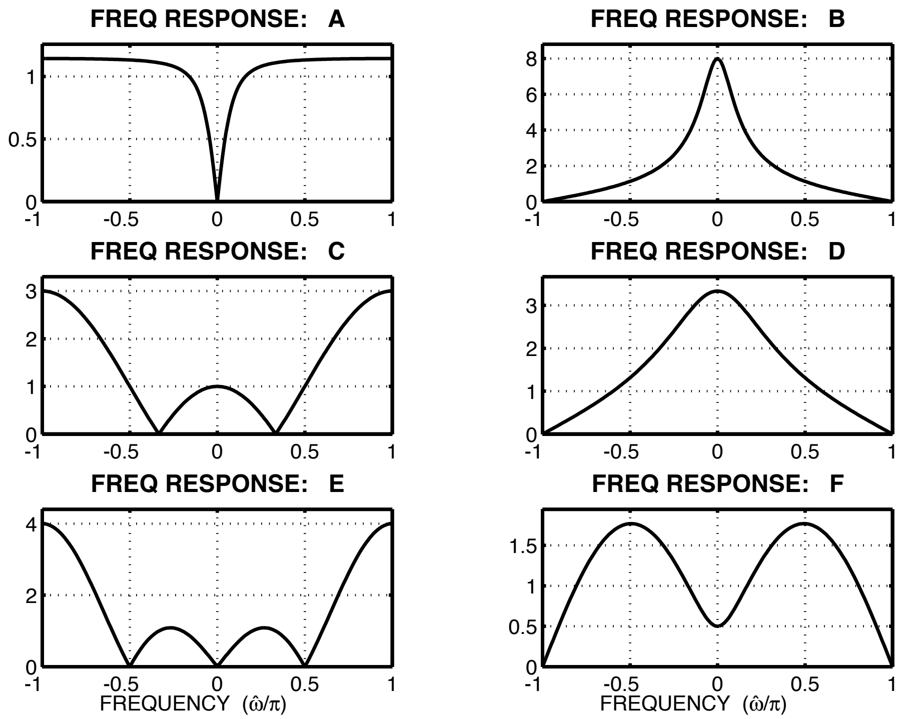
S_6 : $\quad H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$

S_7 : $\quad y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{4}x[n-2]$

S_8 : $\quad H(z) = \frac{1}{3}(1-z^{-1})^3$

[2]These 8 systems are exactly the same as the other matching problems.

PROBLEM 9.5:



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems[3] (specified by either an $H(z)$ or a difference equation) matches the frequency response (magnitude only). NOTE: frequency axis is **normalized**; it is $\hat{\omega}/\pi$. In addition, derive a formula for the magnitude-squared of the frequency response, $|H(e^{j\hat{\omega}})|^2$, for S_3 and S_4.

S_1 : $y[n] = 0.4y[n-1] + x[n] + x[n-1]$

S_2 : $H(z) = \dfrac{1+z^{-1}}{1-0.75z^{-1}}$

S_3 : $y[n] = -0.75y[n-1] + x[n] - x[n-1]$

S_4 : $H(z) = \dfrac{1-z^{-1}}{1-0.75z^{-1}}$

S_5 : $y[n] = x[n] - x[n-1] + x[n-2]$

S_6 : $H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$

S_7 : $y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{4}x[n-2]$

S_8 : $H(z) = \frac{1}{3}(1-z^{-1})^3$

[3] These 8 systems are exactly the same as the other matching problems.