GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 1999
Problem Set #2

Assigned: 3 September 1999
Due Date: 10 September 1999 (FRIDAY)

Quiz #1 will be held in lecture on Monday 20-September-99. It will cover material from Chapters 2 and 3, as represented in Problem Sets #1, #2 and #3.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$, both sides)

Reading: In *DSP First*, all of Chapter 3 on *Spectrum Representation*, especially pp. 48–73.

The web site: `http://classweb.gatech.edu:8080/SCRIPT/ECE2025/scripts/serve_home`
You should change your password; look under `COURSE TOOLS`. Please check the "Bulletin Board" often.

$\Longrightarrow$ **Look for another on-line HW this week.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

After the beginning of your assigned lecture on Friday (either 11am or 12pm), the homework is considered late and will be given a zero.

PROBLEM 2.1:

Simplify the following and give the answer as a single sinusoid. Draw the vector diagram of the complex amplitudes (phasors) to show how you obtained the answer.

(a) $x_a(t) = \sqrt{2}\cos(2\pi t + 3\pi/4) - \cos(2\pi t + \pi/4)$

(b) $x_b(t) = \cos(11t + 17\pi) + \sqrt{3}\cos(11t + \pi/3) + \sqrt{3}\cos(11t - \pi/3)$

(c) $x_c(t) = \cos(\pi t + 3\pi/4) + \cos(\pi t + 5\pi/4) + \cos(\pi t - \pi/4) + 2\cos(\pi t + \pi/4)$

PROBLEM 2.2*:

Define $x(t)$ as

$$x(t) = 20\cos(200\pi t + \pi/2) + A\cos(200\pi t + \phi) \quad (1)$$

How should A and ϕ be chosen so that

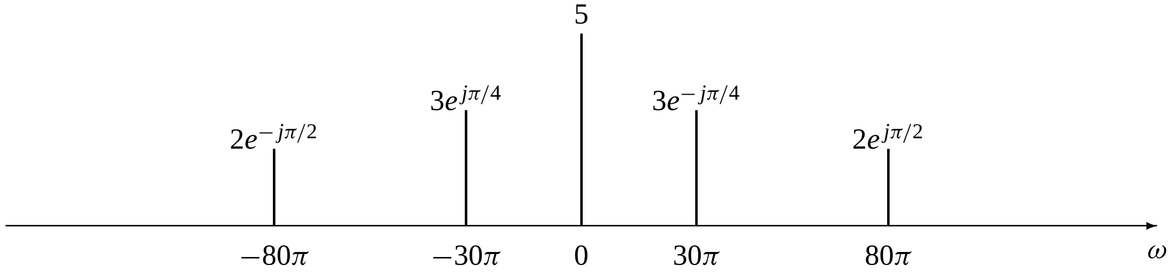
$$x(t) = B\cos(200\pi t), \quad (2)$$

where B is a positive real number? What is the value of B for your choice of A and ϕ?

Hint: There are many correct answers to this problem. To solve this problem try a graphical approach. To get a numerical answer, you will have to fix one of the unknowns A or ϕ and solve for the other.

PROBLEM 2.3*:

A real signal $x(t)$ has the following two-sided spectrum:



(a) Write an equation for $x(t)$ as a sum of cosines.

(b) Explain why the "negative" frequencies are necessary to obtain a real signal.

(c) Plot the spectrum of the signal $y(t) = 2x(t) - 4\cos(50\pi(t - 0.004))$.

PROBLEM 2.4*:

The two-sided spectrum of a signal $x(t)$ is given in the following table:

frequency (ω)	complex phasor
-150π	X_{-2}
-90π	$3e^{j\pi/4}$
0	5
ω_1	X_1
150π	$1 + \sqrt{3}j$

(a) If $x(t)$ is a real signal, what are X_1, X_{-2}, and ω_1?

(b) Write an expression for $x(t)$ involving only real numbers and cosine functions.

PROBLEM 2.5*:

In AM radio, the transmitted signal is voice (or music) mixed with a *carrier signal*. The carrier is a sinusoid at the assigned broadcast frequency of the AM station. For example, WSB in Atlanta has a *carrier frequency* of 750 kHz. If we use the notation $v(t)$ to denote the voice/music signal, then the actual transmitted signal for WSB might be:

$$x(t) = (v(t) + A)\cos(2\pi(750 \times 10^3)t)$$

where A is a constant. (A is introduced to make the AM receiver design easier, in which case A must be chosen to be larger than the maximum value of $v(t)$.)

(a) Voice-band signals tend to contain frequencies less than 4000 Hz (4 kHz). Suppose that $v(t)$ is a 1 kHz sinusoid, $v(t) = \cos(2\pi(1000)t)$. Draw the spectrum for $v(t)$.

(b) Now draw the spectrum for $x(t)$, assuming a carrier at 750 kHz. Use $v(t)$ from part (a) and assume that $A = 2$. *Hint: Substitute for $v(t)$ and expand $x(t)$ into a sum of cosine terms of three different frequencies.*

(c) How would the spectrum of the AM radio signal change if the carrier frequency is changed to 680 kHz (WCNN) and $v(t)$ and A are the same as defined in parts (a) and (b).