ECE 2025 Solutions for Homework Set 1

Prof. Paul Hasler

1 Problem 1.4

Simplify the following complex-valued expressions. Give your answer in either rectangular or polar form, whichever is more convenient. In parts (a)-(d) assume that A, a, and ϕ are positive real numbers. your ansers to parts (a) - (d) will be in terms of these quantitites.

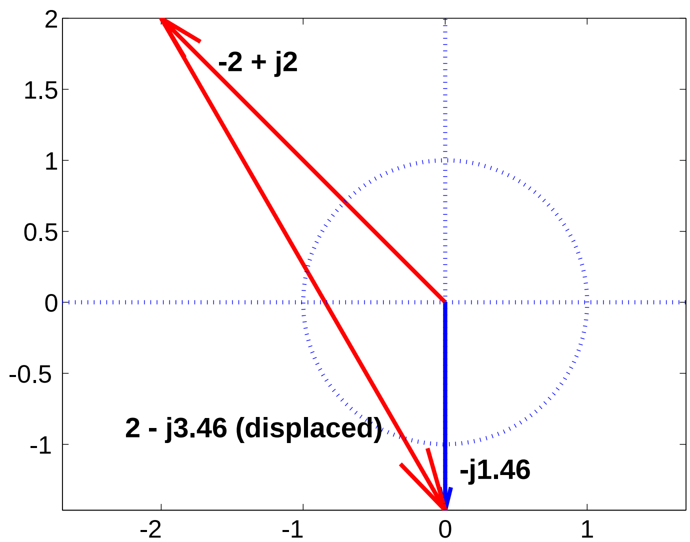

Figure 1: Plot of the two vectors $-2+2j$, and $2-j2\sqrt{3}$ and their sum.

(a) For $z = Ae^{j\pi/3}$, determine an expression for Imaginary$\{z^*\}$.

$z^* = Ae^{-j\pi/3}$

The imaginary part of z is $A\sin(-\pi/3) = -A\frac{\sqrt{3}}{2}$.

(b) For $z = Ae^{-j\pi/3}$, determine an expression for $|z|/z$.

$z/|z| = e^{-j\pi/3}$, therefore, $|z|/z = e^{j\pi/3}$.

(c) For $z = 10e^{j\phi}$, determine an expression for $\Re e\{jz\}$.

$\Re e\{jz\} = \Re e\{j(10\cos\phi + j10\sin\phi)\} = \Re e\{j10\cos\phi + j^2 10\sin\phi\} = -10\sin\phi$.

(d) For $z = -a - ja$, determine an expression for z in polar form.

$z = a\sqrt{2}e^{j(5\pi/4)}$

(e) For $z_1 = -2+2j$, and $z_2 = 4e^{-j(\pi/3)}$, evaluate $z_3 = z_1+z_2$, and plot all three complex numbers in complex plane.

$z_2 = 4(\cos(-\pi/3) + j\sin(-\pi/3)) = 2 - j2\sqrt{3}$.

$z_3 = (-2+2j) + (2 - j2\sqrt{3}) = 2j - j2\sqrt{3} = -j1.4641$.

Figure 1 shows the addition of these two vectors in the complex plane.

2 Problem 1.5

Simplify the following and give the answer in polar form. Make a plot of the vectors involved in the complex addition.

(a) $z_a = 2e^{j(2\pi/3)} + e^{j(5\pi/4)}$

$$
\begin{aligned}
2e^{j(2\pi/3)} + e^{j(5\pi/4)} &= \\
2e^{j(2\pi/3)} - e^{j(\pi/4)} &= \\
2(-\frac{1}{2} + j(\frac{\sqrt{3}}{2}) - (\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}) &= \\
-1 + j\sqrt{3} - (\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}) &= \\
-1.707 + 1.025j & \\
1.9912e^{j(0.8279\pi)} &
\end{aligned}
$$

Figure 2 shows the graphical solution to this problem.

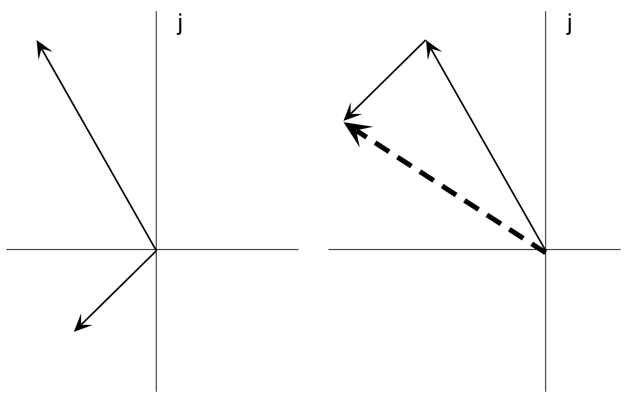

Figure 2: A graphical solution of the phasor additions: $2e^{j(2\pi/3)} + e^{j(5\pi/4)}$. The solution is the dashed vector.

(b) $z_b = \sqrt{2}e^{j\pi/4} + \sqrt{2}e^{-j\pi/4} - 1$

$$
\begin{aligned}
\sqrt{2}e^{j\pi/4} + \sqrt{2}e^{-j\pi/4} - 1 &= \\
\sqrt{2}\left(e^{j\pi/4} + e^{-j\pi/4}\right) - 1 &= \\
\sqrt{2}\left(2\cos(\pi/4)\right) - 1 &= \\
\sqrt{2}\left(\sqrt{2}\right) - 1 &= \\
2 - 1 = 1 &
\end{aligned}
$$

Figure 3 graphically shows this solution.

(c) In addition, write the MATLAB statements that will perform the addition and display the magnitude and phase of the result. Use these to check your hand calculations in parts (a) and (b).

For part (a), we used the following code:

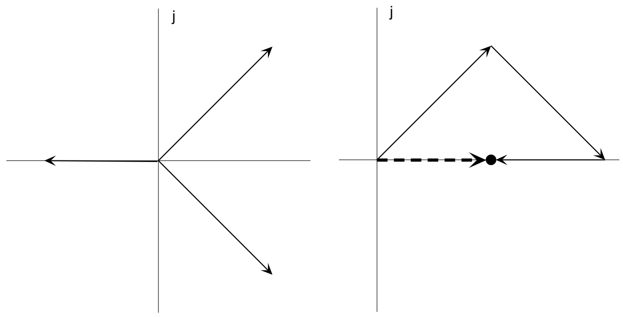

Figure 3: A graphical solution of the phasor addition: $\sqrt{2}e^{j\pi/4} + \sqrt{2}e^{-j\pi/4} - 1$

```
a = 2*exp(j*(2*pi/3));
b = exp(j*(5*pi/4));
z = a + b;
magz = abs(z)
angz = angle(z)/pi
```

and obtained the following answers:

```
magz = 1.0660
angz = 0.4114
```

For part (b), we used the following code:

```
a = sqrt(2)*exp(j*(pi/4));
b = sqrt(2)*exp(j*(-pi/4));
z = a + b -1;
magz = abs(z)
angz = angle(z)/pi
```

and obtained the following answers:

```
magz = 1.0000
  angz = 0
```

3 Problem 1.6

Suppose that MATLAB is used to plot a sinusoidal signal. The following MATLAB code generates the signal and makes the plot. Draw a sketch of the plot that will be done by MATLAB. Determine the period of the sinusoid and label the period on your plot.

```
dt = 0.001;
tt = -0.05:dt:0.15;
Fo = 10;
Z = sqrt(2) * (1 + j);
xx = real( Z*exp(2j*pi*Fo*tt ) );
plot(tt,xx);
grid;

title('SECTION of a SINUSOID');
xlabel('Time (sec)');
```

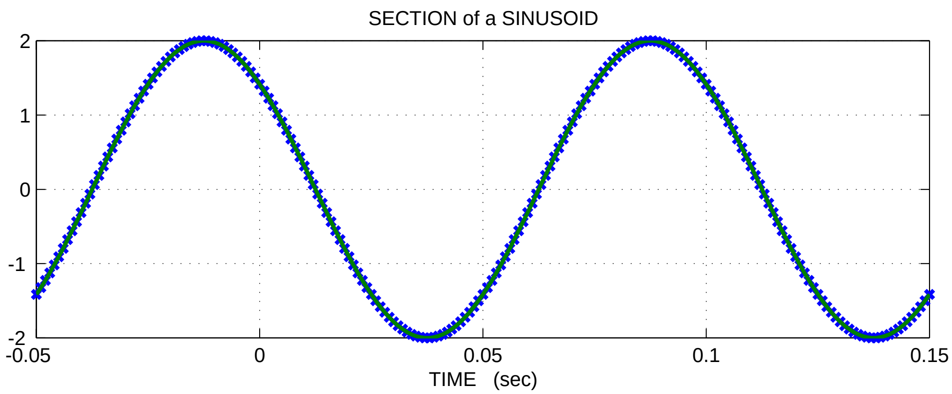

Figure 4: A plot of yet another sinusoidal signal.

Figure 4 is generated by the commands:

```
subplot(2,1,1)
plot(tt,xx,'x',tt,2*cos(20*pi*tt + (pi/4)));grid;
title('SECTION of a SINUSOID');
xlabel('Time (sec)');
```

This plot shows that the amplitude is 2, the frequency is 10 Hz, and the phase is $\phi = +\pi/4$. Note that the time of a positive max is $t_m = -0.0125$ s, so phase is computed via:

$$\phi = -\omega t_m = -(2\pi)(10)(-0.0125) = +0.25\pi$$

4 Problem 1.7

The signal goes between -20 and 20; therefore the amplitude (A) is 20. We see successive peaks at -0.5s and 2.0s; therefore the frequency is

$$f = 1/T = 1/(2.5 \times 10^{-3}\text{s}) = 400 \text{ Hz}$$

or $\omega = 2\pi(400) = 800\pi = 2513.27$ rad/sec. Since the first peak can be found at $t_m = -0.5$ millisec. instead of $t = 0$, the phase shift is

$$\phi = -\omega t_m = -(800\pi)(-0.5 \times 10^{-3}) = 0.4\pi = 2\pi/5$$

We plot the resulting function in Fig. 5.

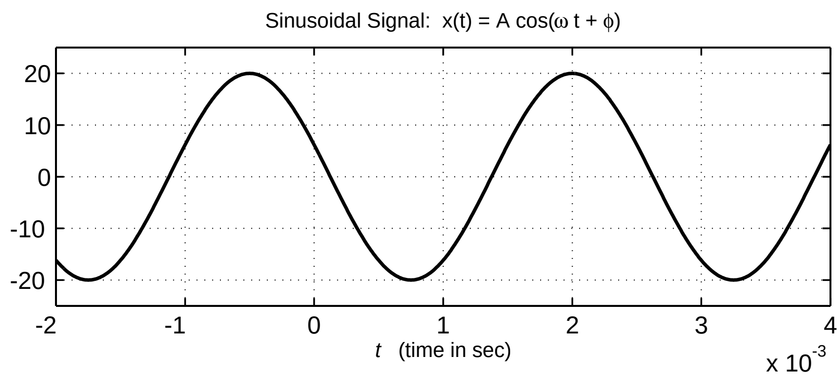

Figure 5: A sinusoidal signal with an amplitude of 20, a frequency of 0.4Hz, and a phase shift of $2\pi/5$.

5 Problem 1.8

(a) The above figure shows a plot of a sinusoidal wave x(t). From the plot, determine the values of A, ω_o, and $-\pi < \phi < \pi$

The period is $T = 0.02$ secs; and the time of the nearest positive peak nearest to $t = 0$ is $t_m = 0.005$s. Therefore, the amplitude is 35, frequency is 50Hz, and $\phi = -2\pi f t_m = -2\pi(50)($ is $-\pi/2$. Note that the plot is a "sine wave" so we expect the phase to be -90 degrees.

(b) Determine Z for the complex signal $z(t) = Ze^{j\omega_o t}$ such that $x(t) = Re(z(t))$.

$$Z = 35e^{-j\pi/2}$$

(c) On the axes provided above, sketch the signal $y(t) = 2x(t - 0.005)$.

Figure 6 shows the plots of sinusoidal signals for this problem.

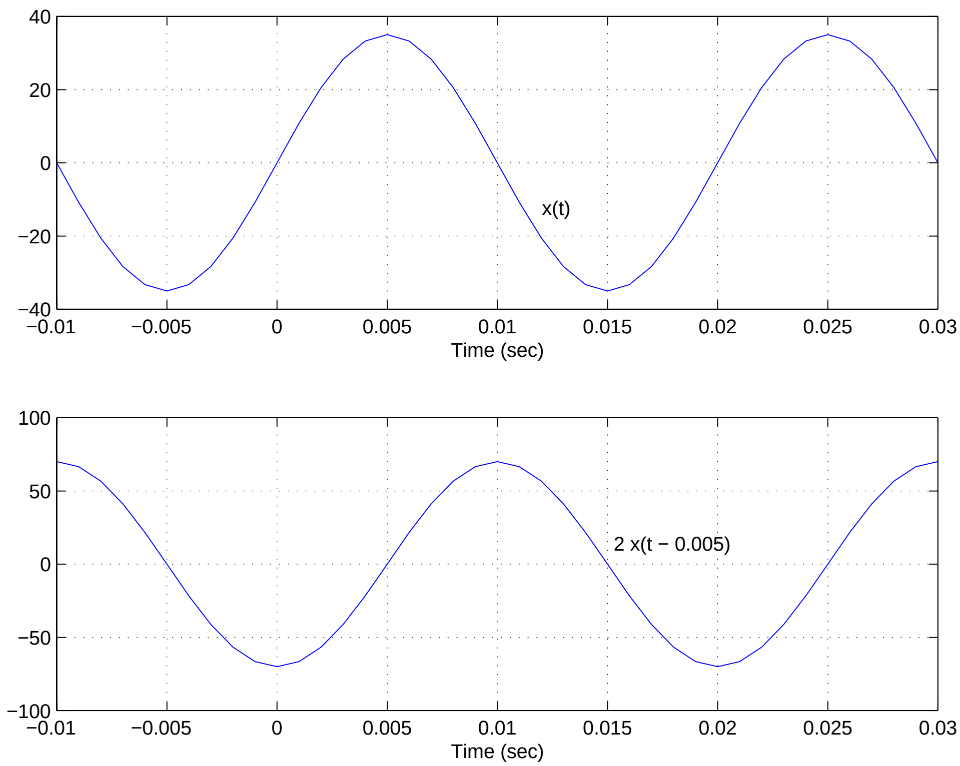

Figure 6: Plots of sinusoidal signal for this problem.

6 Problem 1.9

Simplify the following and give the anser as a single sinusoid. Draw the vector diagram fo the complex amplitudes (phasors) to show how you obtained the answer.

(a) $x_a(t) = 2\cos(2\pi t + 2\pi/3) - \cos(2\pi t + \pi/4)$

We have the addition of two phasors:

$$2e^{j(2\pi/3)} - e^{j(\pi/4)} =$$
$$2(\frac{-1}{2} + j(\frac{\sqrt{3}}{2}) - (\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}) =$$
$$-1 + j\sqrt{3} - (\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}) =$$
$$-1.707 + 1.025j =$$
$$1.9912e^{j(0.8279\pi)}$$

Figure 2 shows the graphical solution to this problem.

The resulting sinusoid is (a) $x_a(t) = 1.9912\cos(2\pi t + 0.8279\pi)$.

(b) $x_b(t) = \cos(41t + 17\pi) + \sqrt{2}\cos(41t + \pi/4) + \sqrt{2}\cos(41t - \pi/4)$

We have the addition of three phasors:

$$e^{j(17\pi)} + \sqrt{2}e^{j(\pi/4)} + \sqrt{2}e^{j(-\pi/4)} =$$
$$-1 + \sqrt{2}(2\cos(\pi/4)) =$$
$$-1 + \sqrt{2}(\sqrt{2}) = 1$$

The graphical solution is shown in Fig. 3.

The resulting sinusoid is $x_b(t) = \cos(41t)$.

(c) $x_c(t) = \cos(200\pi t + 3\pi/4) + \cos(200\pi t + 5\pi/4) + 2\cos(200\pi t - \pi/4) + 2\cos(200\pi t + \pi/4)$

We have the addition of four phasors:

$$\begin{aligned}
e^{j(3\pi/4)} + e^{j(5\pi/4)} + 2e^{j(-\pi/4)} + 2e^{j(\pi/4)} &= \\
e^{j(\pi)}\left(e^{j(-\pi/4)} + e^{j(\pi/4)}\right) + 2e^{j(-\pi/4)} + 2e^{j(\pi/4)} &= \\
-\left(e^{j(-\pi/4)} + e^{j(\pi/4)}\right) + 2e^{j(-\pi/4)} + 2e^{j(\pi/4)} &= \\
(2-1)\left(e^{j(-\pi/4)} + e^{j(\pi/4)}\right) &= \\
\left(e^{j(-\pi/4)} + e^{j(\pi/4)}\right) &= 2\cos(\pi/4) = 2\sqrt{2}/2
\end{aligned}$$

Figure 7 shows the graphical solution to the sum of these four phasors.

Therefore, we get $x_c(t) = 1.414\cos(200\pi t)$

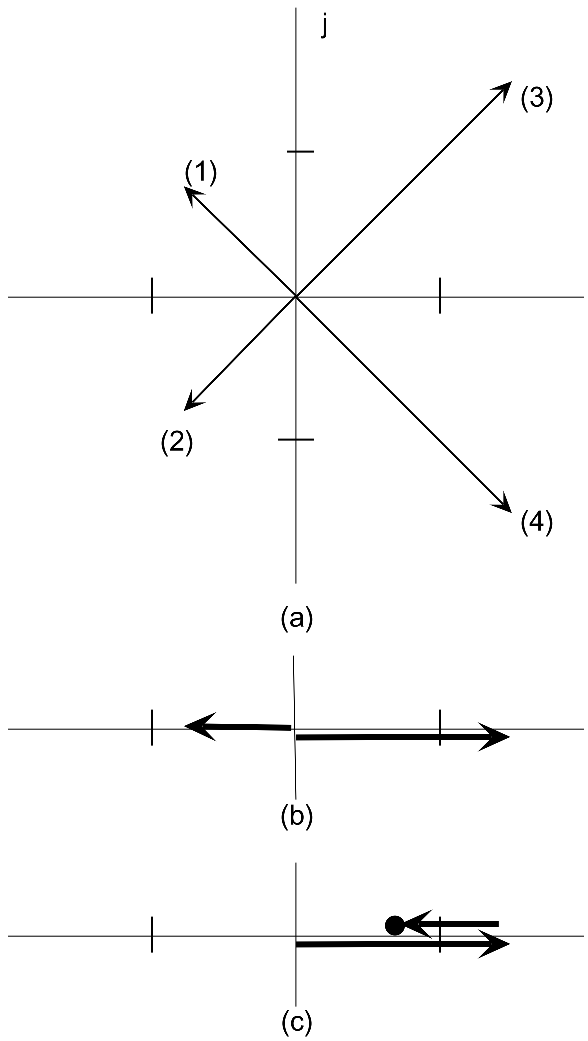

Figure 7: Addition of four phasors. (a) Initial plot corresponding to the phasor addition of $e^{j(3\pi/4)} + e^{j(5\pi/4)} + 2e^{j(-\pi/4)} + 2e^{j(\pi/4)}$. (b) After combining phasors (1) and (2), and combining phasors (3) and (4). (c) Final phasor addition.