

EE-2025

Fall-99

Lecture 8

FIR Filtering Intro

24-Sept-99

Information

- Music Listening next week
 - | Survey next week !!!!!!!
- Problem Set #4 due TODAY
 - | On-Line HW returns next week
- MATLAB help: Wed @ 6pm, VL-456

- Quiz #2 on 25-Oct (Monday)

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READING ASSIGNMENTS

- This Lecture:
 - | Chapter 5, pp. 119–131

- Other Reading:
 - | Recitation: Ch. 5, pp. 127–133, 142–146
 - | **CONVOLUTION**
 - | Next Lecture: Chapter 5, pp. 133–152

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LECTURE OBJECTIVES

- INTRODUCE FILTERING IDEA
 - | **Weighted** Average
 - | **Running** Average

- FINITE IMPULSE RESPONSE FILTERS
 - | **FIR** Filters
 - | Show how to compute the output $y[n]$ from the input signal, $x[n]$

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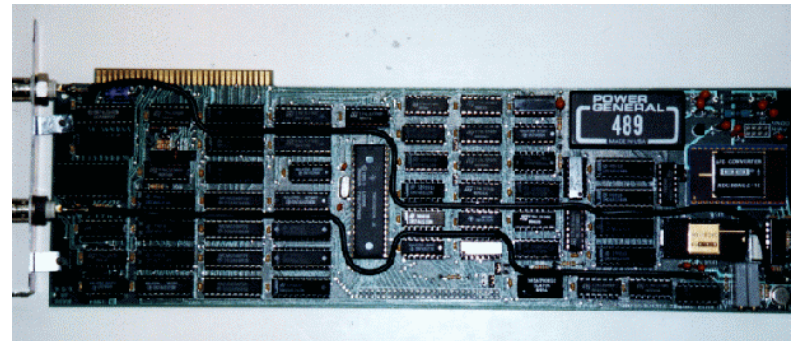
DIGITAL FILTERING



CONCENTRATE on the COMPUTER

- I PROCESSING ALGORITHMS
- I SOFTWARE (MATLAB)
- I HARDWARE: DSP chips, VLSI
- I **DSP: DIGITAL SIGNAL PROCESSING**

The TMS32010, 1983



First PC plug-in board from Atlanta Signal Processors Inc.

Rockland Digital Filter, 1971

Model 4136 PROGRAMMABLE DIGITAL FILTER

Variable-Order Digital Filter for Realizing All Classical Designs

The Rockland Model 4136 Programmable Digital Filter consists of a second-order digital filter section which is multiplexed four ways to achieve eighth-order filtering. Each of the four sections has fully-programmable coefficients which are stored internally in a read-write memory.

Filter input and output words are in 16-bit parallel form at a maximum sampling rate of 80 KHz while internal computations are made with 24-bit accuracy.

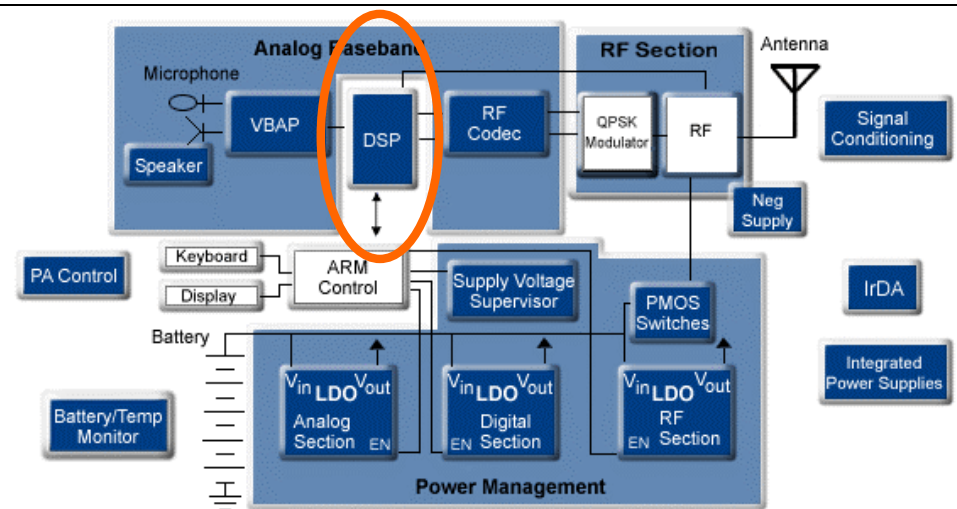
TRANSFER FUNCTION
The transfer function from filter input to filter output in z-transform notation is given by

$$H_n(z) = \prod_{n=1}^N \frac{K_n(1+z^{-1}A1+z^{-2}A2)}{1-z^{-1}B1-z^{-2}B2} \quad (1)$$

where N=0,1,2,3,4 is one-half the filter order selected.

For the price of a small house, you could have one of these.

Digital Cell Phone



Free (?) with 2 year contract

DISCRETE-TIME SYSTEM



- OPERATE on $x[n]$ to get $y[n]$
- WANT a **GENERAL CLASS** of SYSTEMS
 - ANALYZE the SYSTEM
 - TOOLS: TIME-DOMAIN & FREQUENCY-DOMAIN
 - SYNTHESIZE the SYSTEM

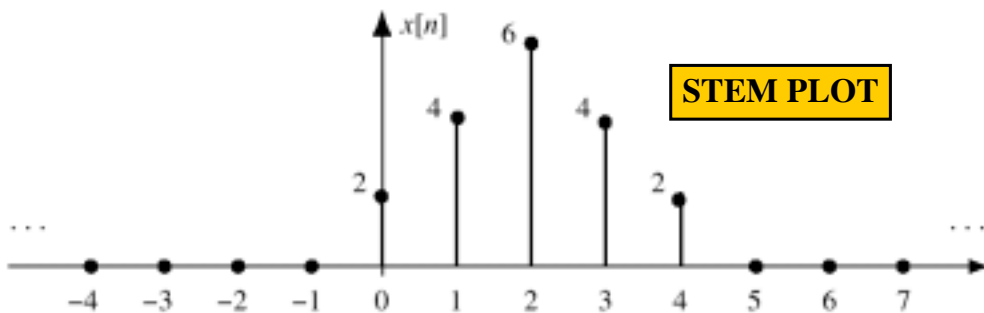
D-T SYSTEM EXAMPLES



- EXAMPLES:
 - POINTWISE OPERATORS
 - SQUARING: $y[n] = (x[n])^2$
 - RUNNING AVERAGE
 - RULE: “the output at time n is the average of three consecutive input values”

DISCRETE-TIME SIGNAL

- $x[n]$ is a LIST of NUMBERS
 - INDEXED by “ n ”



3-PT AVERAGE SYSTEM

- ADD 3 CONSECUTIVE NUMBERS
 - Do this for each “ n ”

the following input–output equation

Make a TABLE

$$y[n] = \frac{1}{3}(x[n] + x[n + 1] + x[n + 2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	$n > 5$
$x[n]$	0	0	0	2	4	6	4	2	0	0
$y[n]$	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	4	2	$\frac{2}{3}$	0	0

$n=0$ $y[0] = \frac{1}{3}(x[0] + x[1] + x[2])$

$n=1$ $y[1] = \frac{1}{3}(x[1] + x[2] + x[3])$

INPUT SIGNAL

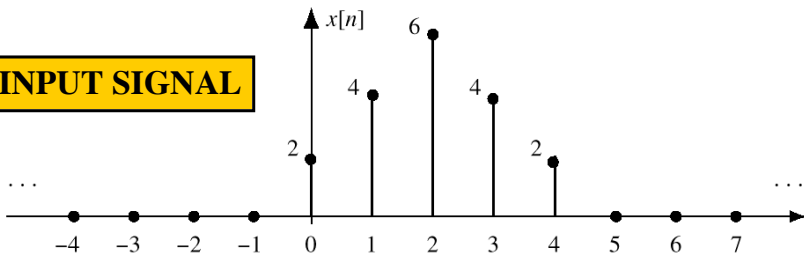
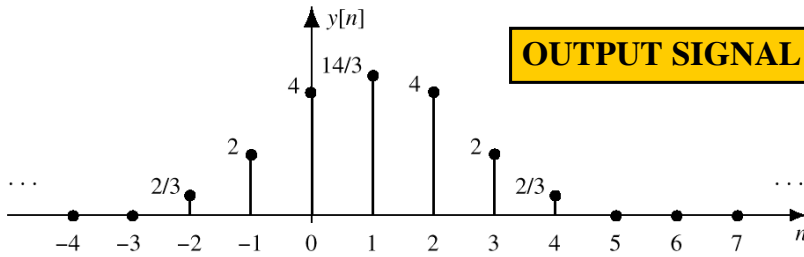


Figure 5.2 Finite-length input signal, $x[n]$.

$$y[n] = \frac{1}{3}(x[n] + x[n + 1] + x[n + 2])$$



OUTPUT SIGNAL

Figure 5.3 Output of running average, $y[n]$.

PAST, PRESENT, FUTURE

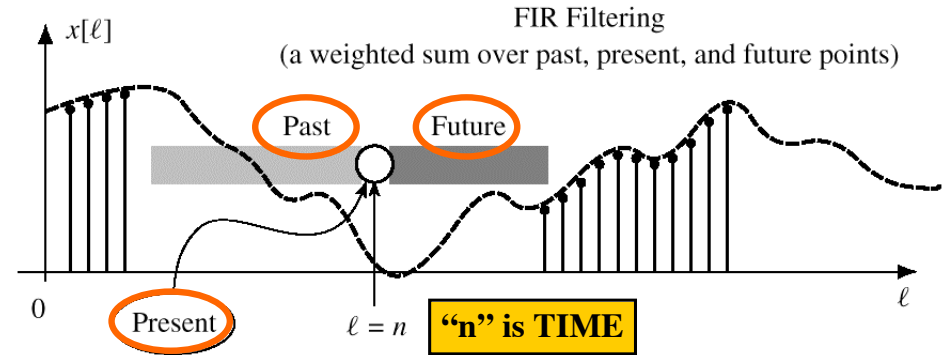


Figure 5.4 The running-average filter calculation at time index n uses values within a sliding window (shaded). Dark shading indicates the future ($\ell > n$); light shading, the past ($\ell < n$).

ANOTHER 3-pt AVERAGER

- Uses “PAST” VALUES of $x[n]$
- IMPORTANT IF “ n ” represents REAL TIME
- WHEN $x[n]$ & $y[n]$ ARE STREAMS

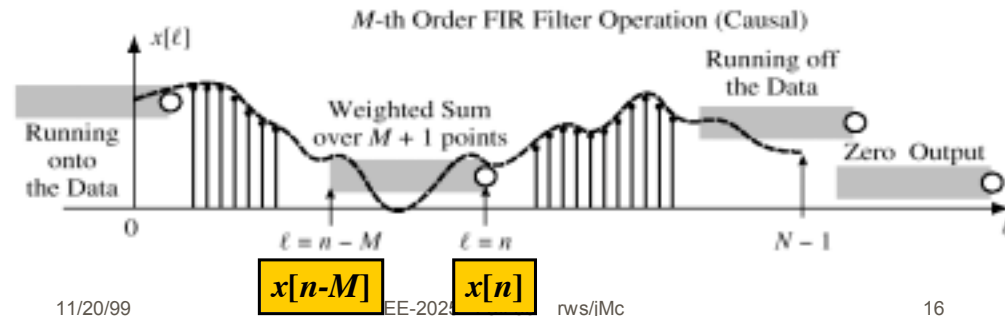
$$y[n] = \frac{1}{3}(x[n] + x[n - 1] + x[n - 2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	6	7	$n > 7$
$x[n]$	0	0	0	2	4	6	4	2	0	0	0	0
$y[n]$	0	0	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	4	2	$\frac{2}{3}$	0	0

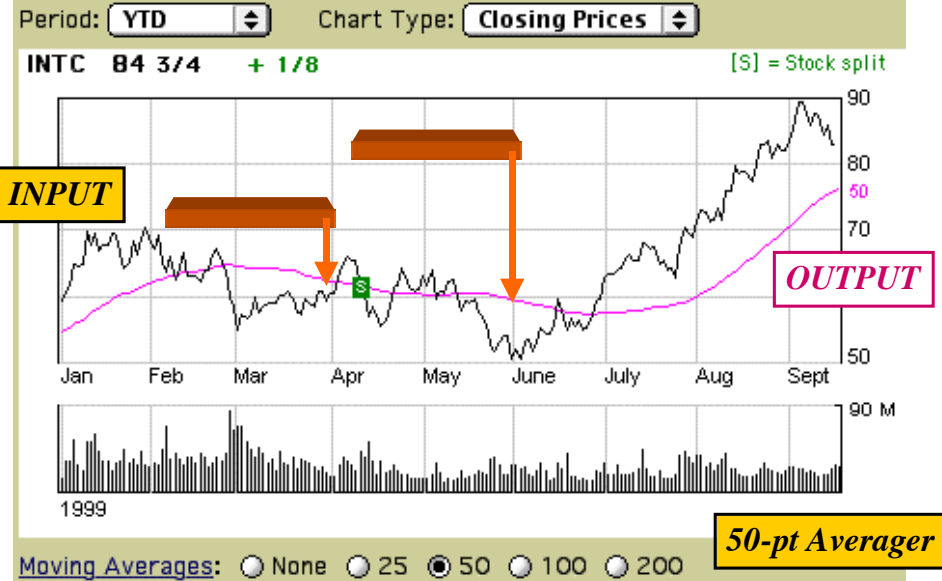
GENERAL FIR FILTER

- SLIDE a WINDOW across $x[n]$

$$y[n] = \sum_{k=0}^M b_k x[n - k]$$



FILTERED STOCK SIGNAL



GENERAL FIR FILTER

■ FILTER COEFFICIENTS $\{b_k\}$

■ DEFINE THE FILTER

$$y[n] = \sum_{k=0}^M b_k x[n - k]$$

■ For example, $\{b_k\} = \{3, -1, 2, 1\}$

$$y[n] = \sum_{k=0}^3 b_k x[n - k]$$

$$= 3x[n] - x[n - 1] + 2x[n - 2] + x[n - 3]$$

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GENERAL FIR FILTER

■ FILTER COEFFICIENTS $\{b_k\}$

$$y[n] = \sum_{k=0}^M b_k x[n - k]$$

■ FILTER ORDER is M

■ FILTER LENGTH is $L = M+1$

■ NUMBER of FILTER COEFFS is L

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FILTERING EXAMPLE

■ 7-point AVERAGER $y_7[n] = \frac{1}{7} \left(\sum_{k=0}^6 x[n - k] \right)$

■ Removes cosine

■ By making its amplitude (A) smaller

■ 3-point AVERAGER

■ Changes A slightly

$$y_3[n] = \frac{1}{3} \left(\sum_{k=0}^2 x[n - k] \right)$$

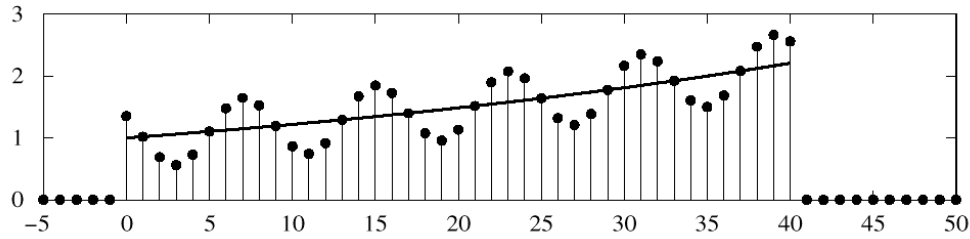
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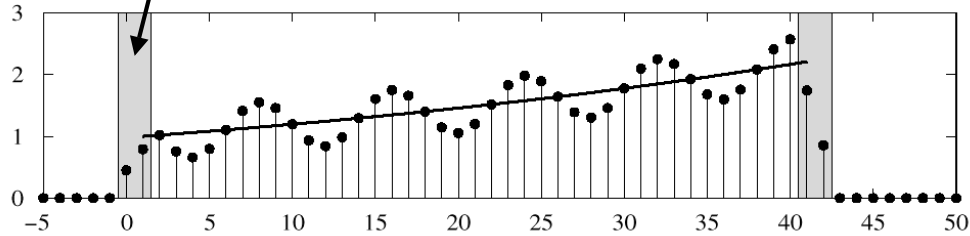
3-pt AVG EXAMPLE

Input Signal: $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$ for $0 \leq n \leq 40$



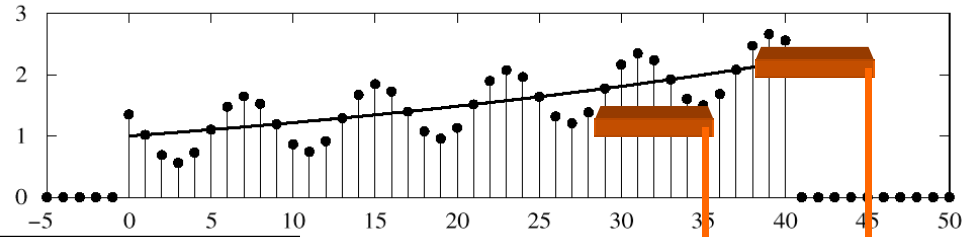
USE PAST VALUES

Output of 3-Point Running-Average Filter



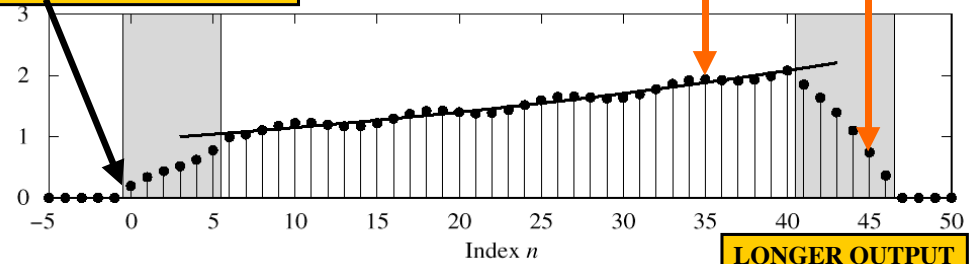
7-pt FIR EXAMPLE (AVG)

Input Signal: $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$ for $0 \leq n \leq 40$



CAUSAL: Use Previous

Output of 7-Point Running-Average Filter



LONGER OUTPUT