EE-2025 Fall-99

Lecture 7

D-to-A Conversion

17-Sept-99

Information

- Check the Bulletin Board for msgs
- Lab #4 posting later today
  - Notes file: airnotes.m (airshort.m)
  - Spectrogram image display info
    - New M-file: plotspec.m
  - FORMAL Lab Report
- Problem Set #4 out today
  - Shorter, with on-Line HW

Quiz Information

- Quiz #1 on 20-Sept (Monday)
- Procedures:
  - Must stay until 11:55 AM
  - Bring your Student ID (Picture)
  - Formula Sheet
  - Calculator
  - Know your Lxx section number !!!
- REVIEW: Sunday nite at 7 PM

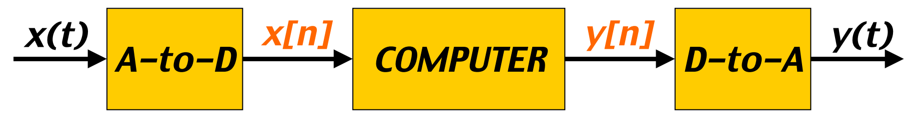
READING ASSIGNMENTS

- This Lecture:
  - Chapter 4, pp. 100-111
- Other Reading:
  - Recitation: Chapter 4, pp. 90-100
    - Strobe Demo
  - Next Lecture: Chapter 5 (beginning)

LECTURE OBJECTIVES

- FOLDING: a type of ALIASING
- DIGITAL–to–ANALOG CONVERSION is
  - Reconstruction from samples
    - SAMPLING THEOREM applies
  - Smooth Interpolation
- Mathematical Model of D–to–A
  - SUM of SHIFTED PULSES
    - Linear Interpolation example

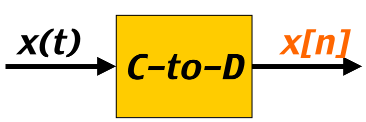
SIGNAL TYPES



- A–to–D
  - Convert x(t) to numbers stored in memory
- D–to–A
  - Convert y[n] back to a "continuous–time" signal, x(t)
  - y[n] is called a "discrete–time" signal

SAMPLING x(t)

- UNIFORM SAMPLING at $t = nT_s$
  - IDEAL: $x[n] = x(nT_s)$



Shannon Sampling Theorem

A continuous-time signal $x(t)$ with frequencies no higher than $f_{\max}$ can be reconstructed exactly from its samples $x[n] = x(nT_s)$, if the samples are taken at a rate $f_s = 1/T_s$ that is greater than $2f_{\max}$.

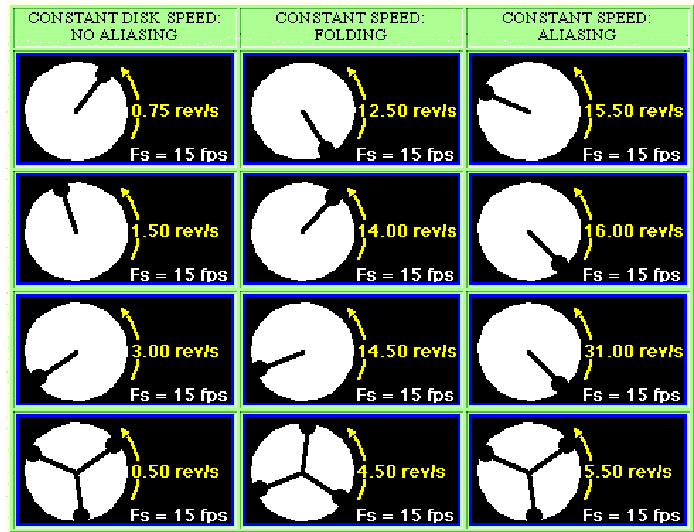
NYQUIST RATE

- "Nyquist Rate" Sampling
  - f_s = TWICE THE HIGHEST FREQUENCY in x(t)
  - "Sampling above the Nyquist rate"
- BANDLIMITED SIGNALS
  - DEF: x(t) has a HIGHEST FREQUENCY COMPONENT in its SPECTRUM
  - NON–BANDLIMITED EXAMPLE
    - TRIANGLE WAVE is NOT BANDLIMITED

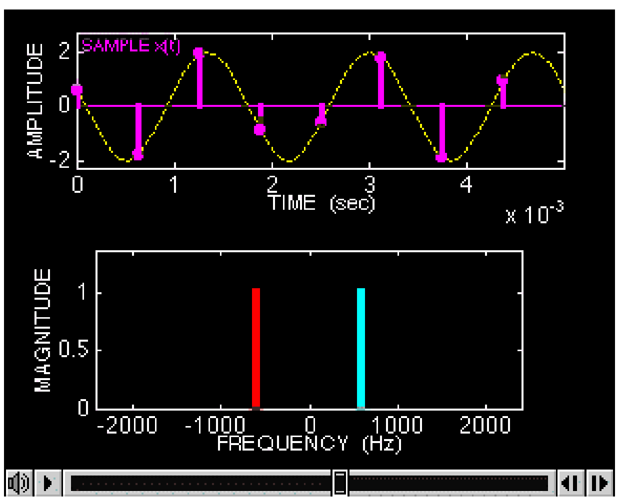
DEMOS from CHAPTER 4

- CD-ROM DEMOS
- SAMPLING DEMO
  - Different Sampling Rates
    - Aliasing of a Sinusoid
- STROBE DEMO
  - Synthetic vs. Real
  - Television SAMPLES at 30 fps
- Sampling & Reconstruction

STROBE DEMO (Synthetic)



SAMPLING DEMO (Ch. 4)



SPECTRUM for x[n]

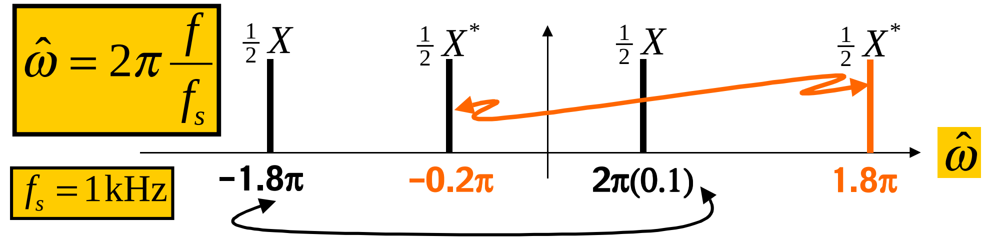
- INCLUDE ALL SPECTRUM LINES
  - ALIASES
    - ADD INTEGER MULTIPLES of 2π and -2π
  - FOLDED ALIASES
    - ALIASES of NEGATIVE FREQS
- PLOT versus NORMALIZED FREQUENCY
  - i.e., DIVIDE f_o and $-f_o$ by f_s

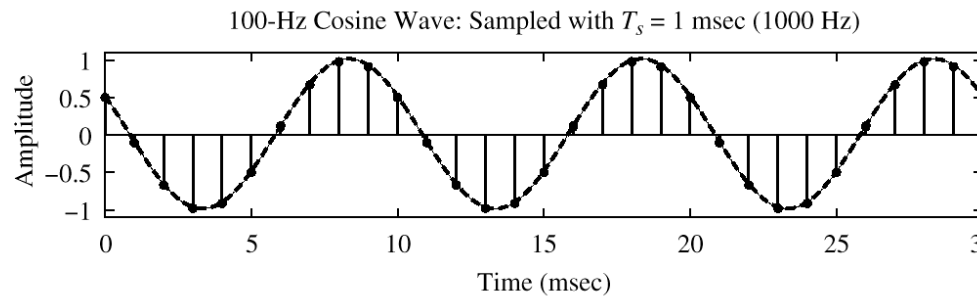
$$\hat{\omega} = 2\pi \frac{f}{f_s}$$

EXAMPLE: SPECTRUM

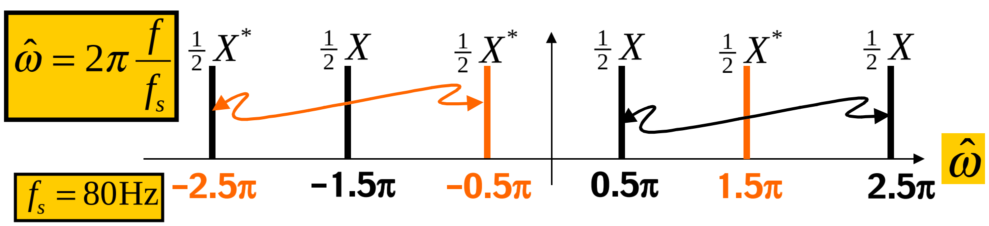
- $x[n] = A\cos(0.2\pi n+\phi)$
- FREQS @ 0.2π and -0.2π
- ALIASES:
  - $\{2.2\pi, 4.2\pi, 6.2\pi, \ldots\}$ & $\{-1.8\pi, -3.8\pi, \ldots\}$
  - EX: $x[n] = A\cos(4.2\pi n+\phi)$
- ALIASES of NEGATIVE FREQ:
  - $\{1.8\pi, 3.8\pi, 5.8\pi, \ldots\}$ & $\{-2.2\pi, -4.2\pi \ldots\}$

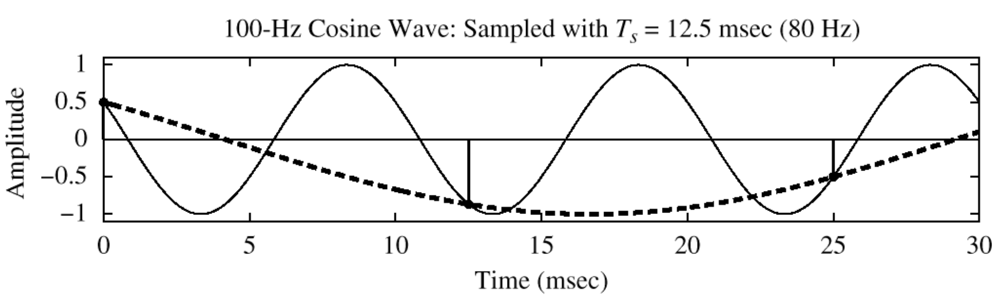
SPECTRUM (MORE LINES)





SPECTRUM (ALIASING CASE)





FOLDING (a type of ALIASING)

- DIFFERENT x(t) give IDENTICAL x[n]
- CAN'T TELL f_o FROM (f_s-f_o)
  - Or, $(2f_s-f_o)$ or, $(3f_s-f_o)$
- EXAMPLE:
  - y(t) has 1200 Hz component
  - SAMPLING FREQ = 1500 Hz
  - 1200 Hz acts the same as ?

$$-1200+1500 \rightarrow 300$$

FOLDING DERIVATION

- Negative Freqs can give the same $\hat{\omega}$

$$x(t) = A\cos(2\pi(-f + \ell f_s)t - \varphi)$$

$$x[n] = x(nT_s) = A\cos(2\pi(-f + \ell f_s)nT_s - \varphi)$$

$$x[n] = A\cos((-2\pi f T_s)n + (2\pi \ell f_s T_s)n - \varphi)$$

$$x[n] = A\cos((2\pi f T_s)n - 2\pi \ell n + \varphi)$$ $\cos(-\theta) = \cos\theta$

$$x[n] = A\cos(\hat{\omega} n + \varphi)$$

SAME DIGITAL SIGNAL

DIGITAL FREQ $\hat{\omega}$ AGAIN

Normalized Radian Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi \ell$$

ALIASING

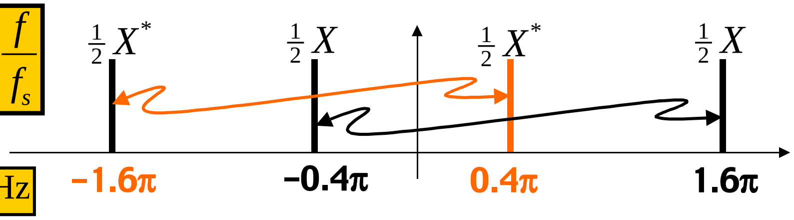
$$\hat{\omega} = \omega T_s = -\frac{2\pi f}{f_s} + 2\pi \ell$$

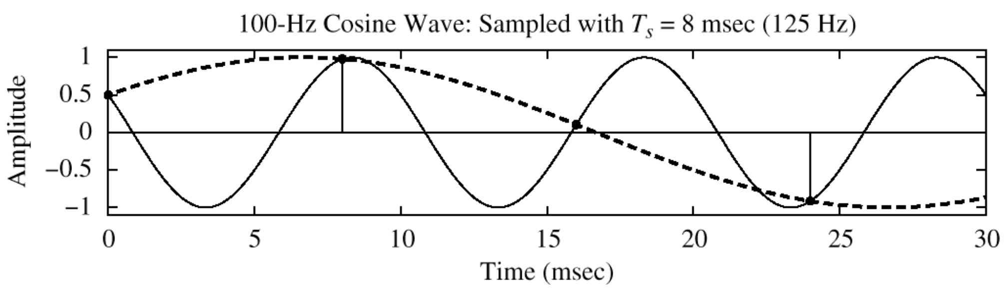
FOLDED ALIAS

SPECTRUM (FOLDING CASE)

$\hat{\omega} = 2\pi \frac{f}{f_s}$

$f_s = 125\text{Hz}$

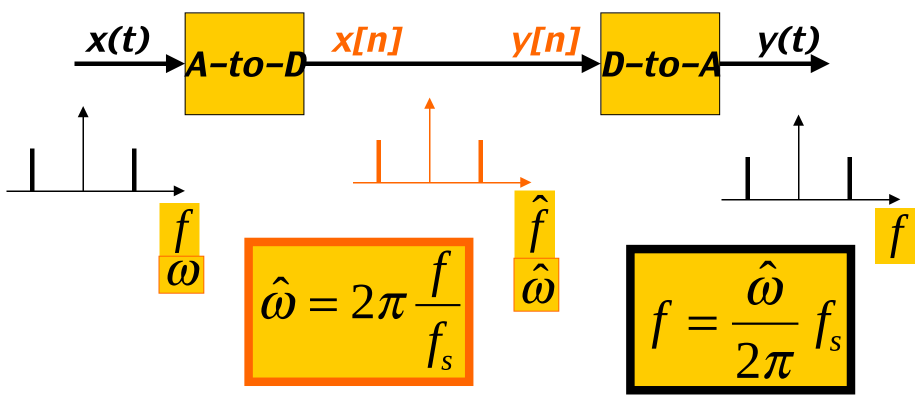




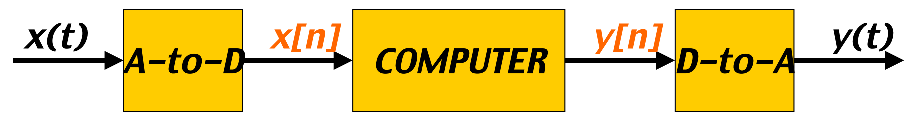
ALIASING & FOLDING

- $x(t)$ = SINUSOID @ f_o
- SAMPLED SIGNAL: $x[n] = x(n/f_s)$
- ALIASING:
  - x[n] COULD HAVE COME FROM
  - $(f_o + f_s)$
  - or $(f_o - f_s)$
  - or $(f_o + 2f_s)$
  - or $(f_o - 2f_s)$, etc.
- FOLDING:
  - A type of ALIASING
  - x[n] COULD BE FROM:
  - $(-f_o + f_s)$
  - or $(-f_o - f_s)$
  - or $(-f_o + 2f_s)$
  - or $(-f_o - 2f_s)$, etc.

FREQUENCY DOMAINS



D-to-A Reconstruction



- Create continuous y(t) from y[n]
  - IDEAL
    - If you have formula for y[n]
  - Replace n in y[n] with $f_s t$
  - y[n] = Acos(0.2πn+φ)   with f_s = 8000 Hz
  - y(t) = Acos(2π(800)t+φ)

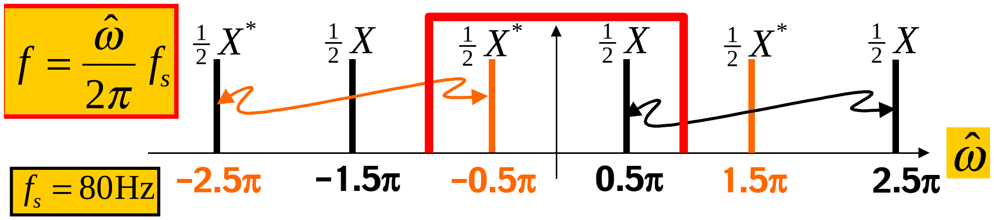
D-to-A Reconstruction

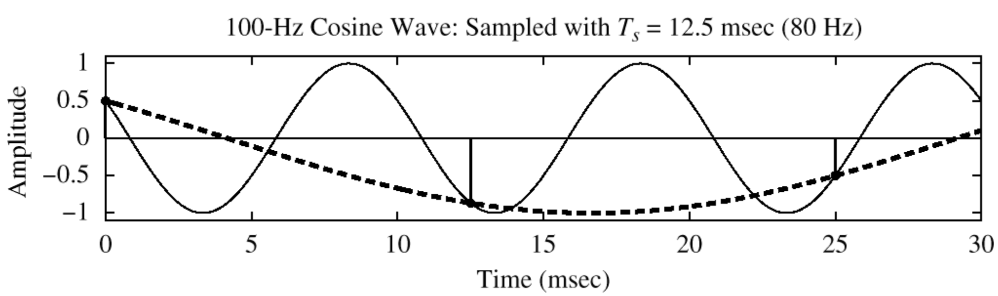
- Create continuous y(t) from y[n]
  - SMOOTH y(t)
    - Use the lowest possible frequency
  - y[n] is a list of numbers
  - How fast?
  - In MATLAB: `soundsc(yy,fs)`

D-to-A is AMBIGUOUS !

- ALIASING
  - Given y[n], which y(t) do we pick ? ? ?
  - INFINITE NUMBER of y(t)
  - D-to-A RECONSTRUCTION MUST CHOOSE ONE OUTPUT
- RECONSTRUCT THE SMOOTHEST ONE
  - THE LOWEST FREQ, if y[n] = sinusoid

SPECTRUM (ALIASING CASE)

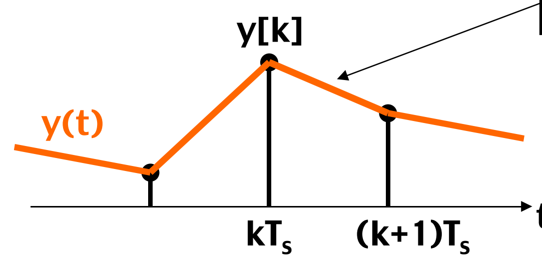




Reconstruction (D-to-A)

- CONVERT STREAM of NUMBERS to x(t)
- "CONNECT THE DOTS"
- INTERPOLATION

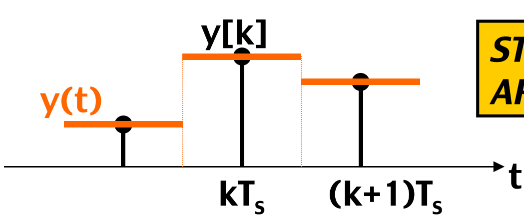
INTUITIVE, conveys the idea



SAMPLE & HOLD DEVICE

- CONVERT y[n] to y(t)
  - y[k] should be the value of y(t) at $t = kT_s$
  - Make y(t) equal to y[k] for
    - $kT_s - 0.5T_s < t < kT_s + 0.5T_s$

STAIR-STEP APPROXIMATION



MATH MODEL for D-to-A

$$y(t) = \sum_{n=-\infty}^{\infty} y[n]p(t - nT_s)$$

SQUARE PULSE:

$$p(t) = \begin{cases} 1 & -\frac{1}{2}T_s < t \le \frac{1}{2}T_s \\ 0 & \text{otherwise} \end{cases}$$

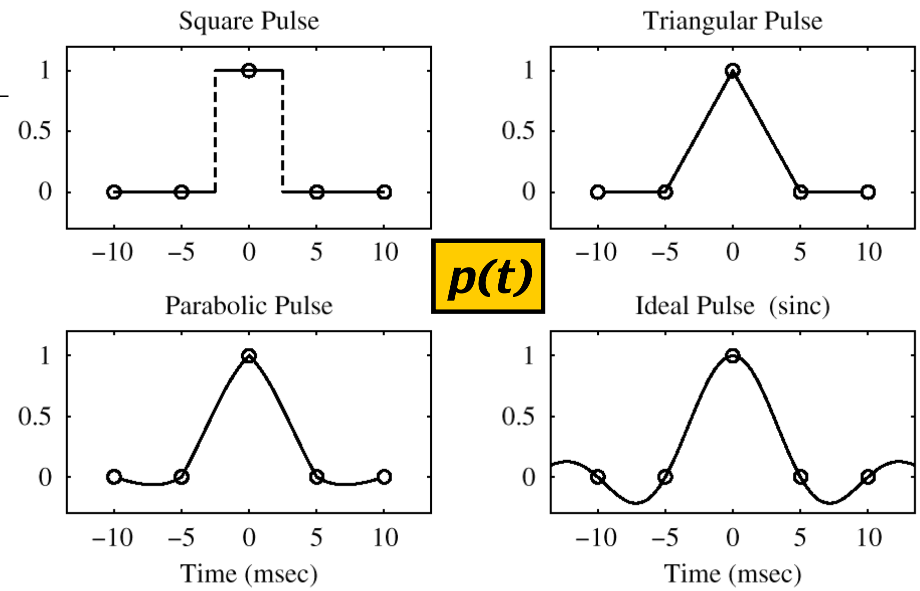


Figure 4.17 Four different pulses for D-to-C conversion. The sampling period is $T_s = 0.005$, i.e., $f_s = 200$ Hz. Note that the duration of each pulse is approximately one or two times T_s.

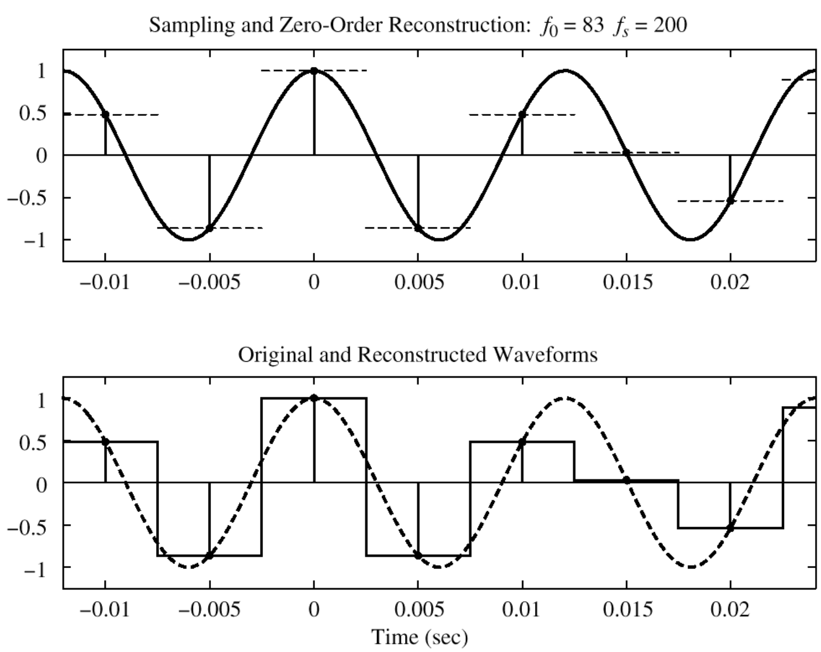
EXPAND the SUMMATION

$$\sum_{n=-\infty}^{\infty} y[n]p(t-nT_s) =$$

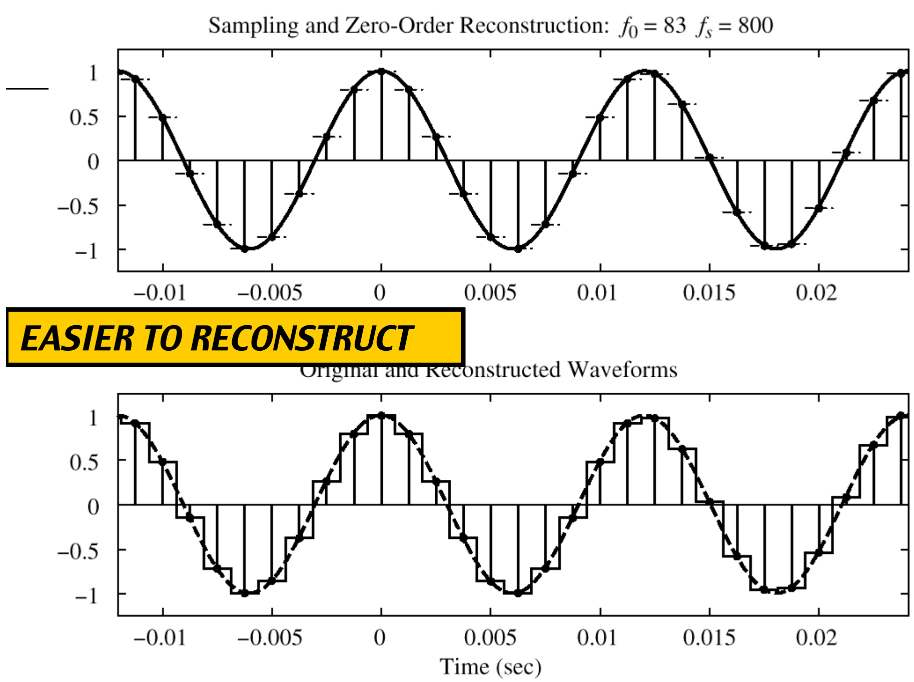
$$\ldots + y[0]p(t) + y[1]p(t-T_s) + y[2]p(t-2T_s) + \ldots$$

- SUM of SHIFTED PULSES $p(t-nT_s)$
  - "WEIGHTED" by y[n]
  - CENTERED at $t=nT_s$
  - SPACED by T_s
    - RESTORES "REAL TIME"

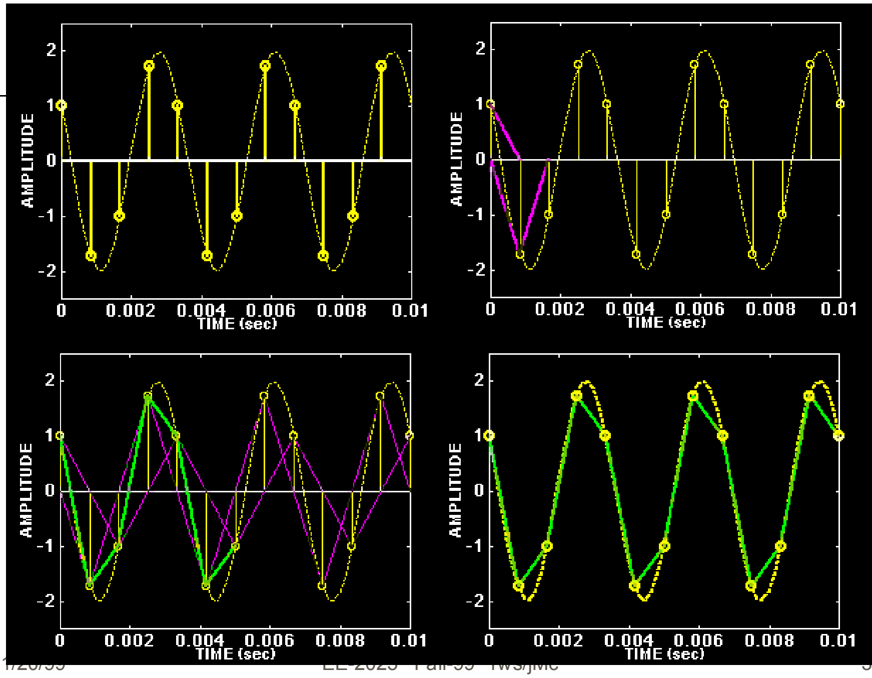
SQUARE PULSE CASE



OVER-SAMPLING CASE

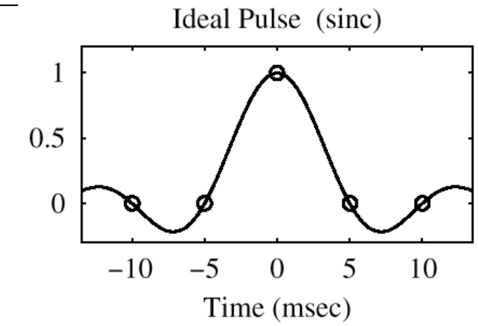


TRIANGULAR PULSE (2X)



OPTIMAL PULSE ?

CALLED “BANDLIMITED INTERPOLATION”



$$p(t) = \frac{\sin \frac{\pi}{T_s} t}{\frac{\pi}{T_s} t} \qquad \text{for } -\infty < t < \infty$$

$$p(t) = 0 \qquad \text{for } t = 0, \pm T_s, \pm 2T_s$$